# Sliding mode variable structure control of RP manipulator based on computational torque

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**Abstract.** Study the control problem of a coupled robotic arm with moving and rotating joints, a PID control method based on calculated torque is proposed. Considering that the robot model is difficult to obtain accurately, only its estimation model can be obtained. The output feedback signal is used to define the sliding mode surface function with error signal. Sliding mode variable structure controller(SMVSC) is designed based on the computational torque control method. Taking the mathematical model of RP robot as an example, Experimental calculations have found that the control accuracy can be guaranteed, but there is a significant chattering situation. In engineering practice, it is necessary to process the symbol function.

**Keywords:** RP manipulators; PID controller; SMVSC; chattering; robust controller.

## 1. Introduction

In recent years, due to the rapid development of scientific and technological level, the development, production and use of robots are in full swing all over the world. It not only represents a country's comprehensive national strength [1]-[2]. China is in the transition stage from a "traditional manufacturing country" to an "emerging manufacturing country". Improving the localization share and technology of robots is the key period to achieve "made in China 2025" [3].

RP (rotation-parallel) industrial robot is one of the key equipment in the manufacturing industry. Because it has the advantages of programmable logic, personification, versatility, direct and simple action path, it is widely used in the fields of acquisition, welding, painting, product packaging and so on [4].

As a common mechanical and electromagnetic coupling system [5], RP robot has strong coupling effect, which is further strengthened by the existence of transmission structure. As a high-speed light component, the electromechanical coupling effect of flexible manipulator has a significant impact on the analysis of its vibration characteristics, so the coupling factors within the mechanical system should be fully considered. The whole mechanical system has the interaction between the driving motor and mechanical parameters, the interaction between electromagnetism and transmission system, and there are also coupling factors such as the interaction between electromagnetism and end load in the system.

## 2. Dynamic modeling of RP robot

For the RP manipulator shown in Fig.1, the base center is defined as A, the rod 1 rotates on the fixed axis, the center of mass is B, the rod length is $l_1$, the mass is $m_1$, and the inertia is $I_1$. The rod 2 moves along the direction of the rod 1. The center of mass is D, the rod length is $l_2$, the mass is $m_2$, the inertia is $I_2$, and the distance to the rotation center of the matrix is $r$. Joint 1 is a rotating joint, and the output $T$ is torque; Rod 2 is a moving joint and its output $F$ is force.

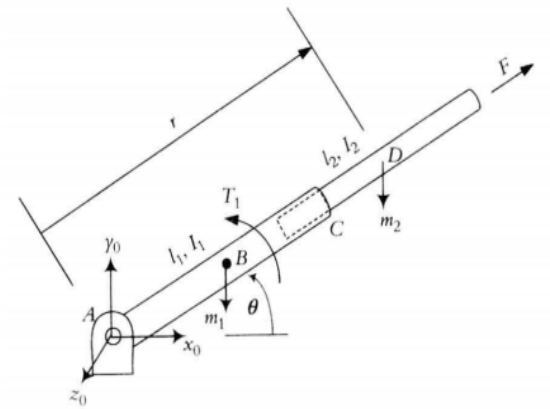



**Fig 1.** RP (rotation-parallel) manipulator.

$E_k$ Is defined as the energy of RP robot, $K_1$ and $K_2$ are the kinetic energy of RP (rotation-parallel) manipulator 1 and RP (rotation-parallel) manipulator 2, respectively, and $\dot{\theta}$ is the joint rotation speed of RP (rotation-parallel) manipulator 1.

$$E_k = K_1 + K_2 \tag{1}$$

Then the kinetic energy of rod 1 is

$$\begin{aligned} K_1 &= \frac{1}{2} I_{1,A} \dot{\theta}^2 \\ &= \frac{1}{2} (\frac{1}{3} m_1 l_1^2) \dot{\theta}^2 \\ &= \frac{1}{6} m_1 l_1^2 \dot{\theta}^2 \end{aligned} \tag{2}$$

Establish the coordinate system as shown in the figure, and make $c\theta = \cos\theta$, $s\theta = \sin\theta$. $x_D$ Is the coordinate of the centroid point of rod 2 on the coordinate axis X, and is the coordinate of the centroid point of rod 2 on the coordinate axis Y.

$$x_D = rc\theta \rightarrow \dot{x}_D = \dot{r}c\theta - r\dot{\theta}s\theta \tag{3}$$

$$y_D = rs\theta \rightarrow \dot{y}_D = \dot{r}s\theta + r\dot{\theta}c\theta \tag{4}$$

$$v_D^2 = \dot{r}^2 + r^2\dot{\theta}^2 \tag{5}$$

Then the kinetic energy of rod 2 is

$$\begin{aligned} K_2 &= \frac{1}{2} I_{2,D} \dot{\theta}^2 + \frac{1}{2} m_2 v_D^2 \\ &= \frac{1}{2} (\frac{1}{12} m_2 l_2^2) \dot{\theta}^2 + \frac{1}{2} m_2 (\dot{r}^2 + r^2 \dot{\theta}^2) \end{aligned} \tag{6}$$

Then the total kinetic energy is

$$E_k = \left( \frac{1}{6} m_1 l_1^2 + \frac{1}{24} m_2 l_2^2 + \frac{1}{2} m_2 r^2 \right) \dot{\theta}^2 + \frac{1}{2} m_2 \dot{r}^2 \tag{7}$$

Gravitational potential energy is

$$E_p = m_1 g \frac{l_1}{2} s\theta + m_2 g r s\theta \tag{8}$$

Then

$$\begin{aligned} L &= E_k - E_p \\ &= \left(\frac{1}{6}m_1 l_1^2 + \frac{1}{24}m_2 l_2^2 + \frac{1}{2}m_2 r^2\right)\dot{\theta}^2 + \frac{1}{2}m_2 \dot{r}^2 \\ &\quad -(m_1 g\frac{l_1}{2} + m_2 g r)s\theta \end{aligned} \tag{9}$$

Substituting into Lagrange equation

$$\begin{cases} T = \frac{\mathrm{d}}{\mathrm{d}t}(\frac{\partial L}{\partial \dot{\theta}}) - \frac{\partial L}{\partial \theta} \\ F = \frac{\mathrm{d}}{\mathrm{d}t}(\frac{\partial L}{\partial \dot{r}}) - \frac{\partial L}{\partial r} \end{cases} \tag{10}$$

Then

$$\tau = M\ddot{q} + C\dot{q} + G \tag{11}$$

Where

$\dot{q} = \begin{bmatrix} \dot{\theta} \\ \dot{r} \end{bmatrix}$, $M = \begin{bmatrix} \frac{1}{3}m_1 l_1^2 + \frac{1}{12}m_2 l_2^2 + m_2 r^2 & 0 \\ 0 & m_2 \end{bmatrix}$, $C = \begin{bmatrix} m_2 r\dot{r} & m_2 r\dot{\theta} \\ -m_2 r & 0 \end{bmatrix}$, $G = \begin{bmatrix} (m_1 g\frac{l_1}{2} + m_2 g r)c\theta \\ m_2 g s\theta \end{bmatrix}$, $\tau = \begin{bmatrix} T \\ F \end{bmatrix}$.

## 3. SMVS control based on computational torque method

Considering that the actual system can lead to model uncertainty due to various uncertainties, SMVS control system design can be adopted, and the control law is

$$\tau = \hat{M}v + \hat{C}\dot{q} + \hat{G} \tag{12}$$

Where $\hat{M}, \hat{C}$ and $\hat{G}$ are estimate of $M, C$ and $G$. $v$ Is sliding mode control term.

The system equation becomes

$$M\ddot{q} + C\dot{q} + G = \hat{M}v + \hat{C}\dot{q} + \hat{G} \tag{13}$$

Then

$$\begin{aligned} M\ddot{q} &= \hat{M}v + \hat{C}\dot{q} - C\dot{q} + G - G \\ &= \hat{M}v - \tilde{C}\dot{q} - \tilde{G} \end{aligned} \tag{14}$$

Where $\tilde{M} = M - \hat{M}$, $\tilde{C} = C - \hat{C}$, $\tilde{G} = G - \hat{G}$

Subtracting $\tilde{M}\ddot{g}$ in the two sides of the above formula respectively, get

$$\begin{aligned} \hat{M}\ddot{q} &= \hat{M}v - [\tilde{M}\ddot{q} + \tilde{C}\dot{q} + \tilde{G}] \\ &= \hat{M}v - Y(q, \dot{q}, \ddot{q})\tilde{p} \end{aligned} \tag{15}$$

Where $\tilde{p} = p - \hat{p}$.

If $\hat{p}$ makes $\hat{M}(q)$ reversible, the system equation (17) can be written:

$$\begin{aligned}\ddot{q} &= v-[\hat{M}(q)]^{-1}Y(q,\dot{q},\ddot{q})\tilde{p} \\ &= v-\varphi(q,\dot{q},\ddot{q},\hat{p})\tilde{p}\end{aligned} \tag{16}$$

If

$$\varphi(q,\dot{q},\ddot{q},\hat{p})\tilde{p}=\tilde{d} \tag{17}$$

Where $\tilde{d}=[\tilde{d}_1,\cdots,\tilde{d}_n]^T, d=[d_1,\ldots,d_n]^T$

If $e=q_d-q$

Then sliding mode function can be

$$s=\dot{e}+\Lambda e \tag{18}$$

So $\dot{e}=\dot{q}_d-\dot{q}, s=[s_1,\cdots,s_n]^T, \Lambda$ is a positive definite angle matrix.

Because

$$\dot{s}=\ddot{e}+\Lambda\dot{e} \tag{19}$$

$$\begin{aligned}\dot{s} &= (\ddot{q}_d-\ddot{q})+\Lambda\dot{e} \\ &= \ddot{q}_d-v+\tilde{d}+\Lambda\dot{e}\end{aligned} \tag{20}$$

Variable structure items are designed as

$$v=\ddot{q}_d+\Lambda\dot{e}+d \tag{21}$$

Where $d$ is a robust term that needs to be designed through the sliding mode principle?

Then

$$\dot{s}=\tilde{d}-d \tag{22}$$

Selecting

$$d=(\bar{d}+\eta)\,\mathrm{sgn}(s) \tag{23}$$

Where $\left\|\tilde{d}\right\|\le\bar{d}, \eta>0$

**Theorem:** for the RP robot system (11), the stability of the control system of RP (rotation-parallel) industrial robot can be guaranteed by using the computed torque controller (12) and SMVC (21).

Proof**:**

Defining Lyapunov function:

$$v(t)=\frac{1}{2}s^Ts \tag{24}$$

Then

$$\dot{v}(t)=\dot{s}s \tag{25}$$

Substituting formula (22) into (25)

$$\begin{aligned}\dot{v}(t) &= (\tilde{d}-d)s \\ &= \tilde{d}s-\bar{d}\,\mathrm{sgn}(s)s-\eta\,\mathrm{sgn}(s)s \\ &\le -\eta|s| \\ &\le 0\end{aligned} \tag{26}$$

According to LaSalle's invariance principle, we can get when $\dot{V}\equiv 0$, so when $t\to\infty$, $s\to\infty$.

## 4. Finite Element Solution and Analysis

RP (rotation-parallel) industrial robot is simulated by MATLAB. $l_1 = 1.5m$, $m_1 = 4.0kg$, $l_2 = 1.5m$, $m_2 = 1.0kg$.

The objective function for robots to achieve two joint position tracking is $\theta_d = \sin(2\pi t)$, $r = \sin(2\pi t)$

The designed controller parameters are $k_d = \begin{pmatrix} 100 & 0 \\ 0 & 100 \end{pmatrix}$, $\Lambda = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$.

Through numerical simulation, experimental research was conducted on the joint position and torque of the robot, and the following results were obtained:

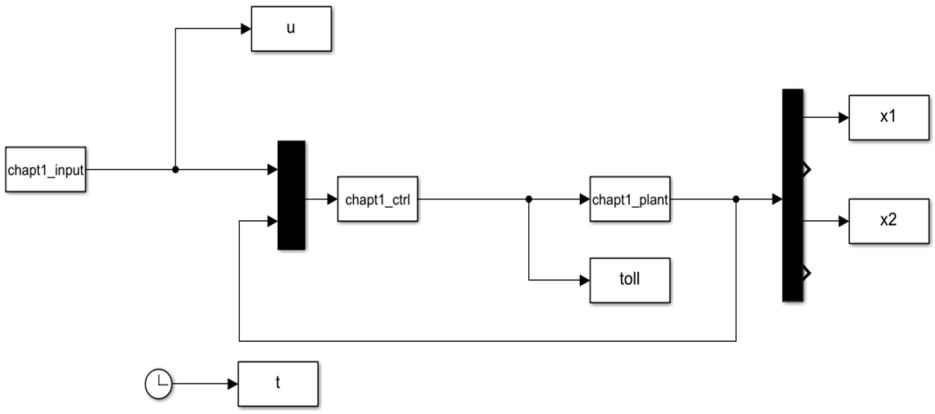



**Fig 2.** simulation block diagram.

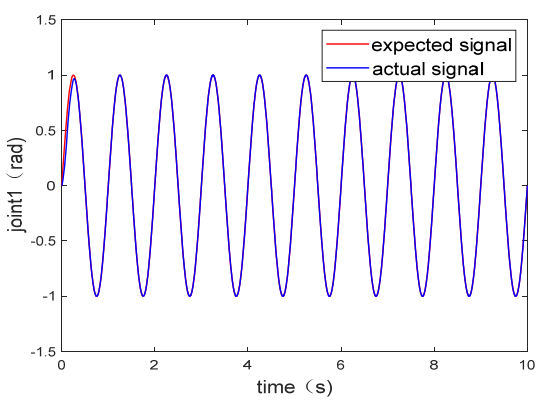



**Fig 3.** Angular position tracking track of joint 1.

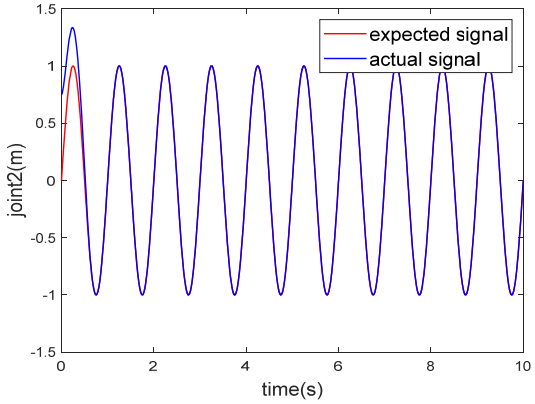



**Fig 4.** Position tracking track of joint 2.

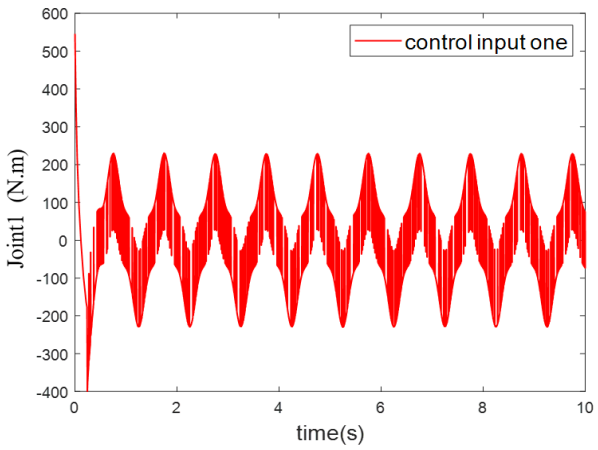



**Fig 5.** Control input of joint 1.

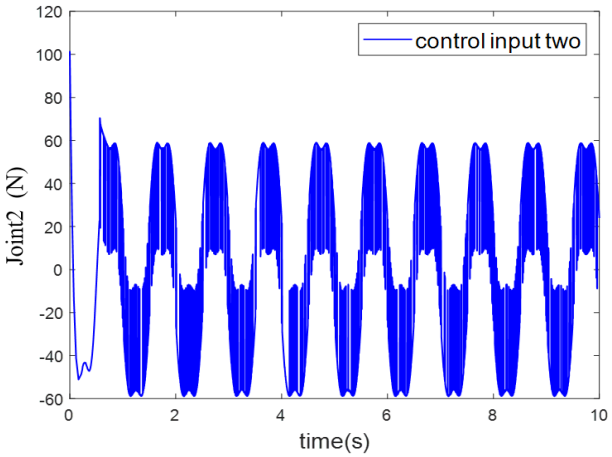



**Fig 6.** Control input of joint 2.

From the trajectory tracking control effect of joint 1 and joint 2 (Figure 3- Figure 4), the control effect is good, with an error of 0.5 and a stabilization time of approximately 1 second. There is chattering in the control torque, which is a control characteristic of sliding mode variable structure. In the future, saturation functions can be used instead of symbolic. The motor output torque of joint 1 is relatively high, at 250N.m, while the motor torque of joint 2 is 60N.m.

## 5. Conclusions

Because RP robot manipulator has a good application engineering scene, a PID method based on calculated torque is proposed. Because the precise mathematical model of the robot is difficult to obtain, using its estimation model, a SMVSC based on calculated torque is designed, taking the mathematical model of RP robot as an example, the simulation verification is carried out. The experimental calculation results show that both rotating joints and moving joints can achieve good tracking effect, and there is a certain "chattering" problem in the torque and force output of the joint, which is caused by the use of the symbolic function in the sliding mode term. Next it is necessary to eliminate the chattering of the control torque output.

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