Optimizing Multibeam Survey Line Layouts Through Geometric Calculations and Target Optimization

Haotian Xu [#], Jiezheng Xing [*, #], Yongyong Gao

School of Information Science and Technology, Dalian Maritime University, Dalian, China, 116026

* Corresponding author: xjz003311@dlmu.edu.cn

[#]These authors contributed equally.

Abstract. The multi-beam bathymetric system enables the simultaneous collection of multiple seabed depth values within a swath, facilitating a transition from point measurements to line and area measurements. In this study, this article selected a sloped maritime area as our survey target and proposed a hybrid survey line model that incorporates both east-west and north-south orientations. The goal was to design an optimal survey line layout with the shortest total length. To achieve this, this article established a mathematical model for a nonlinear programming problem by considering geometric relationships and aggregating overlap widths, with the objective of minimizing the total length of survey lines. Employing nonlinear regression methods, in this article, the optimal number of survey lines and their respective lengths were determined to meet coverage requirements, and optimal survey line planning was conducted. This article successfully addressed the challenge of optimal path planning for vessels conducting explorations in unknown maritime areas, ultimately concluding that the hybrid model required shorter total survey line lengths. Through analysis, this article discovered that substituting the last survey line with north-south-oriented lines yielded a superior solution, significantly enhancing vessel detection efficiency.

Keywords: Multibeam Detection, Computational Geometry, Geometric Calculations, Target Optimization.

1. Introduction

Ma Kai, Xu Weiming, Xu Jian, and Dong Zhouyang [1] introduced a technique for sound velocity profile inversion and seafloor bathymetry correction based on underwater acoustic observations. This method leverages the information regarding beam incidence angles and one-way echo travel times to establish a relationship between beam displacement and sound velocity profile deviations. This approach effectively mitigates the issue of seafloor relief distortion induced by sound velocity errors in multibeam bathymetric surveys.

WEN Jiaxin, LI Jinghan,XING Ruixing,DU Jiawei,LIU Xusheng proposed a novel multibeam bathymetric data decimation algorithm [2], which involves classifying seabed topography complexity and applying distinct decimation strategies accordingly. This approach is designed to ensure navigational safety while preserving essential seabed topographical features. When compared to commonly employed decimation algorithms, this method excels in maintaining the fidelity of seabed representation and avoiding data gaps. It effectively fulfills the data requirements for subsequent nautical charting and seabed topography modeling.

ZHANG Yunxin, ZHAO Weili, and WANG Qun conducted measurements, navigation, and data acquisition in a large artificial river valley reservoir using a multibeam real-time bathymetric system and an inertial navigation system. This setup enables the rapid generation of underwater topographic maps, including 1:500 scale underwater topographic maps and seabed cross-sectional profiles [3]. This approach offers significant advantages, including enhanced noise handling capabilities and resistance to interference.

Wang N, Xu YC, and Tao CF harnessed high-precision underwater topographic data acquired through multibeam sonar technology to establish a comprehensive 3D terrain database model. This was achieved through color-texture rendering, employing state-of-the-art three-dimensional modeling software and sophisticated level-of-detail techniques[4]. This innovative approach enables

the authentic three-dimensional representation of seafloor topography through interactive 3D visualization software, introducing a novel virtual reality methodology for the intuitive presentation of underwater terrain data.

In earlier studies, significant advancements were made in underwater bathymetric mapping using multibeam sonar technology. However, there were still notable limitations that warranted further refinement. Firstly, early methodologies often adopted a single survey line strategy, such as exclusively measuring along the north-south or east-west directions. This limited a comprehensive consideration of seabed complexity and non-uniformity, potentially leading to incomplete or inaccurate measurements. Secondly, previous approaches inadequately addressed the issue of measurement noise, particularly in the context of the final survey line. Additionally, past methods did not comprehensively account for multiple key factors, including the size of the survey area, the beamforming angles of the multibeam transducers, coverage width, and overlap rates. This suboptimal survey design resulted in inefficiencies and resource and time wastage. Finally, earlier methods often lacked robust mathematical modeling and optimization support, relying more on trial-and-error and empirical decision-making, making it challenging to attain optimal solutions. Therefore, further improvements are needed in these areas to enhance the efficiency and accuracy of underwater bathymetric data acquisition.

This paper combines multiple key factors, including the size of the survey area, the beam angles of the multibeam transducers, coverage width, and overlap rates. It determines the optimal survey line layout through mathematical modeling and nonlinear programming optimization. Firstly, it employs two survey line strategies, oriented north-south and east-west, while considering different overlap rate requirements to ensure comprehensive coverage of the seabed terrain in various regions. Secondly, by introducing a mixed survey line model that accounts for potential measurement noise in the final survey line, it achieves superior solutions. This comprehensive consideration and mathematical solving approach effectively address the optimization challenge of survey line layout in the maritime domain, ensuring efficient and accurate underwater topographic data collection. This method holds significant importance for the application of multibeam bathymetric systems, especially in complex maritime conditions, as it can enhance exploration efficiency and data quality.

2. Optimization of Maritime Coverage Using Multibeam Measurements

Given the known variation in seafloor depth from west to east in the marine area and the assumption of a sloping seabed gradient from the central point towards the east, our objective is to identify a set of survey lines capable of comprehensively covering the entire marine region. These survey lines should also meet the stipulated requirement for overlap rates falling within the range of 10% to 20%.

Firstly, this article considers the factors of coverage width and overlap rates. To ensure the complete coverage of the entire area under investigation, it is essential to determine the coverage width for each sonar beam and the degree of overlap between adjacent survey strips. By considering the dimensions of the marine area and the beam angles of the multibeam transducer, In this article, the appropriate coverage width for each sonar beam can be calculated. In accordance with the specified requirements. In this article, a suitable range of overlap rates can then be selected to determine the spacing between the beams. [5]

In order to minimize the total survey length, this study explores two distinct survey approaches: along the longer side and the shorter side of the marine region. This analysis, combined with the requirement for coverage within a predefined interval, allows us to make a comparative assessment of these two scenarios.

2.1. Data Preparation

The marine region exhibits a depth gradient from west to east, with the slope decreasing as one moves away from the central point. Let D represent the central depth, D_actual denote the actual

seafloor depth at a specific location, and θ represent the slope. Using the tangent theorem and geometric calculations. In this article, the relationship can be established:

$$D = D_0 - \mathrm{d} \cdot \tan\alpha \tag{1}$$

In addition, given a slope of 1.5 degrees and a beam angle of 120 degrees for the multibeam transducer. In this article, a graphical representation has been generated displaying the relationship between horizontal distance from the central point and seafloor depth variation. This graphical representation is depicted in Figure 1.

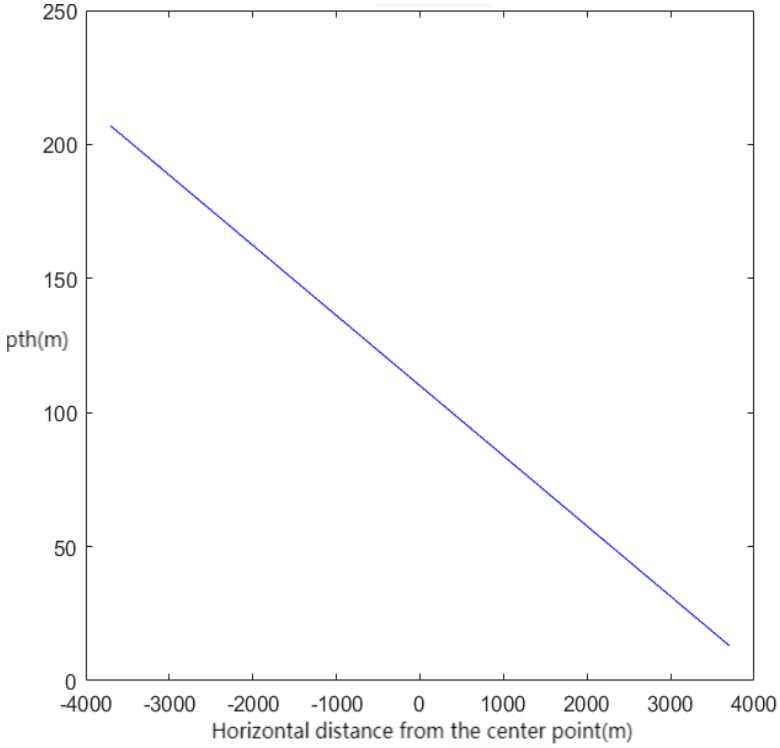


Figure 1. Seafloor Depth Variation with Distance

2.2. Designing a Model for Survey Line Plans

MODEL 1: Design of Survey Line Layout Along the East-West Axis

Step 1: Determining Coverage Width and Line Spacing

As depicted in Figure 8, in this article, the coverage width is defined as 'W,' and the line spacing as 'd.' A Schematic Diagram of the East-West Oriented Survey Line Model is shown in Figure 2.

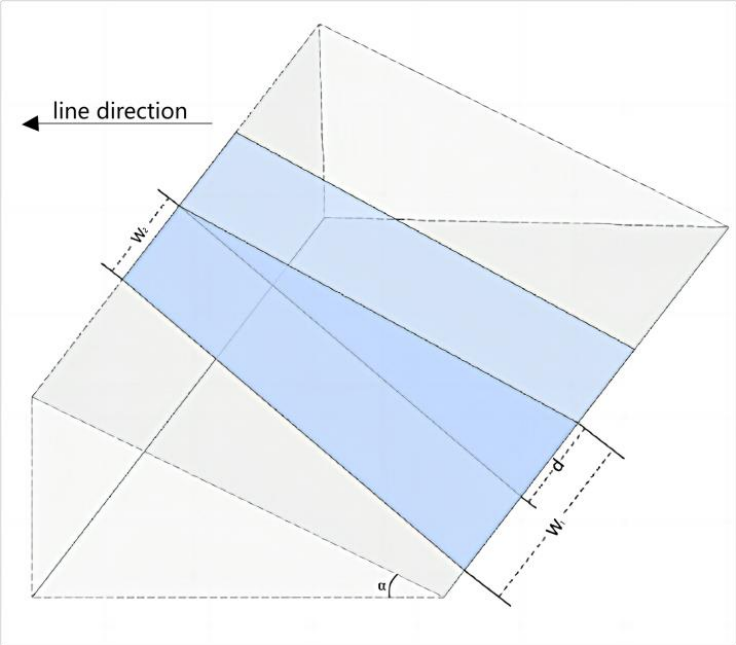


Figure 2. Schematic Diagram of the East-West Oriented Survey Line Model

The formula for calculating coverage width (W) and line overlap width can be derived through geometric calculations as follows: [6]

$$W = 2 \cdot D \cdot \tan(\frac{\theta}{2}); \quad \mathrm{d} = W \cdot (1-\eta) \tag{2}$$

Step 2: Determining the Number of Survey Lines

To ensure complete coverage of the entire marine area under investigation, the number of survey lines is determined by the minimum coverage width within a set of survey lines. Let's denote:

- W_{ew} as the width of the marine area from east to west (the east-west extent of the marine area).
- W_{min} as the minimum coverage width required for the east-west-oriented survey lines.

The minimum number of survey lines needed for complete coverage can be calculated as:

$$N = \frac{W_{ew}}{W_{\min}} \tag{3}$$

This calculation ensures that the entire marine area can be fully covered by the specified number of survey lines, considering the minimum required coverage width.

The calculation results are as shown in Table 1:

Table 1. Calculation Results of East-West Survey Lines

Location	depth(m)	coverage area(m)	coverage rate(%)
Easternmost	62	45	0
Central Area	110	429	/
Westernmost	158	717	94

Upon ensuring complete coverage of the entire marine area, it has come to attention that the westernmost survey line attains a coverage rate of 94%. This notably exceeds the stipulated limit of 20%. Excessive overlap rates can lead to substantial data redundancy, rendering this model unsuitable for the intended task.

MODEL 2: Survey Lines Oriented North-South

As shown in Figure 3, in the case of survey lines oriented along the north-south axis, each survey line generates an equal coverage width. This implies that the region covered by each survey line on

the sloping terrain should exhibit a rectangular shape. [7]

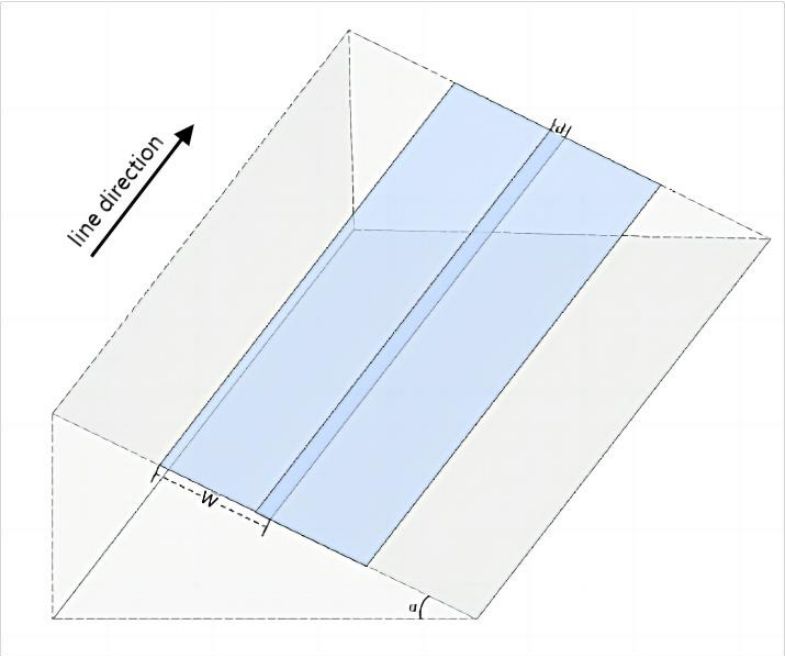


Figure 3. Schematic Diagram of North-South Oriented Survey Line Model

Step 1. Determining Coverage Width and Line Spacing for North-South-Oriented Survey Lines

Through mathematical analysis and calculations, the relationship can be established between the coverage width (W_n) of the nth survey line and the water depth (D_n). Additionally, the formula for calculating the line spacing can be derived as follows in this article: [8]

$$W_n = \frac{D_n \cdot \tan\frac{\theta}{2}}{\cos(\frac{\theta}{2}-\alpha)} + \frac{D_n \cdot \sin\frac{\theta}{2}}{\cos(\frac{\theta}{2}-\alpha)} \qquad \mathrm{d} = W_n \cdot (1-\eta) \tag{4}$$

Step 2. Determining the Number of Survey Lines

In accordance with the requirements of the task, it is necessary to ensure complete coverage of the entire target marine area and maintain coverage rates between adjacent coverage widths within a specified range. To achieve this, the sum of coverage widths should exceed the north-south length of the sea surface, which can be expressed as follows

$$(1-\eta)\cdot(W_1 + W_2 + \cdots + W_N) > Len \tag{5}$$

To minimize the total length of survey lines, it is desirable to keep the coverage rate as low as possible. Therefore, the relationship can be expressed as follows: $\eta = 10\%$.

To solve this problem using Python code, a minimum of 10 survey lines is required. The specific results are shown in Figure 4, and the visualization of the results is presented in Table 2.

Table 2. Model Survey Line Results along the North-South Direction

distance from the center point (n mile)	width of coverage(m)
-0.86	526.40
-0.58	504.87
-0.32	462.68
-0.08	423.11
0.16	386.87
0.34	353.74
0.52	323.45
0.69	295.75
0.84	270.42
0.98	247.26

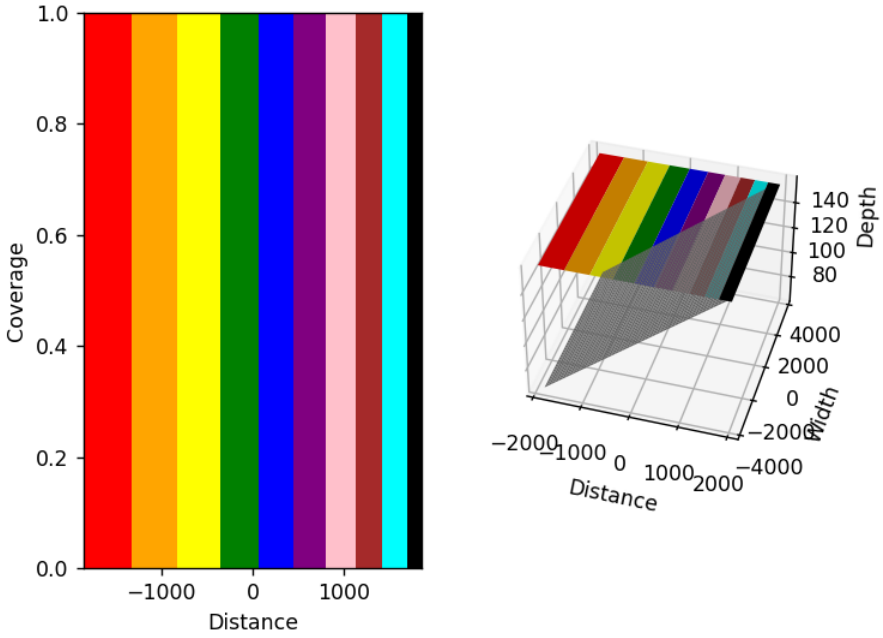


Figure 4. Visualization Diagram of Survey Lines for the Model Along the North-South Direction

From figure 4, the total length of survey lines (Cost) in this scenario is calculated as the product of the total number of required survey lines (N) and the East-West width (W_{ew}):

$$Cost = N \times W_{ew} \tag{6}$$

In conclusion, when survey lines are oriented in a north-south direction, a minimum of ten survey lines, each with a length of 40 nautical miles, are required.

MODEL 3. Survey lines along a combined east-west and north-south direction

After researching the above two models, this study found that when survey lines are oriented north-south, there is wider coverage and higher efficiency on the eastern side of the maritime area, while efficiency significantly decreases on the western side. Similarly, when survey lines run east-west, the overlap rate on the western side meets the requirements. Therefore, this paper considers adopting a combined east-west and north-south survey line orientation model, as shown in Figure 5.

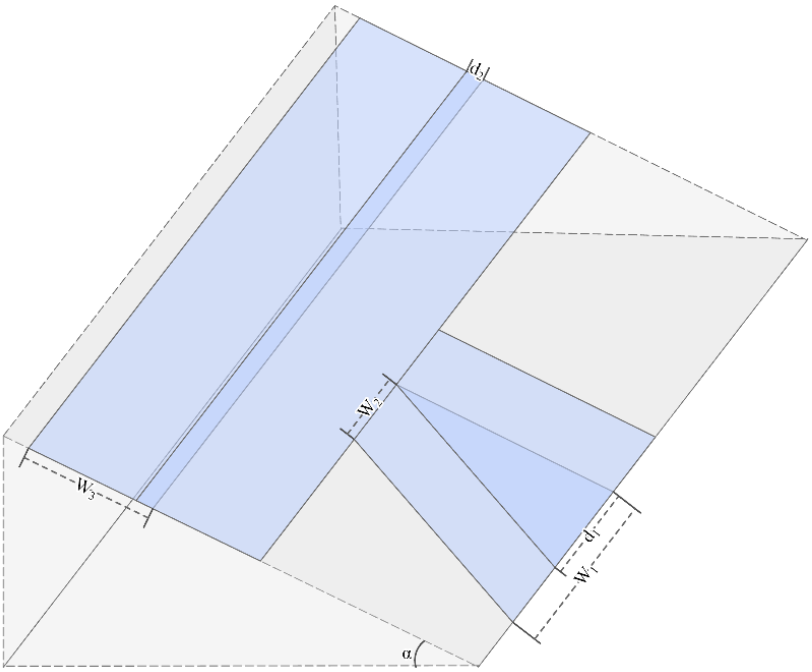


Figure 5. Combined East-West and North-South Survey Line Model Diagram

When survey lines are oriented east-west, the longest coverage width is denoted as W_1 the shortest coverage width is W_2,the width of the overlap section is d_1,the area of overlap is S_1', and the overlap rate is η_1. When survey lines are oriented north-south, the coverage width is W_3, the width of the overlap section is d_2, the area of overlap is S_2, and the overlap rate is η_2.It is assumed that the east-west orientation occupies a total width of L_1 in the east-west direction, while the north-south orientation occupies a total width of L_2 in the east-west direction . [9]

Firstly, L1 and L2 must meet the following conditions:

$$L_1 + L_2 = M \tag{7}$$

For the east-west orientation section, we can derive the following conclusions based on geometric relationships:

$$\begin{cases} \mathrm{d}_1 = W_1 \cdot \eta_1 \\ W_2 = W_1 \cdot (1-\eta_1) \end{cases} \qquad D_n = \frac{W}{2 \cdot \tan(\frac{\theta}{2})}$$

$$\begin{cases} N_1 = \dfrac{L\mathrm{en}}{W_2} \\ L_1 = \dfrac{D_1 - D_2}{\tan\alpha} \end{cases} \Rightarrow S_1 = \frac{N_1 \cdot \mathrm{d}_1 \cdot L_1}{2} \tag{8}$$

Secondly, regarding the overlap width of the north-south survey lines, the following results apply:

$$d_2 = W_3 \cdot \eta_2 \tag{9}$$

Summing the widths of all north-south oriented overlapping sections,

$$\begin{aligned}
&\because \sum_{i=1}^{n} d_2^i = \sum_{i=1}^{n} W_i \cdot \eta_2 \\
&\therefore \sum_{i=1}^{n} d_2^i = (L_2 + \sum_{i=1}^{n} d_2^i) \cdot \eta_2 \\
&\therefore \sum_{i=1}^{n} d_2^i = \frac{L_2 \cdot \eta_2}{1-\eta_2} \\
&\therefore S_2 = \sum_{i=1}^{n} d_2^i \cdot Len = \frac{L_2 \cdot \eta_2 \cdot Len}{1-\eta_2}
\end{aligned} \tag{10}$$

The objective function is$(S_1 + S_2)_{min}$,

$$\frac{Len \cdot \eta_1^{\,2} \cdot W_1}{(1-\eta_1) \cdot 2 \cdot \tan\left(\frac{\theta}{2}\right) \cdot \sin\alpha} + \left(Len - \frac{W_1 \cdot \eta_1}{2 \cdot \tan\left(\frac{\theta}{2}\right) \cdot \sin\alpha} \right) \cdot \frac{\eta_2}{1-\eta_2} \cdot M \tag{11}$$

By employing a non-linear programming approach, the minimum repeated coverage area is achieved when η=0.1, and the specific lengths are provided in Table 3.

Table 3. Length Results of Survey Lines for the Mixed Orientation Model

Survey Line Orientation	number of lines	length for each line
North-South Orientation	12	0.43NM
East-West Orientation	7	4NM

3. Conclusion

Taking into consideration the geographical characteristics of the marine area, this research has established three distinct survey line models, which are based on north-south orientation, east-west orientation, and a combination of both. The primary goal of these models is to optimize the survey line layout, ensuring comprehensive coverage of the designated marine region while adhering to specified overlap rate requirements.

Throughout the study, it has been observed that survey lines oriented both north-south and east-west are susceptible to noticeable measurement noise issues. To address this challenge, constraints have been introduced, restricting the overlap rate to fall within the range of greater than 10% but less than 20%. Furthermore, the originally formulated objective function, which aimed at minimizing survey line length$(1 - \eta) \cdot (W_1 + W_2 + \cdots + W_n) > Len$, has been redefined to minimize overlap area$(S_1 + S_2)_{min}$.

In order to tackle these challenges, especially when adhering to the constraint of minimizing overlap area as the primary objective, in this article, a single-objective nonlinear programming approach was employed while considering various constraints.[10] After conducting a comparative analysis of the three models, the following conclusions have been derived:

Firstly, the north-south-oriented model demonstrates relatively inferior performance and necessitates a larger number of survey lines to meet the coverage requirements.

Secondly, the east-west-oriented model and the combined east-west/north-south model result in similar total survey line lengths. However, the hybrid model featuring both orientations achieves a shorter total survey line length. This enhancement can be attributed to the resolution of potential noise issues concerning coverage width in the final survey line through substitution with a north-south-oriented survey line.

These findings suggest that the combined model offers an improved solution for optimizing the

survey line layout, ensuring comprehensive coverage while minimizing the overall length of survey lines.

In summary, this study, through an in-depth analysis of various survey line models, has proposed an effective survey line layout strategy aimed at enhancing the efficiency and accuracy of underwater terrain data collection. This has significant implications for fields such as marine exploration and resource management, providing robust support for future advancements in marine scientific research.

References

[1] Ma Kai, Xu Weiming, Xu Jian, et al. A Technique for Multibeam Sonar Sound Speed Profile Inversion and Seafloor Topography Correction [J]. Journal of Wuhan University (Information Science Edition), 2019, 44 (04): 525 - 531+600.

[2] Wen Jiaxin, Li Jinghan, Xing Ruixing, et al. A Multibeam Bathymetric Data Down Sampling Algorithm Considering Terrain Complexity [J]. Journal of Geomatics Science and Technology, 2018, 35 (04): 435 - 440.

[3] Zhang Yunxin, Zhao Weili, Wang Qun. Research on Multibeam Underwater Bathymetric Survey for Seafloor Topography Measurement [J]. Geomatics and Spatial Information Technology, 2022, 45 (S1): 302 - 304.

[4] Wang Nan, Xu Yongchen, Tao Changfei. Research on Multibeam Seafloor Topography Three-Dimensional Virtual Simulation [J]. Coastal Engineering, 2019, 38 (03): 203 - 209.

[5] Zhang Tao, Zhang Chi, Zhang Jiayu. A Multibeam Underwater Terrain Matching Method Based on Improved Genetic Algorithm. [J] Journal of Chinese Inertial Technology 2022, 30 (04): 485 - 491 - 500.

[6] Liu Yijun. Research on the Application of Single-Beam and Multibeam Bathymetric Systems in Shallow Water Submarine Topography Measurement. [J] Geospatial Information, 2021 (03): 4 - 6.

[7] Ma Dinghen. Research on the Application of Multibeam Bathymetric Systems in Submarine Topography Measurement [J]. Geospatial Information., 2021 (03): 21 - 25.

[8] Xu Zhaohua, Wang Zhiliang, Zhang Ping. Application of Multibeam Bathymetric Technology in Port Topography Measurement [J]. Surveying Standardization, 2021, 37 (01): 83 - 86.

[9] Yu Gang. Application of Unmanned Vessels Equipped with Multibeam Bathymetric Measurement Technology in Submarine Topography Measurement [J]. Henan Water Resources and South-to-North Water Diversion, 2022, 51 (06): 97 - 99.

[10] Zheng Bozhen, Zhou Changjiang, Ge Juntao. Multibeam Ocean Topography Measurement Based on Combination Positioning Technology. Surveying Standardization [J]. 2022, 38 (03): 57 - 60.