Study on the relationship between thermal conductivity of plain weave fabrics and thermal conductivity of single fibers based on differential equations and goal planning models

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Abstract. Fabrics with high thermal insulation capacity have very common applications in aerospace direction. In order to obtain the relationship between the overall thermal conductivity of the fabric and the thermal conductivity of a single fiber, and then through the optimization of various parameters of the fabric, to obtain the fabric with stronger heat insulation ability, this paper considers the fiber heat transfer and the gas heat transfer, the fiber heat transfer and the gas heat transfer as a parallel model, and considers the influence of the difference between the volume of the fibers and the volume of the air on the overall heat transfer effect in the entire heat transfer process, and takes the ratio of the volume of the fibers and the ratio of the air The proportion of fiber volume and the proportion of air volume are taken as the weight influence factors in the parallel model, and the mathematical model of the relationship between the overall thermal conductivity of plain fabric and the thermal conductivity of single fiber is established. Then the objective planning model is established, and the optimization algorithm is used to solve the problem, so as to obtain the minimum thermal conductivity of a single fiber and the corresponding structural parameters.

Keywords: One-dimensional heat conduction modeling, thermal conductivity calculations, heat convection.

1. Introduction

Due to its excellent thermal insulation properties, the new thermal insulation material has a wide range of applications in aerospace, military industry, construction and transportation, and other high-tech fields, so the study of its thermal conductivity properties is of great importance.[1-2] The focus of this research is to establish a model of the heat transfer mechanism between the thermal conductivity of a single insulation fiber and the overall thermal conductivity of the fabric. However, due to the small diameter of a single fiber of the insulation material, it is not possible to measure its thermal conductivity directly, so it is necessary to establish a model to solve this problem[3]. The model not only needs to determine the thermal conductivity of single A fiber, but also needs to regulate the weaving structure of the fabric and optimize the design of the thermal conductivity of the fabric[4], so as to realize the accurate calculation and optimal design of the thermal conductivity of single fiber and the fabric as a whole, and to satisfy the requirements of high-performance heat-insulating materials in the field of high technology[5].

2. Modeling and Solving

2.1. Definition of thermal conductivity

Thermal conductivity is specifically defined as the thermal conductivity of a substance in the interior of an object perpendicular to the direction of thermal conductivity by taking two parallel planes 1 meter apart with an area of 1 square meter, if the temperature difference between the two planes is 1K, then the amount of heat conducted from one plane to the other in 1 second is specified as the thermal conductivity of the substance in units of W/(m·K). Also known as thermal conductivity, it reflects the ability of a substance to conduct heat, and is defined by Fourier's law (see heat conduction) as the amount of heat transferred per unit temperature gradient (a 1K decrease in temperature over a 1m length) per unit of time through a unit of heat-conducting surface[6-7].

2.2. One-dimensional heat conduction equation

The one-dimensional heat conduction equation ∂T(x,t)/∂t=(∂^2 T(x,t))/(∂x^2) can be used to describe the heat conduction problem in an infinite plane, and the fabric in this problem can be considered to be an infinite plane and satisfy the conditions for its applicability.

Partial differential equations need to solve the initial conditions T(x,0)=F(x), the boundary conditions are divided into three categories[8-9].

The first category has a known boundary temperature:

$$T(x_d, t) = G(t) \tag{1}$$

The second type of known boundary heat flow density: (i.e., temperature gradient function):

$$\frac{\partial T(x_d,t)}{\partial x} = H(t) \tag{2}$$

The third type of boundary is convective heat transfer:

$$-\lambda \frac{\partial T(x_d,t)}{\partial x} = h\big(T(x_d, t) - T_f\big) \tag{3}$$

where x_d is the boundary, is the thermal conductivity where λ is the thermal conductivity coefficient, h is the convective heat transfer coefficient, and T_f is the fluid temperature.

Noting that the temperature of position x_j at moment t_i, $T(x_j, t_i) = T_i^j$, calculating the first-order partial derivation of the temperature with respect to time and the second-order partial derivation of the temperature with respect to space can be obtained[10].

Discretization of one-dimensional heat conduction:

$$\frac{T_{i+1}^j - T_i^j}{\Delta t} = \alpha \frac{T_i^{j+1} - 2T_i^j + T_i^{j-1}}{\Delta x} \tag{4}$$

After sorting it out we get:

$$T_{i+1}^j = rT_i^{j+1} + (1-2r)T_i^j + rT_i^{j-1} \tag{5}$$

included among these:

$$r = \alpha \frac{\Delta t}{(\Delta x)^2} \tag{6}$$

The final discretized equation is:

$$\begin{gathered} T_{i+1}^j = rT_i^{j+1} + (1-2r)T_i^j + rT_i^{j-1} \\ T_{i+1}^e = G(t_{i+1}) \\ T_1^j = F(x_j) \end{gathered} \tag{7}$$

where e is the boundary after value discretization.

Assuming that the temperature on the other side of the fabric heat source remains at $25^\circ C$ in $t \in [0,0.1]$, a three-point method heat transfer model can be developed as shown in Fig 1.

$$\begin{gathered} T_{i+1}^j = rT_i^{j+1} + (1-2r)T_i^j + rT_i^{j-1} \\ T_{i+1}^3 = 25 \\ T(0,t) = 25; t \in [0,0.1] \end{gathered} \tag{8}$$

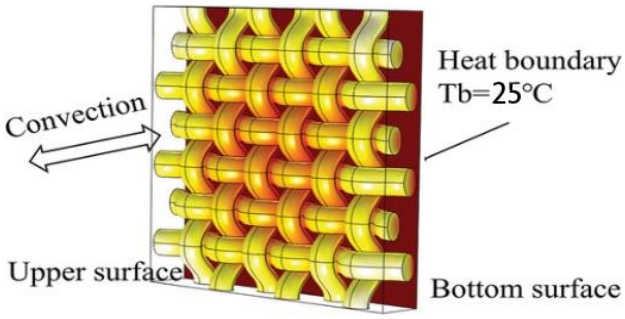


Fig 1 Three-point method heat transfer model

The data is imported into MATLAB and the MATLAB code is used to solve the equations and the output is shown in Fig 1 and Fig 2 and the discretized results of the solution are shown in Table 1.

Table 1 Discrete results

	t_0	t_1	t_2	t_3	t_4	t_5
x_0	25	25.5751	25.6930	25.8069	25.8960	25.9709
x_1	25	25.0000	25.0809	25.1556	25.2269	25.2934
x_2	25	25.0000	25.0000	25.0114	25.0301	25.0538
x_3	25	25.0000	25.0000	25.0000	25.0016	25.0054
x_4	25	25.0000	25.0000	25.0000	25.0000	25.0000

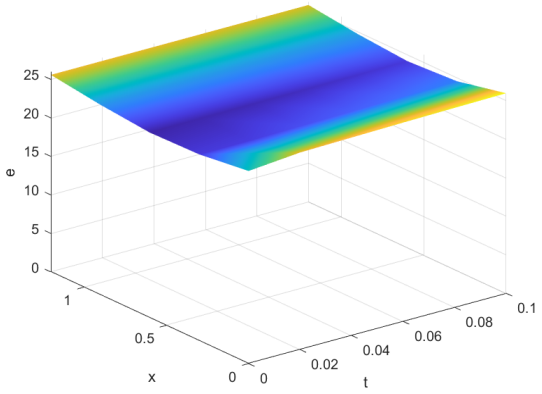


Fig. 2 Rendering

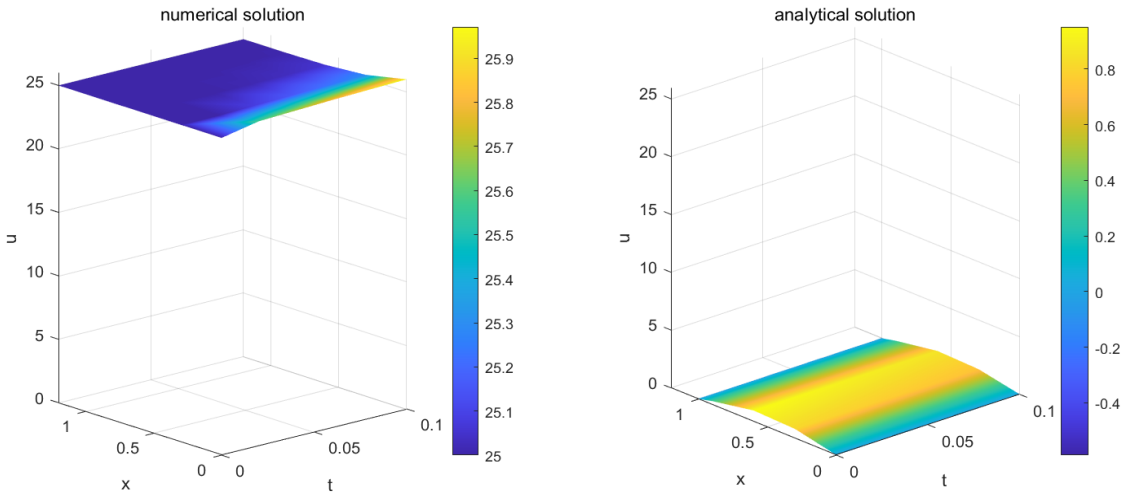


Fig. 3 Rendering

2.3. Determination of fiber bending path

The bending path of a fiber is defined by determining its three-dimensional spatial position through the centerline of the fiber, which is a correlation function of the axial direction of the fiber. In previous

studies, most of them use the sine-cosine curve or multiple discounted ends to represent the fiber centerline, but their fitting effects are not ideal, in this paper, we extracted five characteristic points (A, B, C, D, E) on the fiber centerline, and fitted them by using Newton's interpolation formula, and finally we can get the fiber centerline curve equation. The specific establishment process of fiber centerline equation for plain fabric is as follows:

Assuming that the diameters of a single fiber in the warp and weft directions are d, the distance between adjacent fiber centerlines is ρ_i, and the flexural wave height of the centerline is d, the fiber centerline equations are established for the fibers in the warp direction (Fig. 4) and the weft direction (Fig. 5) respectively under the right-angle coordinate system.

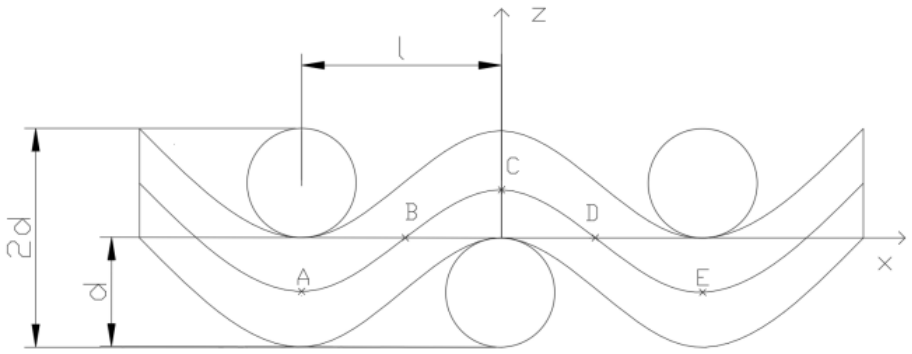


Fig. 4 Longitudinal coordinates

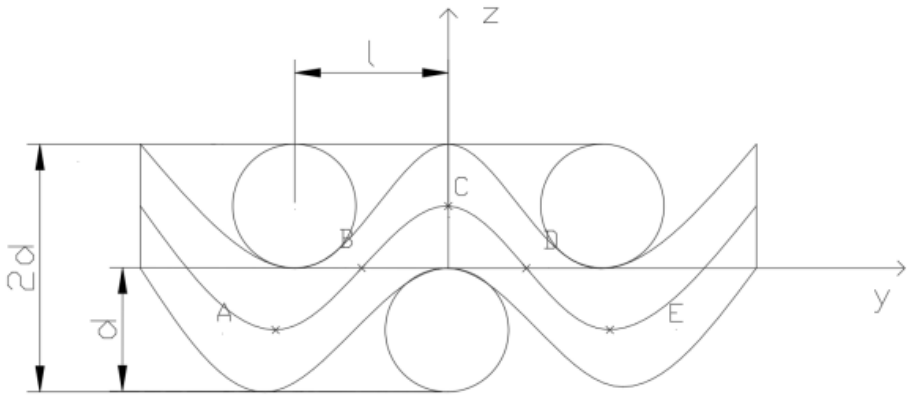


Fig. 5 Latitudinal coordinates

$$Z_{lx} = f(x)$$
$$Z_{ly} = f(y) \tag{9}$$

Based on the coordinates of the five feature points we selected, a quadratic Newton interpolation polynomial is constructed and solved using the MATLAB code to obtain the curve equation of the fiber centerline as:

$$Z_{lx} = \frac{324}{3125}x^4 - \frac{63}{125}x^2 + \frac{3}{10}, x \in \left(-\frac{5}{3}, \frac{5}{3}\right)$$
$$Z_{ly} = \frac{1024}{3125}y^4 - \frac{63}{125}y^2 + \frac{3}{10}, y \in \left(-\frac{5}{4}, \frac{5}{4}\right) \tag{10}$$

2.4. Determination of fiber volume

Assuming that the cross-sectional shape of each fiber at any place is a circle with diameter d, in the previous subsection we have obtained the curve equation of the centerline of the fiber, we only need to calculate the length L of the curve of the centerline, and multiply it by the cross-sectional area of the fiber, S, to calculate the volume of the warp and weft fibers, V_1,V_2. Since the centerlines of the warp and weft are symmetric, we only need to take the warp coordinates(x_1, x_2), and the weft

coordinates (x_1, x_2), (y_1, y_2), weft coordinates (y_1, y_2) can be, considering the overall fabric thickness is h, we can get in (x_1, x_2), (y_1, y_2) the overall computational domain volume $V_{total} = (x_2 - x_1)(y_2 - y_1)h$, the overall computational domain of the volume of the air in the $V_{air} = V_{total} - V_1 - V_2$.

The length L of the curve of the fiber centerline can be expressed as:

$$L_1 = \int_{x_1}^{x_2} \sqrt{1 + \left(f'(x)\right)^2}\, dx$$

$$L_2 = \int_{y_1}^{y_2} \sqrt{1 + \left(f'(y)\right)^2}\, dy \tag{11}$$

The data was imported into Matlab and calculated as:

$$L_1 = 3.6110mm$$

$$L_2 = 2.8554mm \tag{12}$$

Then, according to the formula for calculating the cross sectional area of a cylinder, we get:

$$S = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = 0.2827mm \tag{13}$$

The cylindrical volume formula calculates:

$$\begin{gathered} V_1 = L_1 \cdot S \\ = 3.6110mm \times 0.2827mm \\ \approx 1.0208mm^3 \\ V_2 = L_2 \cdot S \\ = 2.8554mm \times 0.2827mm \\ \approx 0.8072mm^3 \end{gathered} \tag{14}$$

And the volume of the overall computational domain:

$$\begin{gathered} V_{total} = |x_1 - x_2| \cdot |y_1 - y_2| \cdot h \\ = 10mm^3 \end{gathered} \tag{15}$$

The volume of air in the computational domain is:

$$V_{air} = V_{total} - 2 \times V_1 - 2 \times V_2 = 6.344mm^3 \tag{16}$$

According to the formula $\lambda = \lambda_1\phi_1 + \lambda_{air}(1 - \phi_1)$, Substituting values gives:

$$\lambda_1 = 0.0389W \cdot m^{-1} \cdot K^{-1} \tag{17}$$

2.5. Fiber centerline fitting

For the single warp and weft fibers in the assumed computational domain, the Newton interpolation method is used to construct the centerline curve equations for their centerlines, and the volume of a single fiber in the computational domain is calculated, and finally the expression of ϕ_1 can be obtained. The objective function ϕ_1 contains variables such as d, ρ, θ, etc. We also need to introduce the relationship equation between the parameters, and introduce the structural parameters into the objective function in order to determine the specific values of the structural parameters when reaching the optimal solution, so we introduce the tangent equation of the centerline curve equation at the place, and the slope of the tangent equation is equal to the tangent value of the bending angle sita, which is the same as the tangent value of the bending angle sita.Combined with the Newton interpolation formula K order mean deviation formula.

$$f[y_0, y_1, \cdots, y_R] = \frac{f[y_0, y_1, \cdots, y_{k-1}] - f[y_1, y_2, \cdots, y_k]}{y_0 - y_k} \tag{18}$$

The mean difference of each eigenpoint can be obtained (in the x-z plane, for example):
The centerline equation can be obtained as:

$$Z = f(x) = \frac{2d}{l^4}x^4 - \frac{7d}{2l^2}x^2 + d \tag{19}$$

Similarly in the zoy plane:

$$Z = f(y) = \frac{2d}{l^4}y^4 - \frac{7d}{2l^2}y^2 + d \tag{20}$$

Observing the equation $\lambda = \lambda_1\phi_1 + \lambda_{air}(1-\phi_1)$, it can be seen that λ is affected by the size of ϕ_1, and at the same time, since the warp and weft fibers have the exact same properties, the warp and weft fibers ρ, d, θ should be the same when taken to the extreme value. From the first question, fiber thermal conductivity is greater than static air thermal conductivity, then it can be seen that if λ obtains the minimum value, then ϕ_1 obtains the minimum value $\phi_1 = f(d, \rho, \theta)$, where $10° \leq \theta \geq 26.535°, 0.3 \leq d \geq 0.6$, ρ is constrained by θ, d
The equation of the fitted curve is:

$$f(x) = \frac{2d}{\left(\frac{10}{\rho}\right)^4}x^4 - \frac{7d}{2\left(\frac{10}{\rho}\right)^2}x^2 + d \tag{21}$$

The integration yields a centerline length of:

$$\int_{-\frac{10}{\rho}}^{\frac{10}{\rho}} \frac{2d}{\left(\frac{10}{\rho}\right)^4}x^4 - \frac{7d}{2\left(\frac{10}{\rho}\right)^2}x^2 + d \tag{22}$$

Then the volume of a single fiber is:

$$V_1 = \frac{\pi \cdot d^2}{4}\int_{-\frac{10}{\rho}}^{\frac{10}{\rho}} \frac{2d}{\left(\frac{10}{\rho}\right)^4}x^4 - \frac{7d}{2\left(\frac{10}{\rho}\right)^2}x^2 + d = \frac{7\pi d^3}{6\rho} \tag{23}$$

obtainable:

$$\phi = \frac{4V_1}{V_{total}}$$
$$\phi = \frac{7\pi\rho d^2}{1200} \tag{24}$$

Deriving from x at $x_0 = -\frac{5}{\rho_1}$:

$$f'(x) = \frac{8d}{\left(\frac{10}{\rho}\right)^4}x^3 - \frac{7d}{\left(\frac{10}{\rho}\right)^2}x$$
$$f'(x_0) = -\frac{d\rho_1}{10} + \frac{7d\rho_1}{20} \tag{25}$$

From θ with respect to the tangent line:

$$tan(\theta) = -\frac{d\rho_1}{10} + \frac{7d\rho_1}{20}$$
$$\rho_1 = \frac{4\,tan(\theta)}{d} \tag{26}$$

which when taken into $\phi(\rho, d)$ yields:

$$\phi = \frac{7\pi\tan(\theta)d^2}{300} \tag{27}$$

The objective function is the:

$$min(\phi) \tag{28}$$

2.6. Solving problems using genetic algorithms

Genetic Algorithm (GA) is an optimization algorithm based on natural evolutionary principles for solving search and optimization problems. It simulates the operations of inheritance, crossover and mutation in the process of biological evolution.

The basic principle of genetic algorithm is as follows:

Initialize the population: a set of initial solutions, called the population, is randomly generated.

Evaluate fitness: Each individual in the population is evaluated to determine its fitness, i.e., the quality of the solution, according to the requirements of the problem.

Selection operation: based on the size of the fitness, a selection operation is performed on the individuals. Individuals with higher fitness have a higher probability of being selected, thus retaining a better solution.

Crossover operation: two or more individuals are randomly selected from the selected individuals and new individuals are generated by crossover operation. Crossover simulates the process of hybridization in biological inheritance.

Mutation operation: a mutation operation is performed on the newly generated individuals to introduce randomness in order to increase the exploration of the solution space. Mutation simulates genetic mutation in biological inheritance.

Generate new populations: new populations are generated based on selection, crossover and mutation operations.

Repeat iterations: Repeat the process of evaluating fitness, selection, crossover, mutation, and generating new populations until stopping conditions are met (e.g., the maximum number of iterations is reached or a satisfactory solution is found, etc.).

Output result: outputs the optimal solution obtained during the iteration process.

Through continuous selection, crossover and mutation operations, the genetic algorithm can gradually search for better solutions in the solution space and gradually approach the optimal solution. Genetic algorithms have better performance in solving complex optimization problems, and they are also more adaptable to multi-peak problems and problems that cannot be solved by traditional methods.

Set the variables θ, d to be calculated as genes

Set the length of the gene as 30, the size of the population as 200, the number of iterations as 500, the crossover operator as 0.8, and the variation operator as 0.005.

Through iteration, the optimal solution is obtained as: $\theta_s = \theta_w = 10.041^{\circ}, d = 0.308mm$

3. Conclusions

This paper focuses on the thermal conductivity of planar fabrics, through the establishment of the overall thermal conductivity model of the fabric, describes the relationship between the overall thermal conductivity of the fabric and the thermal conductivity of a single fiber, and then through the optimization of various parameters of the fabric, to obtain the fabric with a stronger thermal insulating ability, the fabric with a strong thermal insulating ability in the aerospace direction has a very common application. At the same time, fabrics with strong thermal insulation ability also have weak electrical conductivity, which can be used in insulated clothing, insulated gloves and other aspects, and have high applicability.

First of all, based on the four times Newton interpolation formula to fit the fiber centerline equation and then establish the fiber physical model, the fabric is considered to be a three-dimensional model formed by the interlocking of curved cylinders, by the fabric model of the fiber and the volume of air ratio, combined with the measured thermal conductivity of the air and the fabric as a whole, to establish the fabric as a whole and the relationship between the single fibers of the thermal conductivity of the mathematical model, and then use based on the four times Newton interpolation formula to fit the fiber centerline equation. formula to fit the fiber centerline equation, and establish the centerline tangent equation to obtain the relationship between diameter, warp density, weft density,

warp bending angle, and weft bending angle. After obtaining the optimization objective function, the optimal solution is obtained by optimizing the diameter, warp density, weft density, bending angle of the model.

References

[1] MA Shiheng,GE Jingran,LIU Kai et al. Study of specific heat capacity and thermal conductivity of glass fiber/epoxy prepregs during the whole curing process[J/OL]. Composites Science and Engineering:1-6[2023-08-28].

[2] Xie Shihong,Gao Jie,Ning Laiyuan et al. Progress of carbon fiber/polymer composites thermal conductivity research in the last decade[J/OL]. Journal of Composite Materials:1-12[2023-08-28].

[3] Guo JY,Chen ZW,Su LJ et al. Effect of radiative heat transfer on the thermal insulation performance of silica aerogel composites[J]. Journal of Silicates,2023,51(07):1811-1818.

[4] Gan Peiyuan,Wei Deshi,Wei Shijiang et al. Effects of rare earth elements La and Ce on the organization and properties of Al-7Si-0.4Cu-1.0Fe die-cast aluminum alloy[J]. Shanghai Metal,2023,45(03):13-19.

[5] H.B. Zhu,Q.P. Du,X.L. Xu et al. Preparation of multistage micro- and nanofiber insulation materials and their thermal protection properties[J/OL]. Journal of Basic Science in Textile Colleges and Universities:1-7[2023-08-28].

[6] Zhang ZY, Peng XG, Huang WG, et al. Advances in the preparation and application of heat-insulating (lightweight) refractories[C]//Chinese Society for Metals Refractories Branch,Sinosteel Group Luoyang Refractories Research Institute Co.,Ltd,State Key Laboratory of Advanced Refractories,Refractories Magazine.Proceedings of the National Refractory Raw Materials Academic Exchange Conference in 2023.Proceedings of the National Refractory Raw Materials Academic Exchange Conference in 2023,2023:6-14.

[7] HE Duanpeng, ZHANG Lei, GAO Hong et al. Relationship between thermal conductivity of carbon fiber and its microstructural parameters[J]. Aerospace Materials Technology,2023,53(02):105-110.

[8] XU Kang,SHI Shanshan,JIANG Tao et al. Research progress of carbon fiber 3D network structure thermally conductive composites[J]. Engineering Plastics Application,2022,50(09):155-159.

[9] Dong Jianhong. Preparation and performance study of lightweight zirconia-based fiber thermal insulation materials[D]. Shandong University,2022.

[10] Zhang Xiaoshan,Wang Bing,Wu Nan et al. Research progress of micro-nano ceramic fibers for high temperature insulation[J]. Journal of Inorganic Materials,2021,36(03):245-256.