Analysis of Chaos Fluctuations in Atmospheric Prediction, Fluid Mechanics and Power System Load Forecasting

Mengxuan Du*

School of Science and Engineering, The Chinese University of Hong Kong, Shen Zhen, China, China

*Corresponding author: 122090093@link.cuhk.edu.cn

Abstract. Chaos fluctuation is one of the hottest topics in differential equations and dynamics systems. On this basis, this study discusses the development of chaos theory, introduces Lorenz's discovery of the butterfly effect, Ruelle's research on strange attractors, and Mandelbrot's contributions to fractals; briefly explains the basic principles of chaos theory and bifurcation theory and analyzes them based on partial differential equations. It provides the Lyapunov exponent method for identifying chaos and illustrates the basic properties and laws of chaotic motion using the Logistic map as an example; focuses on the applications of chaos theory in atmospheric science, fluid mechanics, and load forecasting, explaining the basic principles, research methods, and roles of the theory in solving these three complex fields involving disaster prediction, turbulence analysis, SVM model and Legendre orthogonal basis neural networks. Finally, this paper analyzes the limitations of chaos theory in the study of multiple parameters, multiple scales, and unknown influencing factors. These results express hope for the development of chaos theory in more fields such as communication, quantum computing, and biomedical science, giving prospects for the future.

Keywords: Chaos theory; bifurcation theory; prediction; hydrodynamic.

1. Introduction

Chaos theory is a field of study within mathematics, physics, and other scientific disciplines. It deals with complex systems that exhibit unpredictable behavior, making them difficult or impossible to model accurately. Chaos theory has revolutionized our understanding of dynamic systems, from weather patterns to the stock market. At its core, chaos theory is concerned with understanding how seemingly random processes can lead to long-term order and stability. The key idea is that small changes in initial conditions can lead to vastly different outcomes over time, a phenomenon known as the butterfly effect. The name comes from the idea that the flap of a butterfly's wings in Brazil could ultimately cause a tornado in Texas, illustrating how a tiny change in one part of a system can have a massive impact overall. Chaos theory originated in the late 19th and early 20th centuries with the study of nonlinear dynamics, which looked at systems that were sensitive to initial conditions. However, it wasn't until the 1960s and 70s that the field really took off with the work of mathematicians such as Edward Lorenz and Benoit Mandelbrot.

Lorenz is best known for his discovery of deterministic chaos [1]. He found that even simple systems, like the weather, could exhibit highly complex and unpredictable behavior. His famous "butterfly effect" experiment involved inputting slightly different initial conditions into a computer simulation of a weather system [2]. The result was two vastly different weather patterns, illustrating the sensitivity of weather to initial conditions. In the 1970s, Ruelle became one of the pioneers in the study of chaos theory, a branch of mathematics that deals with complex and unpredictable systems [3]. He, along with his collaborator Floris Takens, introduced the concept of strange attractors, which helped in understanding the chaotic behavior of dynamical systems. Their work has been influential in fields such as meteorology, fluid dynamics, and population biology. Mandelbrot, on the other hand, pioneered the study of fractals, self-repeating patterns that appear at different scales in nature [4]. For example, the branching pattern of tree branches, the structure of snowflakes, and the coastline of Britain all exhibit fractal-like properties. By studying fractals, Mandelbrot was able to show that many seemingly random processes exhibit underlying order and self-similarity.

In the past 20 to 30 years, the numerical analysis methods of nonlinear differential equations, approximate methods, and the rapid development of computer and artificial intelligence technology have promoted people's further understanding of chaos theory and the application of chaos [5]. Chaos applications can be divided into chaos synthesis and chaos analysis [6]. The former uses artificially generated chaos to discover usable functions from chaotic dynamical systems; the latter analyzes chaotic signals and underlying deterministic rules in complex artificial and natural signal interactions. In this way, nonlinear prediction of a series of data can be realized. The application field of chaos theory also has a very wide coverage. The use of new ideas and methods in chaos theory injects new vitality into the study of turbulence in the atmospheric boundary layer; the combination of chaos and neural networks, and their complementary advantages can better solve the problem of negative charge prediction. This paper briefly sumarizes the development of chaos history. Based on analyzing the origin of chaos and bifurcation in partial differential equations, it summarizes the application and scientific research progress of chaos system in atmospheric science, fluid mechanics and electromagnetic system, and summarizes and predicts the future development trend of chaos theory is hoped to play a role in attracting jade.

2. Chaos Theory and Bifurcation Theory

System theory studies the knowledge of the law of development and change of things. The changing law of a system can be expressed by an ordinary differential equation, for example$\dot{x} = f(x(t), u(t), r(t))$:where: x(t) $\in R^n$ represents the state quantity of the system. Here, u(t) $\in R^m$represents the system control, that is, the external input to the system; r(t) $\in R^p$represents the undetermined parameters of the system that change with time, and it can also be a constant. If f is linear with respect to x, u, then it can be expressed as $\dot{x} = A(t)x(t) + B(t)x(t)$. For such a linear system, if it does not change with time, its stability, reliability, and robustness can be studied mathematically. If f is not a linear function of x, u, it will involve nonlinear control theory, namely chaos and bifurcation.

For a general nonlinear system$\dot{x} = f(x(t), u(t), r(t))$, in order to focus on the research object, assuming that the external control variable u is not considered, the initial value x of x is studied (t=0) changes will have an impact on the future development of the system. In the corresponding linear system $\dot{x} = Ax(t)$without control variable u, it can be proved that: $\|x_1(t = k) - x_2(t = k)\| \leq \|x_1(t = 0) - x_2(t = 0)\|e^{\lambda^A_{max}k}$ where $\lambda^A_{max}k$ is the maximum value of the real part of the A eigenvalue. In layman's terms, the difference between the starting point of the system state will not exceed $e^{\lambda^A_{max}k}$ after a finite time k. In the corresponding nonlinear system, however, significant differences appear. The following is the most classical nonlinear system Lorentz attractor $\dot{x} = \sigma(y - x)$ $\dot{y} = x(\rho - z) - y$ $z = x\dot{y} - \beta z$. The state quantity changes with time, each color The lines in are the motion routes developed for different initial values. Near the origin, small changes in the initial value will cause huge differences in long-term behavior. This kind of motion state with randomness that is not caused by random external factors but directly obtained by deterministic equations is called chaos.

When the research object is not the change of the system state with time, but the influence of the system parameter r on the system, the bifurcation theory needs to be applied [7]. Bifurcation theory is a mathematical tool widely used in dynamic system theory to study the stability and behavior changes of the system when the parameters change. It involves stability failures and mode changes in nonlinear dynamic systems, thereby helping to understand the different behaviors that the system may exhibit under different conditions. Logistic mapping is a simple nonlinear dynamic system, often used to show examples of bifurcation phenomena, and was originally used to describe the generational change of insect numbers. It can be represented by a discrete mapping function that describes the evolution of the population over a certain time step. Logistic map can be expressed as the following formula [5]:

$$x_{n+1} = r \cdot x_n \cdot (1 - x_n) \quad (1)$$

In this formula, $x_n$is the proportion or state of the population at step n, $x_{n+1}$is the proportion or state of the population at the next step, and r is the value representing the control parameter, usually in the interval [0,4] Variety. The meaning of this mapping is: at each step, the proportion $x_n$of the population will change according to the given parameter r, the new proportion x_{n+1} depends on the previous proportion x_n, and is subject to the interaction between $x_n$and 1-x_n effect. The parameter r controls the speed and stability of the population growth. When 0<r <1: In this range, the population will converge to a stable value, that is, a fixed ratio. This ratio depends on the initial condition x_0. When 1<r <3, in this range, the population experiences periodic oscillations. The population proportions will change back and forth over different time intervals, but these changes are periodic. When r>3, in this range, the behavior of the population proportion becomes more complex. The system will experience a bifurcation phenomenon, bifurcating from a stable state or a periodic state to emerge a new periodic or chaotic behavior. This is a very important and interesting property of logistic maps. In particular, when r is close to 3, the Logistic map will exhibit biperiodic behavior, and when r increases further, the system may enter a chaotic state, resulting in seemingly random and unpredictable behavior.

3. Identification of Chaos: Lyapunov Exponential Method

The Lyapunov exponent is a parameter used to measure the exponential growth rate between different trajectories in a dynamic system, and quantitatively characterizes the sensitivity of the strange attractor to the initial value [6]. For a time-continuous system, consider two trajectories with small deviations from initial conditions, the distance between which expands over time. The Lyapunov exponent measures the logarithm of the expansion rate, and its positive value indicates that the system expands and folds in a certain "direction", so that the adjacent trajectories in the attractor gradually move away, so that the long-term behavior of the system in the initial state becomes unpredictable, and a negative value indicates that the phase volume of the system is shrinking in this direction, and the movement in this direction is stable. If one considers an initial point (x0, y0) and (x1, y1) after a period of time t in a two-dimensional phase space, then the average exponential divergence rate of adjacent trajectories can be expressed as:

$$rate = log(r(t)) / t \quad (2)$$

Where

$$r(t) = \sqrt{(x1 - x0)^2 + (y1 - y0)^2} \quad (3)$$

If rate > 0, then the system will enter a chaotic state.

4. Application of Chaos Theory in Atmospheric Prediction

Atmospheric prediction is an important research direction in the field of meteorological science, and it has an important impact on many fields of human society, such as agriculture, energy, and transportation. Early implementations of probabilistic methods for numerical weather prediction were based on the analysis of small perturbations to the state of the atmosphere by some variables (humidity, wind, temperature, and air pressure) under initial conditions. However, since the atmosphere is nonlinear, these small perturbations are subsequently amplified by chaotic processes, and each prediction is different from the others (As shown in Fig. 1). Traditional numerical models still have certain limitations in prediction [8]. In 1963, Lorenz published a seminal paper on "deterministic non-periodic flows" that profoundly changed the course of weather and climate prediction and embedded chaos theory at the heart of meteorology. As a method of studying nonlinear dynamical systems, chaos theory has brought new ideas and tools to atmospheric prediction. Chaos theory can be used to improve the construction of atmospheric prediction models. By analyzing the chaotic characteristics

of the system, more accurate initial conditions and boundary conditions can be designed, the stability and prediction accuracy of the model can be improved, and the development of data assimilation methods can be guided to minimize the inconsistency between the observed data and the model. By considering the nonlinear characteristics of the system, data assimilation can better capture the evolution process of the atmospheric system.

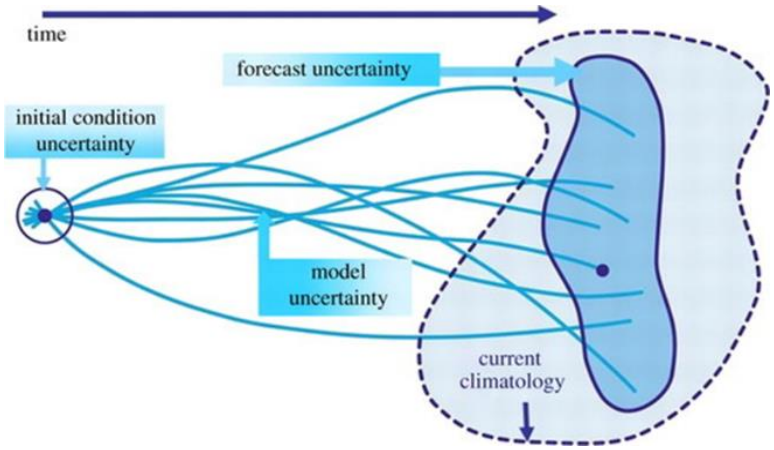


Fig. 1 Schematic of a probabilistic weather forecast using initial condition uncertainties. The blue lines show the trajectories of the individual forecasts that diverge from each other owing to uncertainties in the initial conditions and in the representation of sub-grid scale processes in the model. The dashed, lighter blue envelope represents the range of possible states that the real atmosphere could encompass, and the solid, dark blue envelope represents the range of states sampled by the model predictions.

The predictability of a weather model is usually assessed by determining the error growth rate, i.e., introducing a small perturbation to the initial conditions, allowing the two initial states to evolve through the integral model, and the evolution of their difference, usually the RMS of the error, with the growth rate of the calculation error [8]. Introducing an ensemble of initial perturbations and obtaining the growth rate of the ensemble mean of the errors allows for a better assessment of predictability. The 2021 Nobel Prize in Physics was awarded to two climatologists and a physicist for their "groundbreaking contributions to the understanding of complex physical systems". Among them, two climatologists, Syukuro Manabe and Klaus Hasselman [9], were awarded for their pioneering work in developing climate models. Based on the climate model studied by Manabe, Klaus applied chaos theory to create a model that links weather and climate by describing chaotic weather phenomena with rapidly changing noise. He developed stochastic models inspired by Brownian motion, first using simple models to demonstrate some basic concepts about natural climate variability, and then building more realistic models, coupled ocean-atmosphere circulation models, to demonstrate that a rapidly changing atmosphere would lead to slow changes in the ocean.

It is known that the ever-changing atmospheric phenomenon is a chaotic event, and its strange attractor is the climate [1]. Meteorological disasters are a specific manifestation of atmospheric phenomena, which have the characteristics of "not random but seemingly random. Appearance is closely related to the initial state of the relevant element field. These all show the chaotic characteristics of disasters. Since disasters are chaotic events, one can use the difference equations (groups) describing chaotic events to predict the disaster process [10]. By determining the difference equation describing the chaos and the state equation that is simultaneous with the difference equation, since the complex nonlinear dynamic process experienced by various factors of the chaotic event has not been accurately described, it can be considered to reflect the large-scale, the "atmospheric circulation" alternative basic element that characterizes the state of comprehensive elements for a long time. After the provisional description equation is defined, if the expressed result is consistent with the actual, it can be used as a forecast model, otherwise, the equation should be re-constructed according to the law of chaos until it meets the requirements.

5. Application of Chaos Theory in Hydromechanics

Hydromechanics is a discipline that studies the motion of fluids and the interaction between matter, and it is widely used in engineering and nature. In the study of fluid mechanics, the application of chaos theory has gradually attracted attention. The motion of some fluids shows chaotic behavior, including turbulence, fluctuation, oscillation, etc. These chaotic behaviors are characterized by nonlinearity, complexity and unpredictability. Therefore, the application of chaos theory to fluid mechanics can help us better understand and describe these chaotic behaviors. In fluid motion, there is an extremely close relationship between separation, eddies, and chaos. Regarding the basic properties of the three-dimensional Beltrami flow and the chaotic phenomenon caused by the spherical vortex, the strict solution of the ideal incompressible fluid with chaotic orbit can be found through numerical calculation.

Turbulent flow is one of the most common chaotic behaviors in fluid mechanics. It is a highly complex state of fluid motion, which is characterized by the change of physical quantities such as fluid velocity, pressure, and density with time and space, showing irregular random motion. As a typical fractal phenomenon, the Kolmogorov image of turbulent flow is a large vortex nested within a small vortex, and a smaller vortex nested within the small vortex (seen from Fig. 2). This nested structure is obviously a self-similar structure [11]. In recent years, studying turbulent motion in phase space is the most basic and common method. The Lyon school led by French scientist Temam proved that the solution of the two-dimensional NS equation will be gradually absorbed into a finite-dimensional absorbing set during the time evolution process and proved that this system has a finite-dimensional overall attractor [12]. The dynamic behavior in turbulent flow can be studied by applying chaos analysis method. This includes identifying attractors, bifurcation points, periodic solutions, etc. Although turbulence itself is unpredictable, chaos theory can help us understand the unpredictability in turbulent flow. Analyzing initial condition sensitivities in turbulent systems can provide some qualitative insight into the future behavior of turbulence. Using Wolf's improved algorithm [13], the maximum Lyapunov exponent can be calculated for the reconstruction of atmospheric boundary layer turbulence data, and the maximum Lyapunov exponents of the atmospheric boundary layer are all greater than 0, indicating that the atmospheric boundary layer turbulence has chaotic characteristics and there are strange attractors [6]. The Grassberger-Proccia algorithm uses the original phase space sequence calculation, which can get more stable results.

In describing the one-dimensional turbulence equation, the B—K—dV equation can be used. The Burgers-KPZ (B-K) equation is a partial differential equation describing one-dimensional turbulent behavior [11]. KPZ stands for Kardar-Parisi-Zhang, which is a nonlinear stochastic equation used to describe phenomena such as interfacial growth and surface roughness. The Burgers equation is a one-dimensional nonlinear convection equation, which can be combined with the KPZ equation to obtain the B-K equation, which is used to simulate one-dimensional turbulent flow. The B-K equation combines nonlinear convection and diffusion terms, as well as random noise terms, reflecting the complexity and unpredictability in turbulence phenomena. The solution of this equation can simulate the evolution process of one-dimensional turbulent flow, including vortex generation, interaction and turbulent instability

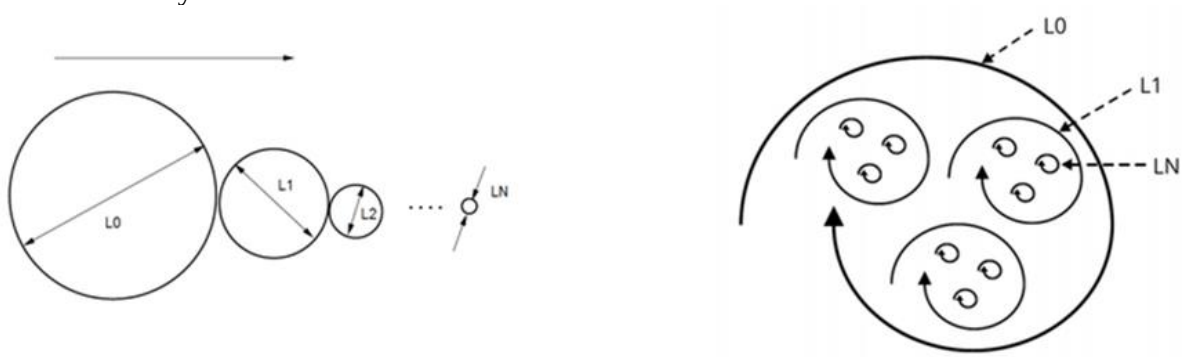


Fig. 2 The Kolmogorov image of turbulent flow.

6. Application of Chaos Theory in Power System Load Forecasting

Power system load forecasting is an important means to ensure the safe, stable and economical operation of the power system. The early load forecasting methods include time series models such as Box-Jenkins and Kalman filtering, and regression analysis, which require a small amount of calculation, but the robustness of the system is not guaranteed. The load change in the actual power system is complex and uncertain, and it is difficult to predict accurately. In the early 20th century, artificial neural networks were introduced to handle load forecasting. Artificial neural networks are highly adaptive and can also model complex nonlinear relationships. However, because there are many hyperparameters that need to be adjusted, improper selection may lead to a decline in model performance [14]; when the network is too complex or the training data is small, the artificial neural network is prone to overfitting, that is, it performs well on the training data, but does not perform well on new data. Afterwards, people improved the neural network and proposed various methods combining neural network and fuzzy set theory. Ref. [15] proposed gray neural network and gray correlation degree, but only applicable to the load index with exponential growth. Another study improves the blood pressure neural network, but its convergence performance is not good [16]. Ref. [17] proposed a short-term load forecasting method.

Based on RBF neural network, but its learning speed is very slow. Chaos theory emphasizes the complexity of nonlinear dynamic systems and the huge effects that small changes may cause, while neural networks could deal with nonlinear relationships. Combining the two can enhance a model's ability to understand and predict the behavior of complex systems. Therefore, the combination of chaos theory and neural network has become a hot spot in load forecasting research. The basic principles of chaos theory include sensitive dependence on initial value, fractal structure and universality. These principles show that nonlinear dynamical systems exhibit complex behaviors that are difficult to predict under certain conditions. In power system load forecasting, the application of chaos theory mainly involves the complexity and uncertainty analysis of load changes, load pattern recognition etc. Chaos theory can help identify nonlinear relationships and dynamic behavior in data. By applying chaos analysis to raw load data, nonlinear features hidden in the data can be revealed. These features can be used as input to a neural network, helping the network better capture complex patterns in the data. Using the NN (neural network) based chaotic learning algorithm for daily load forecasting is significantly better than the BP algorithm in terms of speed, convergence and forecasting accuracy [18]. Niu et al. proposes a model for power load forecasting using support vector machines based on chaotic time series [19]. Compared with BP neural network, SVM has the following advantages: BP algorithm needs to select the number of layers, while SVM only needs to search for optimal parameters. SVM shows better generalization ability than BP network. According to the meaningful features (Lyapunov index, fractal dimension, etc.) extracted from the chaotic time series, it can be used to train the SVM model for load forecasting, and the error and acceptable results obtained are better than the BP algorithm (as illustrated in the Fig. 3).

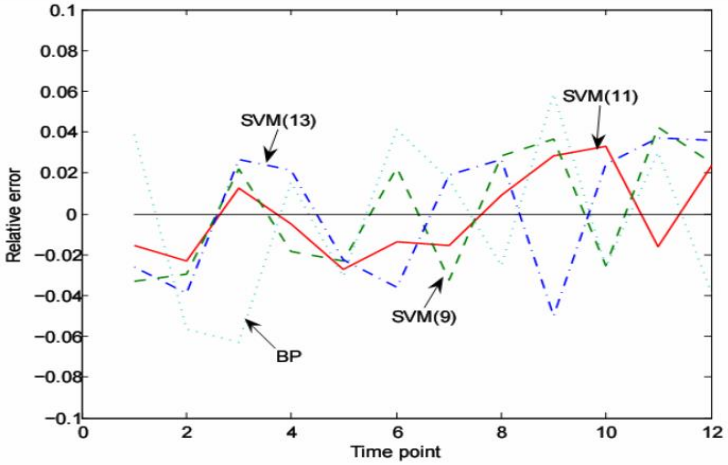


Fig. 3 Error analysis of the proposed method.

For the shortcomings of neural networks that are difficult to adjust due to too many hyperparameters, chaos theory can also be well improved. Chaos optimization algorithms are used to optimize the hyperparameters of neural networks to improve the predictive performance of the model. Another idea of combining chaos and neural networks is to find a suitable embedding dimension and delay time and use the data samples reconstructed from the phase space for statistics and analysis. Legendre Orthogonal Basis Neural Network is a method of applying the orthogonal basis functions of Legendre polynomials to neural network structures. By constructing the prediction model of Legendre neural network based on chaos theory, higher prediction accuracy and prediction effect can be obtained [20].

7. Limitations and Prospect

Although the study of chaotic systems has profoundly revealed the nonlinear behavior of complex dynamical systems, its limitations cannot be ignored. First, the high sensitivity of chaotic systems to initial conditions makes their long-term prediction difficult, limiting their reliability in some practical applications. Second, building accurate mathematical models to describe real-world complex systems is often challenging, especially when multi-parameters, multi-scales, and unknown influencing factors are considered. Furthermore, for some systems, obtaining enough data for in-depth study may be limited, limiting a comprehensive understanding of chaotic behavior. In summary, research on chaotic systems faces limitations in predictability, modeling complexity, and data requirements, which need to be addressed in future research for deeper understanding and applications.

However, the outlook for chaotic systems is also full of potential and opportunity. With the advancement of control theory and complex network research, it is expected to develop more effective chaos control and synchronization methods, which can be applied to the fields of communication, energy systems and biomedicine, such as a computer-aided diagnosis (CAD) system [20], which uses chaos Analysis theory extracts nonlinear and trajectory patterns to help cardiologists accurately diagnose cardiac arrhythmias. for more stable and controllable system behavior. At the same time, the role of chaotic systems in complex network analysis will also become increasingly important, helping to reveal the relationship between network structure and dynamics, thereby improving information transmission and distribution. In addition, the study of quantum chaos has prospects in quantum computing and communication, and the combination of classical chaos and quantum mechanics is expected to achieve more efficient information processing. In addition, the combination of chaotic systems and artificial intelligence may lead to new optimization algorithms and problem-solving methods.

8. Conclusion

This paper analyzes the development of chaos theory, introduces its application in atmospheric forecasting, fluid mechanics and power system load forecasting based on summarizing chaos theory and bifurcation theory, and analyzes the research limitations of chaotic systems. By studying Lyapunov exponents and strange attractors, the error of long-term atmospheric prediction can be reduced, and the turbulence problem can be described accurately; the combination of chaos theory and neural network is also one of the most accurate methods for short-term load forecasting today. Although chaotic system research faces modeling difficulties and challenges related to predictability, with the continuous development of experimental technology, it is expected to better observe and understand the behavior of chaotic systems, thereby promoting its application and innovation in various fields.

References

[1] Lorenz E. The butterfly effect. World Scientific Series on Nonlinear Science Series A, 2000, 39: 91-94.

[2] Vernon J L. Understanding the butterfly effect. American Scientist, 2017, 105(3): 130.

[3] Ruelle D. Strange attractors. In Universality in Chaos, 2nd edition 2017: 35-48.

[4] Mandelbrot B B, Evertsz C J, Gutzwiller M C. Fractals and chaos: the Mandelbrot set and beyond (Vol. 3). New York: Springer, 2004.

[5] Tang W, Li D, Chen X. Research on Chaos Theory and Its Application. Automation of Power Systems, 2000, 24(7): 67-70.

[6] Li X. Chaos Theory and Research on Atmospheric Boundary Layer Turbulence. Advances in Earth Science, 2000, 15(2): 178.

[7] Crawford J D. Introduction to bifurcation theory. Reviews of modern physics, 1991, 63(4): 991.

[8] Slingo J, Palmer T. Uncertainty in weather and climate prediction. Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences, 2011, 369(1956): 4751-4767.

[9] Tyutyunnik V M. Physical modelling of Earth's climate: 2021 Nobel Prize winners in Physics Syukuro Manabe and Klaus Hasselmann. Journal of Physics, 2022, 11.

[10] Lu Z, Bai L, Lu L. A Preliminary Study on Making a Long-term Disaster Forecast Model Using Chaos Theory. Journal of China Agricultural University, 2002, 7(3): 43-46.

[11] Shen X, Chen J. Application of fractal and chaos theory in turbulence research. Journal of Henan University of Science and Technology: Natural Science Edition, 2005, 26(1): 27-30.

[12] Ning Z, Yao Y, Li G, Ge J. Chaos theory and its application in fluid mechanics. Xinjiang Petroleum Geology, 2002, 23(2): 170.

[13] Wolf A, Swift J. Progress in computing Lyapunov exponents from expert indent al data. In: Holton C W, Reichl L E, eds. statistical Physics and Chaos in Fusion Plasma. New York: Wiley Pub, 1984.

[14] Yang H, Ye H, Wang G. Application of Chaos Theory in Power System Load Forecasting. Relay, 2005, 33(16): 26-30.

[15] Wang L, Zhu W, Yang W, et al. Medium and long term daily load curve prediction based on grey neural network and grey correlation degree Journal of Wuhan University (Engineering Edition), 2019, 52 (01): 58-64+70

[16] Peng H, Jiang Q. Application of improved BP neural network in short-term load forecasting of power system Heilongjiang Electric Power, 2018, 40 (05): 414-419.

[17] Peng X, Hu S, Lu D. A review of short-term load forecasting methods based on RBF neural network. Power system protection and control, 2011, 39 (17): 144-148.

[18] Yang H, Zuo S, Tu Y, Wang C. Short-term load forecasting based on chaos theory and Legendre orthogonal basis neural network. Electrical Measurement and Instrumentation, 2015, 52(13): 63-66

[19] Niu D, Wang Y, Duan C, Xing M. A New Short-term Power Load Forecasting Model Based on Chaotic Time Series and SVM. J. Univers. Comput. Sci., 2009, 15(13): 2726-2745.

[20] Ma H, Tang J M. Short-term load forecasting of microgrid based on chaotic particle swarm optimization. Procedia Computer Science, 2020, 166: 546-550.