

Multi-Beam Line Measurement Model Based on Analytic Geometry

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Abstract. The multi-beam sounding system has a wider coverage area and efficiency than the single-beam sounding system, and has been used in a variety of offshore operations. However, the design of flat sea area survey method can not meet the real seabed relief, so it is of great significance to study a multi-beam sounding system line planning model for different seabed topography to improve the measurement efficiency and accuracy in sea area exploration and other applications. In this paper, under the condition of real terrain, the linear model of the Angle of the fixed multi-beam transducer is studied with appropriate simplification. Based on the analytic geometric relationship, the functional expressions between the coverage area and overlap rate and the parameters of the transducer's open Angle, slope Angle and sea water depth are given, and the parameter model of the submarine landform linear model with a certain slope is established.

Keywords: Analytic Geometry, Universal Model, Multi-beam Sounding System.

1. Introduction

Multi-beam sounding system is a kind of acoustic equipment with high efficiency, [1] high precision and high resolution, which has been widely used in seabed topography mapping. [2] In a flat sea area, compared with the single-beam sounding system, which has dense data along the track and no data characteristics between the survey lines, the multi-beam sounding system can measure a full-coverage depth strip with the survey ship's survey line as the axis and a certain width. [3] However, the topography of the real seabed is fluctuating, and if the average depth is used to design the survey line interval, the requirements of measurement efficiency and measurement range cannot be taken into account at the same time. [4] That is, when the overlap rate is constant, the shallow water surface area is prone to be unable to cover by sound waves. When the regions at all depths are covered, the overlapping rate of the deeper regions is larger and the data repetition rate is higher. [5] Therefore, it is of great significance to study an efficient measurement model for different submarine topographic structures. Xavier models the sound field radiated by a multi-beam echo sounder for acoustic impact assessment; [6] Zhao studies directional beam emission for configurable compact multi-source systems; [7] Dinn studied the effect of sound velocity error on the depth accuracy of multi-beam sonar. [8] Lanzoni et al. studied the high-resolution calibration of multi-beam echo sounders; [9] For the convenience of the research, this paper takes the parallel survey line set as the research object. [10] Based on the above background, it establishes the conditions that the opening Angle of the transducer is 120° , the given slope and the sea water depth at the center point of the sea area. The first part calculates the relationship between the coverage width and overlap rate and the position coordinates. In the second part, the relationship between the coverage width and the Angle of projection on the horizontal plane of the survey line and the normal direction of the submarine slope is calculated, and finally the universal model is given.

2. The basic funamental of the multi-beam sounding system model

2.1. Establishment of coordinate system

First, a two-dimensional coordinate system is established. The research plane is defined as the plane perpendicular to the line of sight and passing through the center point of the multi-beam transducer, looking at the ship along the survey line, as shown in Figure 1. The origin and axis of the coordinates are defined as follows: along the vertical direction of the center point, at the depth of 70 m, the position intersecting with the submarine slope is taken as the origin of the coordinates, with the right as the positive direction of the U-axis and the upward as the positive direction of the Y-axis, and a two-dimensional rectangular coordinate system is established, as shown in Figure 2.

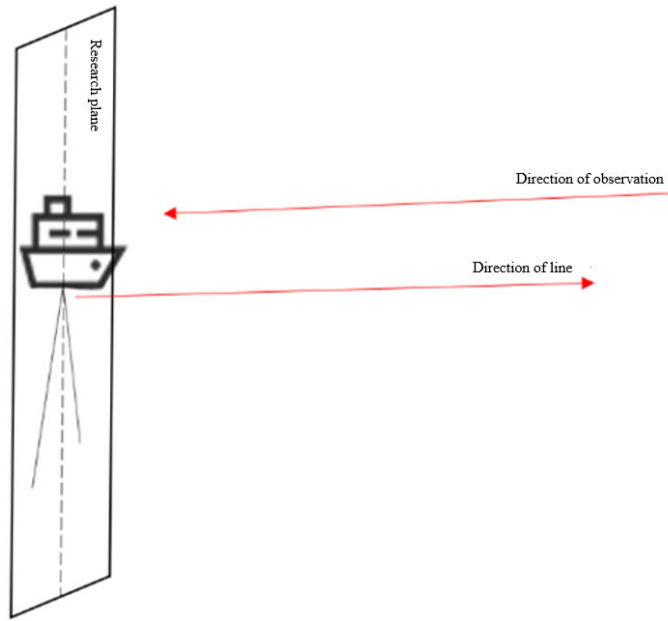


Figure 1. Observation direction and study plane definition diagram

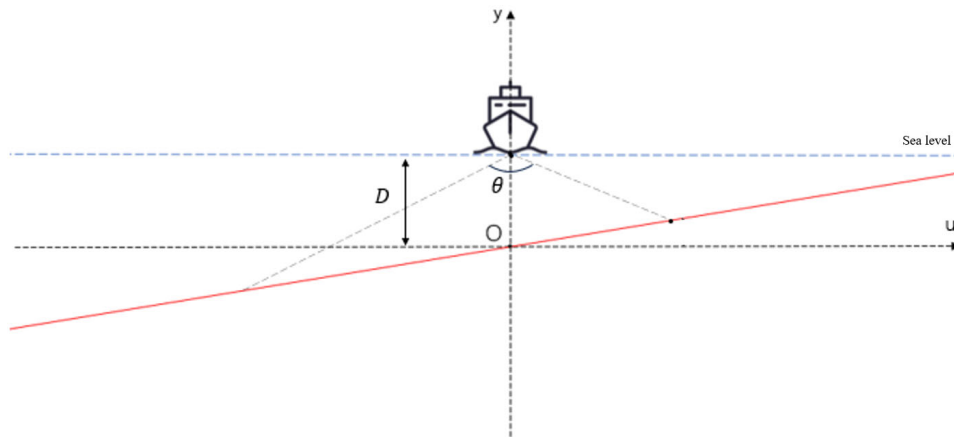


Figure 2. Two-dimensional rectangular coordinate system defined in the study plane

2.2. Definition of parameters

Set D_0 as the sea water depth corresponding to the central point when $u = 0$; Let W be the corresponding coverage width at a measurement point; Let D be the sea water depth along the plumb line at a measurement point; The opening Angle of the multi-beam sounding transducer is set to θ . α_0 is assumed to be the dihedral Angle between the seabed slope and the horizontal plane, while A and B are respectively the intersection point between the farthest beam of the sound wave emitted by multi-beam sounding and the slope, as shown in Figure 3.

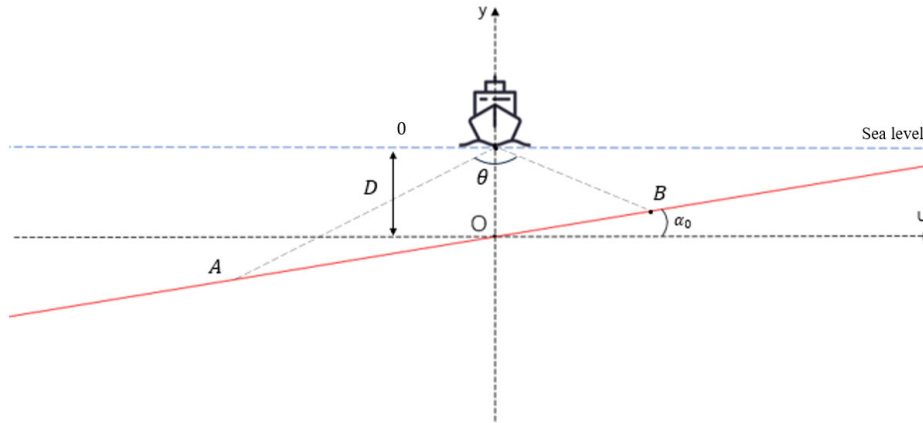


Figure 3. Two-dimensional rectangular coordinate system defined in the study plane

2.3. Establishment of Covering width model

On the intersection line between the sea level and the observation plane, take any position coordinate $P(u, y)$ and two intersection points A and B , and record the intersection line between the slope and the observation plane as l_{AB} , and make a vertical line to the horizontal plane through B , and the intersection plane is at point Q . Perpendicular to AD through P and cross AQ at point C , as shown in Figure 4.

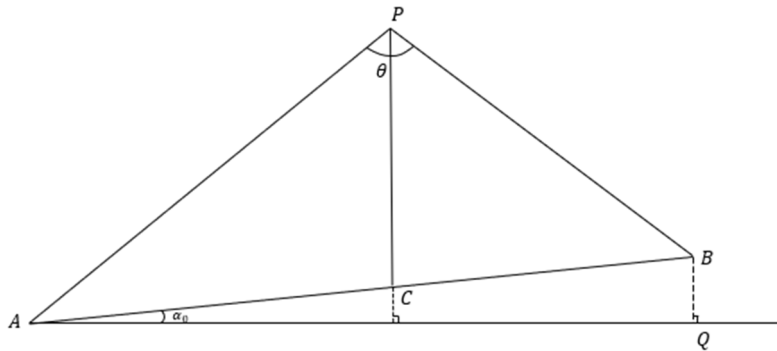


Figure 4. Geometric representations of slopes, levels and triangles covered by sound waves

In $\triangle ACP$, by the sine theorem, there is:

$$\frac{PC}{\sin \angle PAC} = \frac{AC}{\sin \frac{\theta}{2}} \quad (1)$$

In $\triangle BCP$, by the sine theorem, there is:

$$\frac{PC}{\sin \angle PBC} = \frac{BC}{\sin \frac{\theta}{2}} \quad (2)$$

Therefore:

$$AB = AC + CB = PC \cdot \left(\frac{1}{\sin \angle PAC} + \frac{1}{\sin \angle PBC} \right) \cdot \sin \frac{\theta}{2} \quad (3)$$

By the Angle relationship, there is:

$$\angle PAC = 90^\circ - \frac{\theta}{2} - \alpha_0 \quad (4)$$

$$\angle PBC = 90^\circ - \frac{\theta}{2} + \alpha_0 \quad (5)$$

From the relationship between depth and coordinates, there is:

$$PC = D = D_0 - y \quad (6)$$

For any point (u, y) on the line l_{AB} , there is:

$$y = u \cdot \tan \alpha_0 \quad (7)$$

By combining formula (7) and formula (8), the relationship between sea water depth and horizontal coordinate can be obtained as follows:

$$D = D_0 - u \cdot \tan \alpha_0 \quad (8)$$

Substitute formula (1), formula (2), formula (4), formula (5), formula (6), formula (7), formula (8), formula (9) into formula (3), there is:

$$AB = (D_0 - u \cdot \tan \alpha_0) \cdot \left(\frac{1}{\sin(90^\circ - \frac{\theta}{2} - \alpha_0)} + \frac{1}{\sin(90^\circ - \frac{\theta}{2} + \alpha_0)} \right) \cdot \sin \frac{\theta}{2} \quad (9)$$

Simplify the formula (9), and there is:

$$AB = (D_0 - u \cdot \tan \alpha_0) \cdot \sin \frac{\theta}{2} \cdot \left(\frac{\cos(\frac{\theta}{2} + \alpha_0) + \cos(\frac{\theta}{2} - \alpha_0)}{\cos(\frac{\theta}{2} - \alpha_0) \cdot \cos(\frac{\theta}{2} + \alpha_0)} \right) \quad (10)$$

By Prost aphaeresis formulas, there is:

$$AB = 2(D_0 - u \cdot \tan \alpha_0) \cdot \frac{\sin \theta \cos \alpha_0}{\cos \theta + \cos 2\alpha_0} \quad (11)$$

The relation between the coverage width of this position and the lateral movement distance is:

$$W(u) = AQ = AB \cdot \cos \alpha_0 = 2(D_0 - u \cdot \tan \alpha_0) \cdot \frac{\sin \theta \cos^2 \alpha_0}{\cos \theta + \cos 2\alpha_0} \quad (12)$$

Including:

$$\alpha_0 = 1.5^\circ \quad (13)$$

$$D_0 = 70m \quad (14)$$

Formula (13) is the mathematical model of the coverage width of multi-beam sounding obtained in the problem.

2.4. Establishment of Overlapping ratio model

Under the premise above, let d be the vertical distance of two parallel measurement lines, take the geometric representation composed of the coordinates $P_i(u_i, y_i)$, $P_{i+1}(u_{i+1}, y_{i+1})$ of the two adjacent positions in the table, and record the intersection point of $B_i Q_i$ and $A_{i+1} Q_{i+1}$ as B_i' . For the seabed with slope, the restatement overlap rate is defined as follows: For the i and $i + 1$ positions, the overlap rate is equal to the ratio of the projection along the horizontal plane of the farthest beam of the emitted sound wave at the intersection of the slope and the overlapping part of the l_{AB} measured line segment to the coverage width of the position, that is:

$$\eta = \frac{A_{i+1} B_i'}{W(u_{i+1})} \quad (15)$$

As shown in Figure 5.

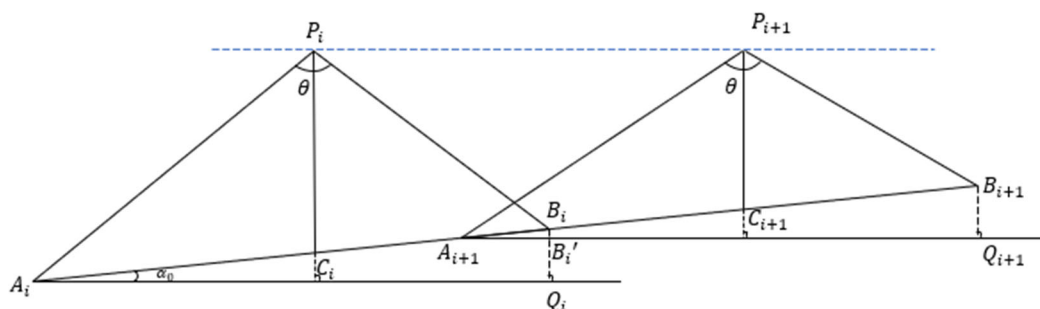


Figure 5. Geometric representation of two adjacent locations

For the two points shown in Figure 5 $P_i(u_i, y_i)$ and $P_{i+1}(u_{i+1}, y_{i+1})$, the covering width should be satisfied:

$$\eta = \frac{(A_{i+1}C_{i+1} + B_iC_i) \cdot \cos\alpha_0 - A_{i+1}Q_{i+1}}{W(u_{i+1})} \quad (16)$$

Including:

$$d = A_{i+1}Q_{i+1} \quad (17)$$

For the two points shown in Figure 5 $P_i(u_i, y_i)$ and $P_{i+1}(u_{i+1}, y_{i+1})$, the corresponding overlap rate should be satisfied:

$$W(u_i) = 2(D_0 - u_i \cdot \tan\alpha_0) \cdot \frac{\sin\theta \cos^2\alpha_0}{\cos\theta + \cos 2\alpha_0} \quad (18)$$

$$W(u_{i+1}) = 2(D_0 - u_{i+1} \cdot \tan\alpha_0) \cdot \frac{\sin\theta \cos^2\alpha_0}{\cos\theta + \cos 2\alpha_0} \quad (19)$$

As shown in Figure 5 and formula (1), formula (2), formula (4), formula (5), formula (6), there is:

$$A_{i+1}C_{i+1} \cdot \cos\alpha_0 = \frac{\cos(\frac{\theta}{2} - \alpha_0)}{2\cos\frac{\theta}{2}\cos\alpha_0} \cdot W(u_{i+1}) \quad (20)$$

$$B_iC_i \cdot \cos\alpha_0 = \frac{\cos(\frac{\theta}{2} + \alpha_0)}{2\cos\frac{\theta}{2}\cos\alpha_0} \cdot W(u_i) \quad (21)$$

Substitute formula (18), formula (19), formula (20), formula (21), formula (22) into formula (17), there is:

$$\eta = \frac{\frac{\cos(\frac{\theta}{2} - \alpha_0)}{2\cos\frac{\theta}{2}\cos\alpha_0} \cdot W(u_{i+1}) + \frac{\cos(\frac{\theta}{2} + \alpha_0)}{2\cos\frac{\theta}{2}\cos\alpha_0} \cdot W(u_i) - d}{W(u_{i+1})} \quad (22)$$

According to the practical significance of overlap rate, order:

$$d = u_{i+1} - u_i \quad (23)$$

Substitute the formula (24) into the formula (23) to simplify, and there is:

$$\eta(u_i, u_{i+1}) = \frac{\cos(\frac{\theta}{2} + \alpha_0) \frac{D_0 - u_i \tan\alpha_0}{D_0 - u_{i+1} \tan\alpha_0} - \frac{(u_{i+1} - u_i)(\cos\theta + \cos 2\alpha_0)}{2(D_0 - u_{i+1} \tan\alpha_0) \sin\theta \cos^2\alpha_0} + \cos(\frac{\theta}{2} - \alpha_0)}{2\cos\frac{\theta}{2}\cos\alpha_0} \quad (24)$$

Including:

$$\alpha_0 = 1.5^\circ \quad (25)$$

Formula (25) is the model of the overlap rate of multi-beam sounding.

3. Results

3.1. The establishment of text model

The result data of the coverage width model and overlap rate model is implemented in the MATLAB software.

For the results of formulas (13) and (25), the following data are given for validation, as shown in Table 1:

Table 1. Test data of the model

Measure the distance of the line from the center point /m		Position i								
		1	2	3	4	5	6	7	8	9
position coordinates	u_i	-800	-600	-400	-200	0	200	400	600	800

3.2. Analysis of text results

The results are shown in Table 2:

Table 2. Result data of the model

Measure the distance between the line and the center point /m	-800	-600	-400	-200	0	200	400	600	800
Sea water depth /m	90.95	85.71	80.47	75.24	70	64.76	59.53	54.29	49.05
Coverage width /m	315.71	297.53	279.35	261.17	242.99	224.81	206.63	188.45	170.27
The overlap rate with the previous line /%	——	35.70	31.51	26.74	21.26	14.89	7.41	-1.53	-12.37

The error between the results and the standard results is less than 1%. The model established in this paper is considered to be good.

4. Conclusion

In order to solve the problem of line planning in multi-beam sounding system, the functional relationship between the coverage width and overlap ratio and the parameters of the transducer's open Angle, slope Angle, initial seawater depth and position is studied, and a universal model is established. Firstly, reasonable assumptions are made to simplify and optimize the problem, improve the universality of the scheme and reduce the complexity of the calculation. Secondly, a mathematical model with the depth and location of seawater as parameters and the coverage width and overlap rate as functions is established. The method of analytic geometry and mathematical analysis is used to solve the function model, and a certain amount of data is substituted for testing. The error between the result and the expected situation is small, so the model is considered to be universal and reliable. The research results of this paper can be used in the path planning work of Marine exploration, search and rescue, hydrological survey and other tasks. The solution of certain variables through this model can optimize the efficiency of path planning, save time, and provide certain reference value for the simulation and calculation of path planning problems.

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