A study of fixed-sun mirror field alignment based on mathematical methods

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Abstract. Different shapes of heliostat mirrors and different arrangements of mirror fields are important factors affecting the thermal power output of tower-type solar power plants. In this paper, for a regional heliostat mirror field, a coordinate system of heliostat mirror field is established, through which the incident vector of sunlight, normal vector of heliostat mirror plane is calculated, and the mirror coordinate system is transformed to calculate the optical cosine efficiency, and the discrete lattice point method and ray-tracing method are used to calculate the shadow blocking efficiency and truncation efficiency. The average annual optical efficiency of the mirror field in this area is 0.609784988, the average annual thermal power output is 37.27373787 MW, and the average annual thermal power output per unit area is 0.593341895 kW/m2, which can be used as a reference basis for the arrangement of the heliostat mirror field in this area.

Keywords: Fixed-sun mirror field, Solar energy utilization, Discrete lattice method, Ray tracing method.

1. Introduction

The adoption of new energy sources, particularly low-carbon clean energy technologies such as solar thermal power generation from towers, is a central strategy for China to meet its carbon reduction targets [1]. This type of power generation relies on a key component called a heliostat, which is used to collect solar energy [2]. A heliostat consists of two rotating shafts and a flat mirror that can be orientated to maximise the amount of sunlight collected [3-5]. These mirrors form an array that concentrates the reflected sunlight into an absorber tower with a collector for energy conversion. The array of heliostats and the absorber tower together form the heliostat field [6]. In order to design a more reasonable mirror field, some scholars have combined the problem of optimal arrangement of heliostat mirror fields with intelligent optimization algorithms, and put forward a variety of flexible and efficient scheduling methods. For example, Atif et al. [7] used a differential evolutionary algorithm to optimise a radially staggered heliostat mirror field with the objective of the annual average efficiency of the mirror field. Li Chao et al [8] proposed a hybrid PSO-GA algorithm by combining genetic algorithm and particle swarm algorithm, and verified the validity of this method by optimally arranging the fixed-sun mirror field in Lhasa area [9-11]. Based on this. A solar simulation model considering the relative position of the sun and the non-parallelism of the sun rays is developed and proposed to solve this problem.

2. Model building

2.1. Establishment of a coordinate system for the heliostat field

Concentration system is the key part of tower-type solar thermal power plant to obtain solar radiation, its role is to focus the sun's rays to the collector tower through the heliostat, so as to achieve

efficient energy conversion. The heliostat field of tower-type photovoltaic power plant is mostly arranged in the circumferential direction with the collector tower as the centre, and it is customary to use the spatial coordinate system with the collector tower as the origin to characterize the orientation of each heliostat in the mirror field. In order to describe the position of the sun and the heliostat at the same time, this paper will use the geometric coordinate system with the intersection of the central axis of the heat-absorbing tower and the heliostat mirror field plane as the origin as the heliostat mirror field coordinate system, i.e., the geometric centre of the heat-absorbing tower is the origin of the coordinate system O, and the east direction is designated as the X semi-axis of the coordinate system, the north direction as the Y semi-axis of the coordinate system, and the zenith direction is designated as the Z-axis of the coordinate system, and the established three-dimensional spatial right-angle coordinate system is used to characterize the position of each heliostat mirror in the mirror field. The established three-dimensional spatial right-angle coordinate system O-xyz is shown in Figure 1.

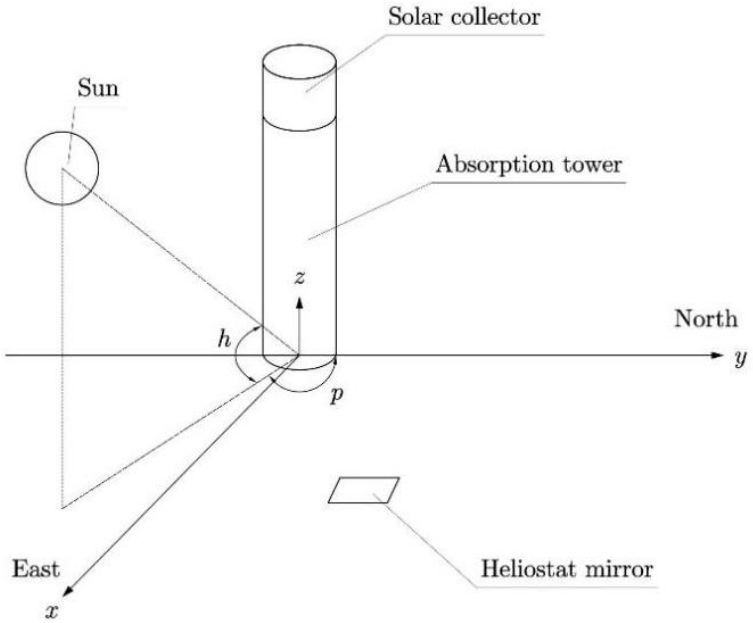


Figure 1. Mirror field coordinate system

It is worth noting that the solar azimuth angle γs (<p on the diagram) is taken as 0- in the direction of due north, and is rotated clockwise to the positive direction. Since the main incident light directed to the centre of the heliostat will be reflected into the centre of the collector, it is easy to establish the unit vector R of the main reflected light.

The unit vector $\vec{R}$ of the main reflected ray is established and calculated as follows:

Step1: Set the collector centre coordinates in the mirror field coordinate system as M0(0, 0, H_1), H_1 refers to the height of the collector centre, which is 80 m at this time.

Step2: Set the coordinates of a heliostat in the mirror coordinate system as M (x, y, h), h specifies the height of the centre of the heliostat, which is 4 m; h specifies the height of the centre of the heliostat, which is 4 m.

Step3: Subtract the coordinates of the two points to get a vector of the refracted ray, divide this vector by its own mode length to get the unit vector of the main reflected ray:

$$\vec{R} = \frac{M_0 - M}{|M_0 - M|} = \frac{(-x, -y, H_1 - H)}{\sqrt{x^2 + y^2 + (H_1 - H)^2}} \quad (1)$$

2.2. Theoretical basis for the non-parallel sunlight model

In many current studies, solar radiation is often treated as parallel light in order to simplify the complexity of simulation calculations. In reality, the observed Sun behaves as a disc anywhere on Earth, so that solar radiation reaches the ground as a cone of light, as shown in the figure below. The difference between the Sun-Earth distance l, which is 1.5×10^8 km, and the radius of the Sun, which

is 6.96×10^5 km, is so large that the angle of the light cone is extremely small. According to the mathematical principle, in the case of a small angle, the sine value and the angle value are almost equal, i.e., $\lim x\to 0 \sin(x) = x$. Therefore, the half-angle α_γ of the solar cone can be obtained:

$$\alpha_\gamma = sin(\alpha_\gamma) = \frac{6.96\times10^5}{1.5\times10^8} = 4.65\ mrad\ \approx\ 0.5° \tag{2}$$

The sun's light is reflected to form a cone of light, and its cone half-angle is the same as the sun's cone half-angle, calculated by similar triangulation, when the heliostat is 300 m away from the collector, the diameter of the aperture can be up to 2.82 m, and this distance is very obvious, which can't be ignored, so when calculating certain values, the sunlight should be regarded as a cone of light, and adopt the model of non-parallel light. The relevant schematics are shown in Figures 2 and 3.

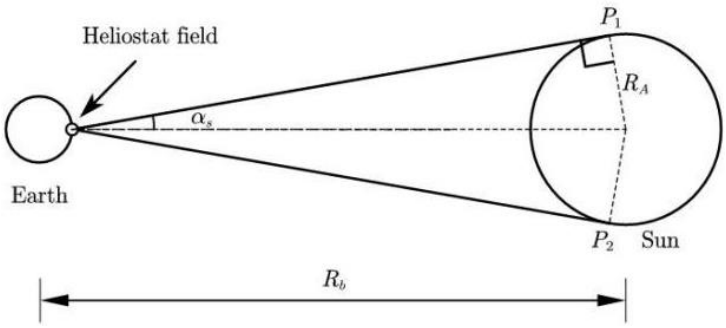


Figure 2. half-angle of a light cone

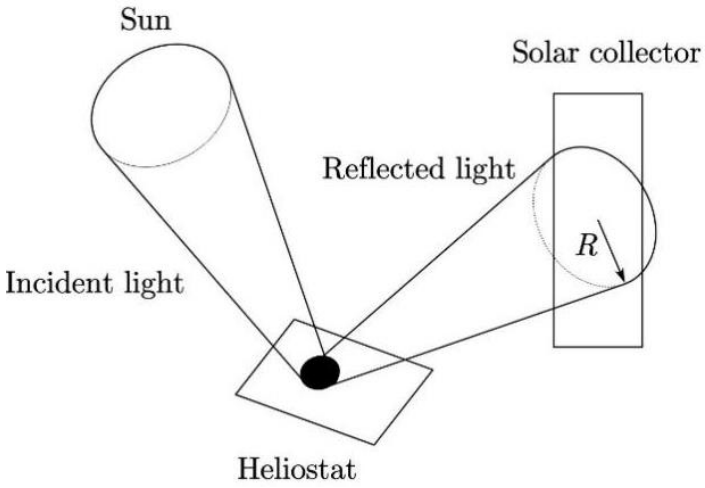


Figure 3. Reflected light cone

2.3. Discrete grid method

Assume that shadow occlusion only occurs within a certain range: This is intuitive and can significantly reduce computational complexity. Consider the 8 nearest heliostats to a specific heliostat A for evaluation. Heliostats that are too far away will not affect the occlusion efficiency of A. The reason for choosing 8 points is that heliostats in 8 directions around A—front, back, left, right, front-left, front-right, back-left, back-right—may affect A, while others do not need to be considered, thus reducing unnecessary calculations.

First, discretize the grid based on the number and distribution of points, as well as the height and width of the heliostats.

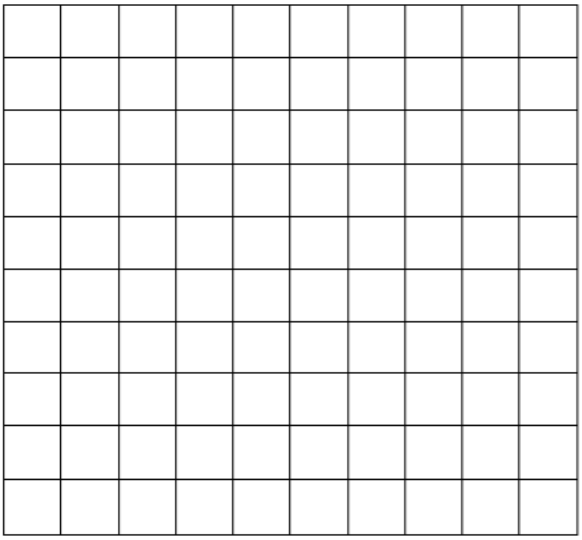


Figure 4. Schematic diagram of the scattering method

As shown in figure 4, divide heliostat i into a×b grids, totaling N grid points. Collect the coordinates of all discrete points on the mirror i in its mirror coordinate system. Calculate the transformation matrix R between the mirror coordinate system of mirror i and the field coordinate system. Then, use the rotation matrix to transform it into coordinates in the field coordinate system. Calculate the 8 nearest mirrors $J = \{j_1, j_2, \ldots\}$ to mirror i using the Euclidean distance formula. Then use the method introduced in section 2.1 to calculate the vector of the reflected light ray and its analytical geometric expression.

2.4. Truncation efficiency

Cut-off efficiency is a measure used to describe the effect of the receiving area of a collector or optical system on the performance of the system. Specifically, an ideal optical system or collector should be able to fully receive all reflected light, but in practice, due to practical issues such as structural constraints and practical installation, some of the reflected light may not be received and may be "truncated". This truncated light cannot be used efficiently by the optical system or collector, so this loss can be described as the truncation efficiency (TEE). This loss can therefore be described as the truncation efficiency. The truncation efficiency has an impact on the efficiency of the energy received by the system and therefore needs to be taken into account when designing and optimising a heliostat collector or optical system. The calculation of the cut-off efficiency is still done using grid points and ray tracing methods to simulate a large amount of light reflecting off the collector, thus simulating the real situation, and when the amount reaches a certain size, it can be shown that the ratio of the number of rays thrown off the collector to the total number of rays is the amount we want to calculate. The key to judge the effective light of the truncation efficiency is that the solution intersecting with the collector is real, that is to say, the quadratic function after the association has real roots, and at the same time, the reflected light which is blocked should be eliminated in the calculation. In addition, when calculating the truncation efficiency, the sun rays are no longer parallel, but conical. Here the cosine loss due to the cylindrical collector is ignored to simplify the model calculation and judgement. The formula is:

$$\eta_{trunk} = 1 - \frac{Number\ of\ rays\ with\ real\ solutions\ in\ the\ target\ area}{Total\ number\ of\ rays} \tag{3}$$

In addition, the truncation efficiency can be approximated in another relatively simple way: since the sunlight will shine on all parts of the body of a heliostat, there will be reflected light cones formed in all areas except the main incident ray in the centre, especially in the border areas, so that the area of the reflected spot will be similar to an ellipse, but with a flat perimeter and arcs at the corners,

which are roughly one-quarter of a circle, each circle and radius can be solved by the triangle similarity and the half-angle of the sunlight cone. circumference, the area and radius of each circle can be solved by the triangle resemblance and the half angle of the solar cone, but there will be cosine loss in the reflection, resulting in the reflected spot can be approximated to an ellipse using the cosine theorem, whose equations are more complicated. However, it is true that the specific equations, and then due to the Gaussian distribution of the spot energy, the spillover efficiency is a double integration of the points falling in the collector and the spot ellipse.

$$\eta_{spill} = \frac{1}{2\pi\sigma^2} \iint e^{-\frac{x^2+y^2}{2\sigma^2}} dxdy = 1 \tag{4}$$

It is sufficient to solve for this equation, where σ can be approximated as the magnitude of the solar tensor half angle.

3. Results

After a series of modelling, the optical efficiency can be solved, and the corresponding annual average optical efficiency, annual average thermal power output, and annual average thermal power output per unit mirror area can be solved. Firstly, the distribution of heliostats in the whole solar thermal plant is drawn, as shown in Figure 5.

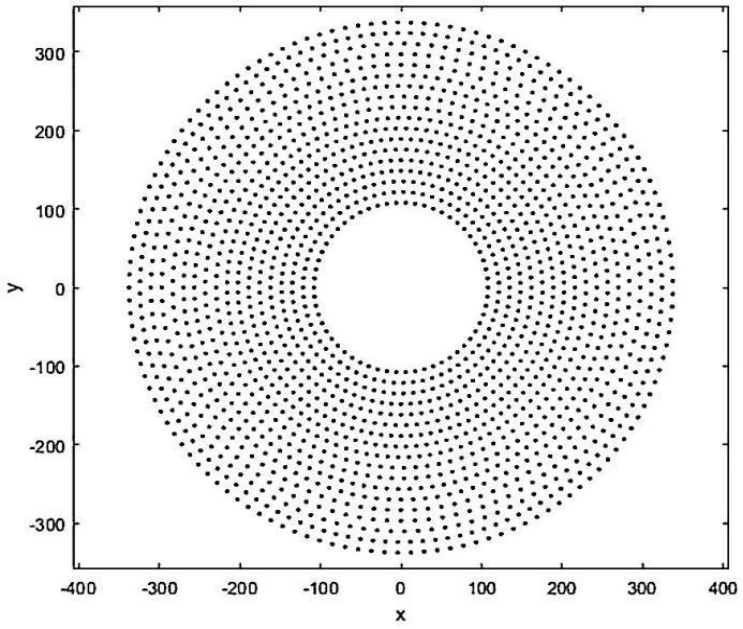


Figure 5. Distribution of heliostats

The annual average optical efficiency can be obtained by summing and averaging the Es of five time periods on the 21st day of each month as the average value for the month, and then averaging the cumulative sums over 12 months to obtain the annual average thermal power.

The output thermal power of the fixed-sun field is:

$$E_{field} = DNI \cdot \sum_i^N A_i \eta_i \tag{5}$$

The annual average thermal power of this index can be obtained by summing and averaging the E of five time periods on the 21st day of each month as the average value of this month, and then summing and averaging the cumulative total of 12 months to obtain the annual average thermal power. The annual average output thermal power per unit mirror area is obtained by dividing the annual average thermal power by the total area of the entire fixed-sun field, i.e., 6×6×1745= 62820 m^2, thus obtaining the annual average output thermal power per unit mirror area.

For the calculation of parameters such as the average optical efficiency for each month (21 days) is included in the annual average calculation, here the monthly average truncation efficiency as an example, write its calculation formula should be:

$$\eta_{trunk} = \frac{1}{N}\sum_{i}^{N} \eta_{trunk,j} \tag{6}$$

Based on the above approach, the relevant results of the calculations are shown in Table.1 below.

Table 1. Average optical efficiency and output power on the 21st of each month

Date	Average Optical Efficiency	Average Cosine Efficiency	Average Shadowing Efficiency	Average Truncation Efficiency	Average Thermal Output Power per Unit Area of Mirror (kW/m^2)
21st Jan	0.57	0.72	0.97	0.93	0.50
21st Feb	0.60	0.74	1.00	0.92	0.56
21st Mar	0.62	0.76	1.00	0.92	0.61
21st Apr	0.63	0.78	1.00	0.92	0.65
21st May	0.64	0.79	1.00	0.92	0.67
21st Jun	0.65	0.79	1.00	0.92	0.68
21st Jul	0.64	0.79	1.00	0.92	0.67
21st Aug	0.63	0.78	1.00	0.92	0.65
21st Sep	0.62	0.76	1.00	0.92	0.61
21st Oct	0.60	0.74	0.99	0.92	0.56
21st Nov	0.57	0.72	0.97	0.93	0.49
21st Dec	0.55	0.71	0.96	0.93	0.46

The above data are processed to obtain the monthly average results. As shown in Table 2.

Table 2. Annual average optical efficiency and output power table

Annual Average Optical Efficiency	Annual Average Cosine Efficiency	Annual Average Shadowing Efficiency	Annual Average Truncation Efficiency	Annual Average Thermal Output Power (MW)	Annual Average Thermal Output Power per Unit Area of Mirror (kW/m^2)
0.61	0.76	0.99	0.92	37.27	0.59

From Table .2, it can be obtained that: the average annual optical efficiency of the mirror field in the region is 0.61, the average annual output thermal power is 37.27 MW, and the average annual output thermal power per unit area is 0.59 kW/m2.

4. Conclusion

In this study, a better scheduling scheme for the fixed-sun mirror field of a tower solar power plant is analysed in depth based on the discrete lattice point method and ray tracing method. And the electric field efficiency of each year is calculated in more depth. The model developed in this study also adopts the compact scheduling method and the fixed-sun mirror field coordinate method, which can effectively reduce the computational volume and memory occupation and improve the computational efficiency. The results of this study can provide a scientific reference basis for the reasonable arrangement of the heliostat field.

References

[1] Gao Bo, Liu Jianxing, Sun Hao et al. Optimal arrangement of heliostat mirror field based on adaptive gravitational search algorithm [J]. Journal of Solar Energy, 2022, 43 (10): 119 - 125.

[2] Xu Hengxiang. Research on irradiation prediction system for ground-based cloud maps of heliostat field [D]. Xi'an University of Technology, 2022.

[3] ZHANG Mogeng, HAN Zhaohui, GU Pengcheng et al. Application of tower solar thermal technology in integrated energy system [J]. Xinjiang Oil and Gas, 2022, 18 (02): 71 - 77.

[4] Sun H. Research and optimisation of fixed-sun mirror field layout based on mixed-strategy whale optimisation algorithm [D]. Lanzhou Jiaotong University, 2022.

[5] Liu Jianxing. Optical efficiency modelling simulation and optimal arrangement of heliostat mirror field for tower-type photovoltaic power plant [D]. Lanzhou Jiaotong University, 2022.

[6] SUN Hao, GAO Bo, LIU Jianxing. Research on the layout of heliostat mirror field of tower solar power station [J]. Power Generation Technology, 2021, 42 (06): 690 - 698.

[7] Gao Weidong, Lv Lixia, Wang Ziqi et al. Research on fixed-sun mirror field scheduling scheme for tower solar power plant based on NSGA-II [J]. Thermal Power Generation, 2021, 50 (05): 94 - 101.

[8] LIU YANGPING, TANG FANG, CHEN SICHOU. Calculation of lighting area and cost of CSP tower power plant with fixed heliostat [J]. Applied Energy Technology, 2019 (06): 38 - 41.

[9] WANG Nan, WANG Xinyu, SUN Feihu et al. Experimental study of tower solar thermal power plant concentrating light during full moon night [J]. New Energy Progress, 2019, 7 (01): 23 - 31.

[10] Yawen Li. Research on the control strategy of tower solar fixed-sun mirror field concentrator system [D]. North China Electric Power University, 2019.

[11] Lv Tongxing. Characterisation and modelling study of tower solar thermal power generation concentrator system [D]. North China Electric Power University, 2019.