Research on Multibeam Bathymetric Strategy Based on Optimization Algorithm and Geometric Analysis

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Abstract. This paper presents a comprehensive examination of the application of multibeam bathymetric strategies, specifically focusing on the transition from two-dimensional cross-sections to three-dimensional space. Firstly, a two-dimensional bathymetric model is established through geometric and mathematical analysis, considering the influence of the seabed slope. This allows for accurate calculation of the coverage width and overlap rate. Secondly, the model is expanded to incorporate three-dimensional sea conditions, considering multiple factors such as the relative angle between the survey line direction and the seabed slope. Lastly, an optimization algorithm is employed to minimize the total length of the survey line. The optimal solution is determined to be 206 survey lines with a spacing of 36.05m between them, resulting in a total length of the shortest survey line of 7425.7m.

Keywords: Multibeam bathymetry, optimization algorithms, coverage width, overlap rate.

1. Introduction

Multibeam sonar surveys (MBES) have been widely used for full-coverage measurements of seafloor depth and topography, but face data redundancy and leakage under complex seafloor topography and inhomogeneous bathymetry conditions [1-3]. This study aims to address this limitation, and the specific objectives include: Constructing a mathematical model of the multibeam coverage width and the overlap rate of adjacent strips; Using the model to perform a quantitative analysis of the coverage width under the given seafloor topography and bathymetry; Optimizing the survey line design to achieve the minimum length with the maximum coverage efficiency. Through this series of studies, this paper aims to improve the accuracy and efficiency of multibeam sonar measurements in complex marine environments.

2. Geometric and mathematical analysis of multibeam bathymetry

2.1. Geometric Relationships and Coverage Modeling

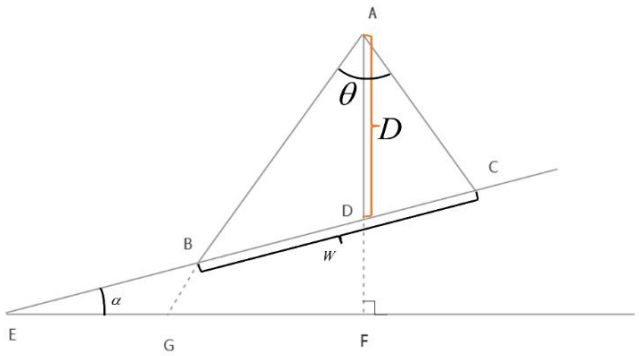


Fig. 1 Relationship between the seafloor slope and the position of the survey line.

In this study, a schematic diagram (shown in **Fig. 1**) was first created to resolve the coverage width of multibeam bathymetry. The diagram shows that the coverage width is composed of two parts, length BD and DC. By analyzing the relationship of angles in the schematic in detail and applying

the sine theorem in the corresponding angles, the lengths BD and DC, which are related to the water depth AD, are successfully solved. The total coverage width of the multibeam bathymetry is obtained by adding these two lengths. By this method, a mathematical model for describing the coverage width of multibeam bathymetry is effectively established in this paper.

In the center, extend the AD line perpendicular to the EF line, the vertical point is point F, extend the AB line and the intersection of the line EF is recorded as point G, because and is the opposite angle, so use the sine theorem can be obtained:

$$BD=\frac{D\sin\frac{\theta}{2}}{\cos\frac{\theta+2\alpha}{2}} \tag{1}$$

From the above analysis $\angle ADB=\pi-\angle BAD-\angle ABD=\frac{\pi+2\alpha}{2}$, then $\angle ACD=\frac{\pi+2\alpha-\theta}{2}$, using the sine theorem:

$$DC=\frac{D\sin(\frac{\theta}{2})}{\cos\frac{2\alpha-\theta}{2}} \tag{2}$$

To summarize, this paper can derive the coverage widths: the

$$\mathrm{W=BD+DC}=\frac{D\sin\frac{\theta}{2}}{\cos\frac{2\alpha+\theta}{2}}+\frac{D\sin(\frac{\theta}{2})}{\cos\frac{2\alpha-\theta}{2}} \tag{3}$$

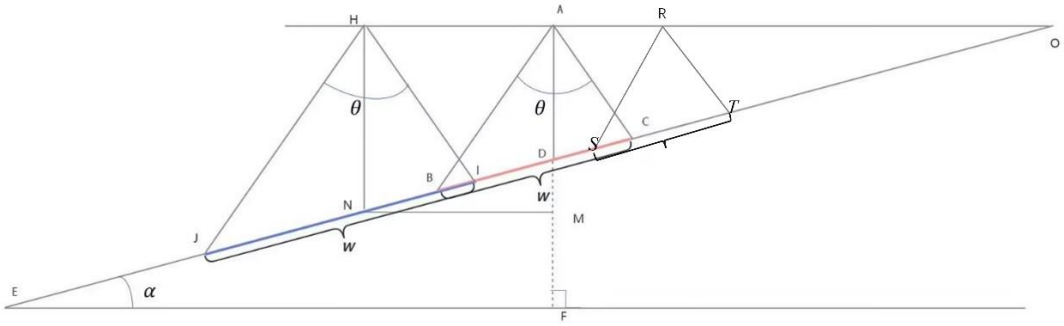


Fig. 2 Schematic diagram of overlapping multiple measurement lines.

As shown in **Fig. 2**, the extension line HA of the survey line and the extension line EC of the slope incline line intersect at point O. In order to facilitate the analysis of the relationship between the coverage width, the spacing of the survey line and the overlap rate, set the seawater depth of each survey line from the center point of the location of the seawater depth D_i (i=0,1,2...8) and ,according to the straight line HO and the straight line EF parallel to the line, can be obtained as $\angle HOE=\angle\alpha$.

In ΔAOD , it is obtained from $\tan\alpha=\frac{AD}{AO}$:

$$AO=\frac{AD}{\tan\alpha}=\frac{D_0}{\tan\alpha} \tag{4}$$

Then $HO = HA + AO = d + \frac{D_0}{\tan\alpha}$, in both ΔHNO and ΔADO triangles, this paper uses the properties of similar triangles to obtain:

$$\frac{AO}{HO} = \frac{AD}{HN} \tag{5}$$

$$HN = \frac{AD \cdot HO}{AO} = \tan\alpha \cdot d + D_0 \tag{6}$$

$$D_i = \tan\alpha + D_0 \tag{7}$$

The following is an analysis of the relationship between coverage η and W is analyzed below in ΔHJI , using the above conclusions, this paper can obtain the relationship between NI and HN, which is obtained by substituting HN:

$$\text{NI} = \frac{\tan\alpha \cdot d \sin(\frac{\theta}{2}) + D_0 \sin(\frac{\theta}{2})}{\cos\frac{2\alpha - \theta}{2}} \tag{8}$$

As in **Fig. 2**, make NM perpendicular to AF and the vertical point is F. In ΔDNM , this is obtained from the triangle cosine function:

$$\cos\alpha = \frac{NM}{ND} \tag{9}$$

$$ND = \frac{NM}{\cos\alpha} = \frac{d}{\cos\alpha} \tag{10}$$

$$NI + BD = \frac{\tan\alpha \cdot d_i \sin(\frac{\theta}{2}) + D_i \sin(\frac{\theta}{2})}{\cos\frac{2\alpha - \theta}{2}} + \frac{D_i \sin\frac{\theta}{2}}{\cos\frac{\theta + 2\alpha}{2}} \tag{11}$$

Get coverage:

$$\eta = 1 - \frac{d}{\text{NI+BD}} = 1 - \frac{d}{\frac{D_2 \sin(\frac{\theta}{2})}{\cos\frac{2\alpha - \theta}{2}} + \frac{D_1 \sin\frac{\theta}{2}}{\cos\frac{\theta + 2\alpha}{2}}} \tag{12}$$

2.2. Overlap rate and coverage width solution

In this paper, taking the center point of the measurement line bureau -800m to 800 as an example, it is known that the opening angle of the multibeam transducer $\theta = 120°$, the slope $\alpha = 1.5°$, the depth of seawater at the center point of the sea D_0 = 70m, according to the different distances of the measurement line from the center point, combined with the relationship between the seawater depth D_i and D_0 derived from the derivation of the above, the relationship between the coverage width W, and the overlap rate of η with the previous article, and the program is written through the MATLAB software to solve the solution, and get the results of the **Table 1** as follows.

Table 1. Overlap rate calculation results.

D/m	Depth of sea /m	Coverage width/m	Overlap rate/%
-800	90.9487	315.8133	///
-600	85.7116	297.6275	0.3470
-400	80.4744	279.4417	0.3058
-200	75.2372	261.2560	0.2590
0	70.0000	243.0702	0.2055
200	64.7628	224.8844	0.1437
400	59.5256	206.6987	0.0714
600	54.2884	188.5129	-0.0142
800	49.0513	170.3271	-0.1172

3. Multifactorial seafloor depth relationships

3.1. Transformation of three-dimensional geometry to two-dimensional planes

In this subsection, a specific angle is first labeled in the given 3D model and this angle is set to be α. Subsequently, this 3D model is transformed into a 2D view like the previous analysis for further study. In this 2D view, the angle between the direction of the survey line and the normal direction of the seafloor slope in the horizontal plane was found, which was labeled as β, i.e., β = 90° - α. The point of intersection with the seafloor directly below the vessel in this scenario was labeled as Point E. The projection of the survey line on the seabed is a line segment, which is labeled as EF, and the projection of the seabed slope normal on the bottom surface is labeled as EG. Through this series of transformations and labels, the foundation is laid for the subsequent mathematical modeling and analysis.

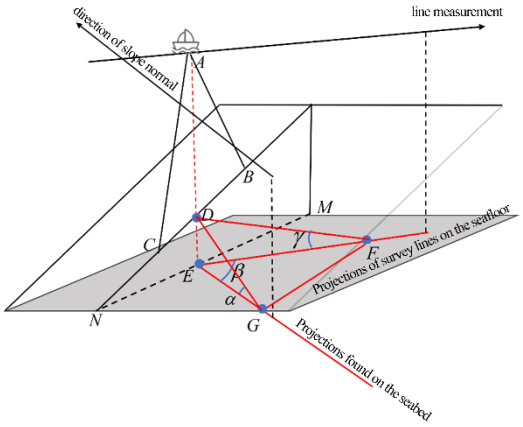


Fig. 3 Position of the three corners under the 3D model

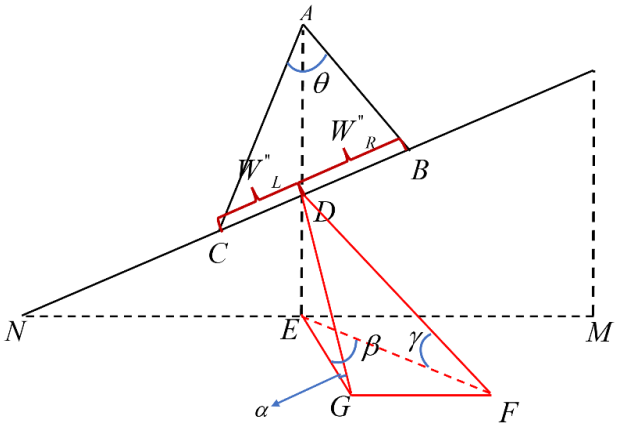


Fig. 4 Positions of 2D plane corners

From planes perpendicular to each other $DE \perp EG$, $DE \perp EF$, by definition:

$$\angle DGE = \alpha, \angle FEG = \beta \tag{1}$$

Suppose $\angle DFE = \gamma$, since point D is the center point of the sea area, $AD = DE$.

$$DE = AD = D \tag{2}$$

Divided from **Fig. 3** and **Fig. 4**, the three triangles are studied separately.

(1) In ΔDEG, $DE=D$, $\angle DGE = \alpha$, $\angle DEG = \frac{\pi}{2}$:

$$EG = \frac{DE}{\tan\alpha} = \frac{D}{\tan\alpha} \tag{3}$$

(2) In ΔEFG, $\angle FEG=\beta$, $FG\perp EG$, $\angle FGE=\frac{\pi}{2}$:

$$EF=\frac{EG}{\cos\beta}=\frac{D\cos\beta}{\tan\alpha} \tag{4}$$

(3) In ΔDEF, $DE=D$, $\angle DFE=\gamma$, $\angle DEF=\frac{\pi}{2}$:

$$\tan\gamma=\frac{DE}{EF}=\frac{\tan\alpha}{\cos\beta} \tag{5}$$

$$\gamma=\arctan\frac{\tan\alpha}{\cos\beta} \tag{6}$$

Based on the analysis in part I, it has been made clear that the width of multibeam coverage is related to several factors, including the depth of the seafloor, the angle to the horizontal formed by the intersection line between the plane perpendicular to the direction of the line of measurement and the seafloor slope, and the opening angle of the transducer. As a result, the following model of multibeam coverage width can be constructed:

$$w=\frac{D\sin(\frac{\theta}{2})}{\cos\frac{2\arctan\frac{\tan\alpha}{\cos\beta}+\theta}{2}}+\frac{D\sin(\frac{\theta}{2})}{\cos\frac{2\arctan\frac{\tan\alpha}{\cos\beta}-\theta}{2}} \tag{7}$$

3.2. Coverage width solution

Again, this paper uses distances from 0 to 2.1 nautical miles as an example, known multi-beam transducer opening angle $\theta=120°$, slope $\alpha=1.5°$, the sea depth at the center point of the sea D_0 = 120m, combined with the previously deduced from the center point of the seabed at different distances from the relationship between the depth of the sea floor can be obtained as follows in **Table 2** of the data. It is obtained by programming using MATLAB software and solved by the model:

Table 2. Calculation of the distance of the measuring vessel from the center of the sea area

Coverage width /m		Distance of the survey vessel from the center of the sea (nautical miles)			
		0	0.3	0.6	0.9
Measuring line direction angle /°	0	416.6918	366.1718	315.6517	265.1316
	45	301.0022	264.5085	228.0147	191.5209
	90	0	0	0	0
	135	301.0022	264.5085	228.0417	191.5209
	180	300.9788	264.4879	227.9970	191.5060
	225	301.0022	264.5085	228.0417	191.5209
	270	0	0	0	0
	315	301.0022	264.5085	228.0147	191.5209
Coverage width /m		Distance of the survey vessel from the center of the sea (nautical miles)			
		1.2	1.5	1.8	2.1
Measuring line direction angle /°	0	214.6116	164.0915	113.5714	63.0514
	45	155.0272	118.5334	82.0397	45.5459
	90	0	0	0	0
	135	155.0272	118.5334	82.0397	45.5459
	180	155.0151	118.5242	82.0333	45.5423
	225	155.0272	118.5334	82.0397	45.5459
	270	0	0	0	0
	315	155.0272	118.5334	82.0397	45.5459

4. Objectives and constraints of optimal coverage

4.1. Objective function for minimum total coverage width

Based on the model above, this paper is designed to design a set of survey lines to minimize the length of the measurements while ensuring complete coverage of the entire sea area to be surveyed. Another constraint is the overlap between neighboring strips, which should be in the range of 10% to 20%. This paper establishes an objective function based on the previous analysis:

$$\min \sum_{i=1}^{n} W'_i \tag{21}$$

Where n is the number of lines.

The constraints are analyzed below:

(1) Constraint 1: Overlap rate η_i.

To ensure ease of measurement and data integrity, the overlap between neighboring bands should be of the order of, where:

$$\eta_i = 1 - \frac{d_i}{W'_i} \tag{22}$$

(2) Constraint 2: water depth D_i.

The relationship between the water depth at the location of each line in a set of survey lines is:

$$D_i = D_0 + d \tan \alpha \tag{23}$$

Where, when I = 0, D0 is the water depth at the center of the sea, d is the distance between adjacent survey lines, and d_j is the distance of each survey line from the center survey line.

(3) Constraint 3: Boundary conditions.

Knowing the north-south length and east-west width of the sea area, the depths at the east-west and north-south boundaries can be calculated from the slope as well as the depth at the center point. Assume that the rectangular sea area is deep in the west and shallow in the east. Let the depths at the east-west and north-south boundaries be D_j, and $D_i \in D_j$, $D_1 < D_2$.

(4) Constraint 4: Coverage width

The survey line requires complete coverage of the entire sea area to be surveyed, and from the conditions given in the question, the width of the entire sea area is 4 nautical miles, i.e., 7,048 meters, so:

$$\sum_{i=1}^{n} W'_i \geq 7408 \tag{24}$$

In summary, the model developed in this paper is as follows:

$$\min \sum_{i=1}^{n} W'_i \tag{25}$$

$$s.t \begin{cases} 10\% \leq \eta_i \leq 20\% \\ D_i = D_{i-1} + d \tan \alpha \\ \sum_{i=1}^{n} W'_i \geq 7408 \\ D_i \in D_j \text{且} D_1 < D_2 \end{cases} \tag{26}$$

4.2. Model Optimization Solution

It is known that the measured rectangular sea area is 2 nautical miles long from north to south and 4 nautical miles wide from east to west, and the depth of seawater at the center point of the sea area is 110 m. The opening angle of the multibeam transducer is 120, and the slope is 1.5. Based on the width and the depth of the center point, the depths at the east-west and north-south boundaries and the widths of coverage can be calculated as shown in **Table 3** below:

Table 3. East-West-South-North Boundary

Frontier	Depth (m)	Coverage width (m)
East	13.0	45.1
West	206.9	718.7
South	61.5	213.5
North	158.5	550.3

Assuming the overlap rate as the key constraint for the solution of this paper, the Python programming traversal algorithm is traversed to solve the total length of the shortest survey line is 7425.7m, which is greater than the width of 4 nautical miles (i.e., 7408m).

For the established optimization model, Python is also used to traverse the solution, and it is concluded that to completely cover the sea area, a total of 206 survey lines are needed, and the distance between each line is 36.05m, and at the same time to meet the coverage rate of between 10% and 20%.

Below is a layout visualization of the survey lines via python [4, 5]. In this visualization, each line represents a survey line, from west to east. Given the relatively large number of lines, only some key lines are presented to simplify the visualization.

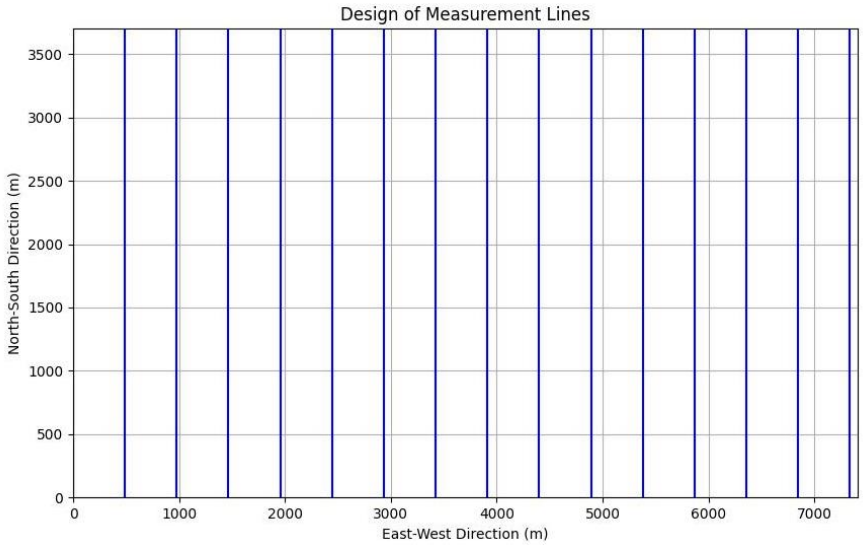


Fig. 5 Visualization of the survey line design

Fig. 5 depicts the design of a survey line. Each straight line in the diagram represents a measurement line running from west to east. Due to the constraints of visualization, this paper only plots 15 survey lines. Through the analysis and calculation, it is ensured that the entire sea area to be measured will be adequately covered, with a coverage rate ranging from 10% to 20%.

5. Summary

The model constructed in this paper is based on multibeam bathymetry technology, which has the advantage of high working efficiency and dense data collection points and is suitable for the accurate acquisition of seabed topography and bathymetry information. The model can realize multi-angle detection in a three-dimensional application scenario and considers the influence of the seafloor slope

on the spacing of survey lines, so that the coverage width and overlap rate can be calculated accurately. Although there are limitations in the assumptions about the direction of the survey line intersecting with the seafloor slope constituting an oblique line, overall, the model is not only applicable to specific research scenarios, but also has strong generalizability. Future model improvements could consider more seafloor water conditions and environmental factors to achieve more comprehensive and accurate bathymetric results.

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