Algorithmic Composition of Music Utilizing the Digits of Pi

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Abstract. As a matter of fact, algorithmic composition, the fusion of computational methods with music creation, has a rich history dating back to Pythagoras. On this basis, this paper explores the innovative intersection of mathematics, algorithms, and music, focusing on the translation of Pi's digits into a musical composition. Through a combination of manual and automated techniques, the research successfully crafts a melody using the minor pentatonic scale and traditional Chinese instruments, enriched by an algorithmically generated counter-melody. According to the analysis, the study reveals both the potential and limitations of this approach, with the fixed nature of the accompaniment and strict adherence to the pentatonic scale constraining musical diversity. Nevertheless, the research offers a novel perspective on mathematical representation in music, contributing to the broader field of algorithmic music composition. The findings open new avenues for personalized music creation, educational tools, and artistic exploration, highlighting the limitless possibilities when art and science converge.

Keywords: algorithmic composition, the Digits of Pi, countermelody, manual and automated technique.

1. Introduction

Algorithmic composition, the application of computational methods in music creation, has a rich and multifaceted history. Its roots can be traced back to the mathematical relationships in music documented by Pythagoras, leading to a structured approach to music composition [1]. During the Middle Ages, Guido d'Arezzo's work in formalizing music notation laid the groundwork for future algorithmic techniques. This was followed by the Renaissance and Baroque periods, where contrapuntal forms and quasi-algorithmic techniques like isorhythm flourished. Notably, composers like Mozart began to experiment with formalized algorithmic composition, such as using dice to create waltzes [1, 2]. The 20th century marked a turning point with the emergence of new compositional methods, including serial composition, atonality, and mathematical and probabilistic approaches. The creation of computer-assisted compositions, such as "The ILLIAC Suite," represented a significant advancement in the field [1]. In contemporary times, algorithmic composition has expanded to include various methods, ranging from traditional compositional procedures to extra-musical algorithms [3]. The integration of AI in music composition has led to both opportunities and challenges, sparking debates about creativity, authenticity, and the future role of human artists [4, 5]. Overall, the history of algorithmic composition reveals a complex interplay between mathematical principles, technological advancements, and artistic creativity. From ancient mathematical models to modern AI-driven techniques, this field has continually evolved, shaping musical forms and challenging traditional notions of creativity. Its ongoing exploration not only enriches the musical landscape but also opens new avenues for research, innovation, and philosophical inquiry [1, 2].

The intersection of mathematics, algorithm, and music has led to interesting and innovative approaches in music composition. Creative compositions such as Mitchell Moffit and Gregory Brown's "The Pi Song" and David Macdonald's "Song From π" exemplify this fusion. Meanwhile, academic works have explored the application of Markov Chains and LSTM Recurrent Neural Networks in this domain. "The Pi Song" is an interesting piece designed to facilitate the memorization of the first 100 digits of π [6]. Through a cheerful and humorous melody, the song translates the π into a singable format, where the lyrics contain the digits of π. "Song From π" uses the digits of pi to construct the melody in a pure piano composition. Each digit of π is assigned to a specific note on the

harmonic minor scale, creating a soft and beautiful song filled with harmonies of left hand [7]. Although the two songs related to π are manually composed, it is worth noting that algorithmic composition is becoming popular and powerful. For example, Markov chains present a mathematical framework for music composition, particularly in rhythm. These chains model sequences where each event's probability depends solely on the preceding one. The structure can be defined by different orders, allowing for complexities in modeling, represented by an N + 1-dimensional state-transition matrix where N is the order of the chain [8]. Moreover, Long Short-Term Memory (LSTM) recurrent neural networks mark a significant direction in machine learning for music composition. LSTM emphasizes constant error flow, utilizing Constant Error Carousels (CECs) to negate challenges faced by traditional RNNs. Its architecture, including distinct input, output, and forget gates, makes LSTM effective in maintaining global musical structure [9, 10]. Nevertheless, both manually composed songs and algorithmic compositions have their constraints. "The Pi Song"'s connection to π is manifested solely through the lyrics, without mathematical correlation to the melody. Although "Song from π" has resolved this issue, its slow tempo limits the number of π's digits that can be represented, failing to fully reveal the randomness and infinity nature of π, an irrational number whose digits do not follow any discernible pattern. Markov chain's core issue lies in its random nature of melody generation, where each note's probability depends only on the preceding note. This can lead to sequences lacking thematic coherence or intention, resulting in compositions that may be harmonious but feel disconnected and artistically hollow. The major shortcoming in using the LSTM model for composition is its lack of individuality or unique voice. Since it learns from existing works, it may only replicate patterns, lacking distinctiveness and originality, and falling short in reflecting a composer's individual artistic vision. Accordingly, these works reveal an exciting intersection of math, algorithm, and music and also point to the potential for further exploration and refinement in this field.

Imagine expressing the mathematical complexity of Pi through the melody of music, exploring the hidden beauty of mathematics from a new perspective. This project embarks on a creative journey to manually and algorithmically compose a musical piece using the digits of Pi, employing the minor pentatonic scale and traditional Chinese instruments. Infused with Chinese characteristics, this composition seeks to bridge the gap between mathematics and music, capturing the intrigue of Pi in a uniquely Chinese musical context. The motivation for this endeavor stems from an adventurous curiosity to overcome the challenge of translating Pi into an aesthetically pleasing melody, offering a fresh insight into the artistic expression of mathematical concepts. The framework of this paper is structured to provide a comprehensive exploration of the topic. As for Section 2, the focus will be on the data and algorithms used in the study, including an in-depth look at the specific software, libraries, and parameters employed. This section will lay the groundwork for understanding the technical aspects of the research. Section 3 will delve into the core findings of the study. It will include a detailed analysis of the composition of the pi song and the development of countermelody. This section will be supported by more than three charts or graphs, providing a visual representation of the research outcomes. Section 4 will address the constraints of the current study and provide a perspective on potential future research directions. This section will offer a balanced view of the research, acknowledging its boundaries and suggesting areas for further exploration and innovation. Finally, Section 5 will encapsulate the key findings, limitations, future prospects, and overall significance of the study. It will provide a concise summary that ties together the various threads of the research, highlighting its contributions to the fields of mathematics, computer science, and music. Through this structured approach, the paper aims to contribute to the growing body of knowledge at the nexus of mathematics and music, offering novel insights and methodologies that may inspire further research and artistic experimentation. The exploration of π as a source of musical composition is not only a mathematical curiosity but also a testament to the limitless possibilities of human creativity when art and science converge.

2. Data And Method

2.1. Manually composition part

Converting Pi's Digits to Music Notes: The first 1000 digits of Pi were converted into musical notes using a number-to-midi-number mapping, which maps the numbers 0 to 9 to midi numbers 61 to 82. Utilizing the pentatonic scale, this mapping ensures that all the musical notes derived from Pi's digits will correspond to C sharp, D sharp, F sharp, G sharp, and A sharp. Finding the right key to represent the endless, non-repeating digits of Pi involved experimentation with major, minor, and pentatonic scales. The melodies generated using major and minor keys resulted in a continuously changing and discordant sound. Surprisingly, the pentatonic scale aligned well with Pi's characteristics; the melody played on this scale maintained a pleasing sound regardless of the note sequence. As a result, the pentatonic scale was selected as the final scale, providing a harmonious melody that connects mathematics with music. Through extensive experimentation, it became clear that a steady looping rhythm would be more effective than a dynamic one. This choice was necessitated by the fact that the melody, based on Pi, is fixed and cannot be altered according to our whims, unlike in conventional music composition. Consequently, after numerous trials and considering these unique constraints, a lively traditional Chinese-style rhythm was selected, harmonizing well with the unchangeable nature of the melody. Selecting chords followed a similar approach to rhythm, focusing on a looping chord progression. However, the performance of the chord progression was made dynamic through techniques such as broken chords and block chords, adding variability and expression to the piece. The piano was selected to perform the chord progression. To capture the essence of traditional Chinese style, instruments such as guzheng, pipa, and suona were synthesized using Nyquist. The guzheng was designated to carry the theme, while the pipa and suona were employed to add texture and complexity, enhancing the overall musical color and interest. Lastly, meticulous adjustments were made to the arrangement of chords and harmonies, as well as the timing within the composition. This careful consideration ensured a cohesive musical experience, taking into account dynamics, transitions, and thematic development for a balanced and well-structured Pi song (without countermelody).

2.2. Automated composition part

The process of composing music using the digits of pi in this research involves several integral functions and logic. The translate_nyquist_score function translates the given Nyquist score into a more accessible structure. It utilizes regular expressions to match specific patterns within the score, extracting essential note components such as start time, duration, instrument, pitch, and velocity. The extracted information is then structured into a Python dictionary containing the translated notes. The normalize_to_octave function simplifies the MIDI pitch value by normalizing it to a single octave, ranging from 0 to 11. This normalization is achieved by taking the modulo of the pitch with 12. The is_minor_pentatonic function determines whether the melody adheres to a minor pentatonic scale. It normalizes the pitch values to a single octave and then compares them with a predefined minor pentatonic pattern (e.g., {1, 3, 6, 8, 10}). If all pitches match this pattern, the melody is considered a minor pentatonic. The generate_dynamic_counter_melody function constructs a countermelody based on the original melody's pitch. The normalize_pitch sub-function re-arranges the pitch to fit within a specific range (1 to 12). The select_counter_pitch selects an appropriate counter-melody pitch. If the theme pitch is low, it selects from a high range; if high, it selects from a low range. It also avoids neighbor pitches and the theme pitch itself. A pitch is then randomly chosen from this range. Finally, the translate_to_nyquist_score function translates the created countermelody back into the Nyquist score format, providing a string representation suitable for further musical rendering or synthesis.

3. Results And Discussion

3.1. Composition of Pi Song (manual composition)

The conversion of Pi's digits to musical notes was achieved through a specific number-to-midi-number mapping. The mapping ensures that the numbers 0 to 9 correspond to midi numbers 61 to 82, aligning with the pentatonic scale. The pentatonic scale was chosen to ensure that all the musical notes derived from Pi's digits would correspond to specific notes (C sharp, D sharp, F sharp, G sharp, and A sharp), creating a harmonious melody.

The process of finding the right key to represent the endless, non-repeating digits of Pi involved experimentation with different scales, specifically major, minor, and pentatonic scales. The first 100 digits of Pi were converted into musical notes using the previously defined mapping, and melodies were generated using each of these scales:

- Major Scale: While it provided a recognizable musical structure, the continuously changing and discordant sound did not align well with the desired representation of Pi.
- Minor Scale: Similar to the major scale, the minor scale resulted in a sound that lacked the desired coherence and harmony.
- Pentatonic Scale: Surprisingly, the pentatonic scale aligned well with Pi's characteristics; the melody played on this scale maintained a pleasing sound regardless of the note sequence.

After careful evaluation of melodies generated based on these scales, the pentatonic scale was selected as the final scale for the composition. This choice was based on the comfort and harmony that the pentatonic scale provided, resonating well with the mathematical nature of Pi. The pentatonic scale's ability to create a harmonious melody that connects mathematics with music played a crucial role in this selection, offering a novel perspective on mathematical representation in music.

The selection of rhythm for the Pi song required extensive experimentation to find a pattern that would complement the unchangeable nature of the melody, based on Pi. The challenge was to create a rhythm that would be more effective than a dynamic one, considering that the melody is fixed and cannot be altered according to conventional music composition practices. After numerous trials, a pleasing rhythm pattern was applied to the pentatonic scale Pi melody, resulting in a lively traditional Chinese-style rhythm that harmonized well with the unchangeable nature of the melody. The selection of chords for the Pi song followed a similar approach to rhythm, focusing on creating a looping chord progression that would complement the melody. The performance of the chord progression was made dynamic through techniques such as broken chords and block chords, adding variability and expression to the piece. The result illustrates the dynamic performance of the chord progression, adding depth and complexity to the overall composition.

The selection of instruments played a vital role in capturing the essence of traditional Chinese style and enhancing the overall musical color and interest of the Pi song. Three specific instruments were synthesized using Nyquist, and their sound samples and timbral characteristics are provided as following. The pipa was employed to add texture to the piece. The plot of the pipa's sound wave, illustrating its timbral characteristics as shown in Fig. 1. The guzheng was designated to carry the theme of the composition. The plot of the guzheng's sound wave is shown in Fig. 2. The suona was employed to enhance the complexity of the piece. The plot of the suona's sound wave is shown in Fig. 3. These instruments were carefully selected to perform different roles within the composition, each contributing unique timbral characteristics that aligned with the desired musical style.

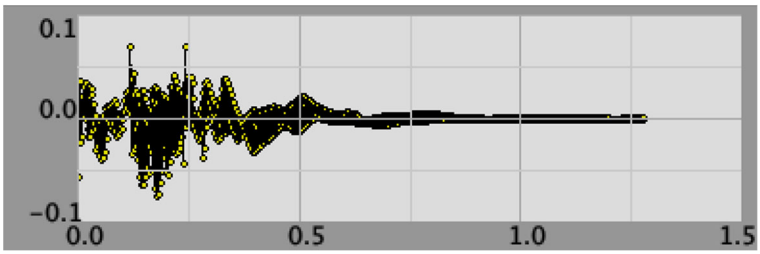


Figure 1. Pipa tremolo sound wave at pitch c4(Photo/Picture credit: Original).

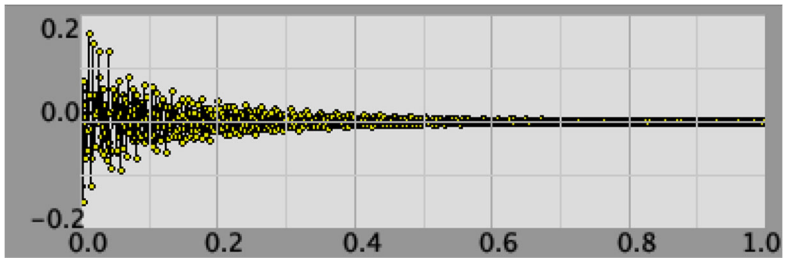


Figure 2. Guzheng sound wave at pitch c4 (Photo/Picture credit: Original).

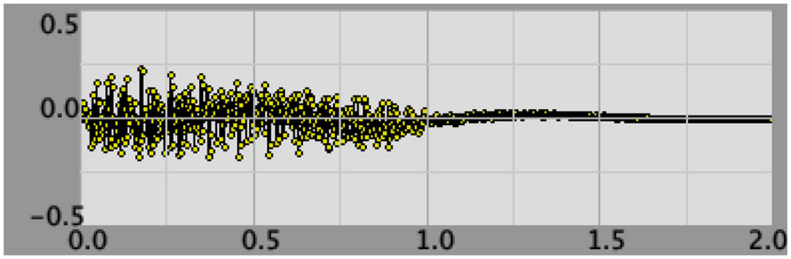


Figure 3. Suona sound wave at pitch c5 (Photo/Picture credit: Original).

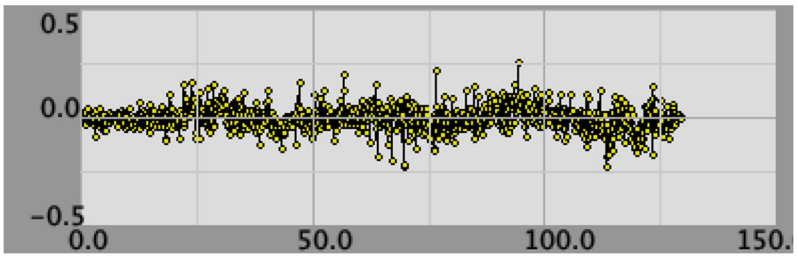


Figure 4. Entire pi song without countermelody (Photo/Picture credit: Original).

The Pi song's arrangement and timing were refined, with attention to dynamics, transitions, and themes, to create a balanced composition. The timing code was used to synchronize various elements of the composition, such as melody, harmony, chords, and instrumental parts. By adjusting the timing and volume of each element, the arrangement was crafted to highlight the unique characteristics of the chosen instruments and the mathematical precision of the melody based on Pi. The specific timing adjustments were made to create a harmonious blend of the pentatonic song, harmony, suona chords, broken chords, pipa chords, and other elements. These adjustments allowed for a seamless integration of different musical components, resulting in a rich and engaging musical experience. The culmination of the manual composition process is the final Pi song, which reflects the innovative approach to converting Pi's digits into musical notes, selecting the key, rhythm, chords, instruments, and careful arrangement and timing. The plot of entire sound sample of the Pi song, without countermelody as illustrated in Fig 4. This final result showcases the successful integration of mathematical concepts with artistic expression, offering a novel perspective on mathematical representation in music.

3.2. Automated composition

The process of translating the Nyquist score to a Python score is a crucial step in the algorithmic composition of the countermelody. This translation allows for the manipulation and analysis of the musical data within a programming environment, enabling the application of algorithmic techniques. Using the translate_nyquist_score function, the Python score is created, represented as a list of dictionaries, where each dictionary contains information about the start time, duration, instrument, pitch, and velocity of each note. This translation process facilitates the application of algorithmic methods to generate the countermelody, providing a structured and accessible representation of the musical data. The algorithm takes into account the original melody's pitch and rhythm, and through a series of functions, it constructs a countermelody that harmonizes with the original theme. The algorithm for generating the countermelody involves several key functions, including normalizing

the pitch to a single octave, identifying the minor pentatonic scale, and dynamically generating the countermelody. The algorithm ensures that the countermelody is in harmony with the original melody, while also introducing variability and complexity.

Building upon the algorithmically generated countermelody, the final version of the Pi song was crafted. The algorithm was applied to the entire Pi song, and multiple iterations were conducted to generate different countermelodies. Among the various versions, the one that best complemented the original melody and fulfilled the artistic vision was selected. The entire sound sample, including the original melody and the selected countermelody, can be accessed in Fig. 5. This final version represents the culmination of the research, showcasing the successful integration of manual composition and algorithmic techniques.

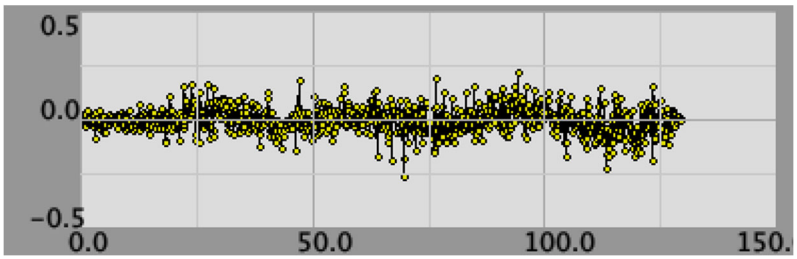


Figure 5. entire pi song with countermelody (Photo/Picture credit: Original).

3.3. Analysis

After discovering the limitations of automatic music generation using algorithms in related academic works, this research decided to combine manual and automatic composition. The manual composition process successfully translated Pi's digits into a musical form, with the pentatonic scale providing a harmonious connection between mathematics and music. Experimentation with rhythm, chords, and traditional Chinese instruments added depth and complexity to the piece. The arrangement and timing were meticulously crafted to create a balanced composition that highlighted the mathematical precision of the melody. The introduction of a countermelody through algorithmic composition added a dynamic layer to the Pi song. The translation from Nyquist to Python score enabled manipulation within a programming environment, illustrating an innovative approach to music composition. The algorithmically generated countermelody, though challenging to harmonize with the original theme, successfully introduced variability and complexity. The integration of manual and algorithmic methods showcases an innovative approach to musical representation of mathematical concepts. The research's strength lies in creatively using algorithmic methods, bridging mathematics with music, and exploring new avenues for musical composition. The findings contribute to the understanding of translating mathematical concepts into artistic expressions and offer insights into the creative possibilities of mathematical representation in music.

In conclusion, this research presents a novel approach to composing music based on Pi's digits, blending manual composition with algorithmic techniques. Through careful selection and arrangement, a harmonious melody was crafted, enriched by an algorithmically generated countermelody. The combination of manual and algorithmic composition allowed for a more nuanced and expressive piece, overcoming the limitations of purely algorithmic methods.

4. Limitations And Prospects

The research presented in this paper, while innovative in its approach, is not without limitations. One of the most significant constraints lies in the fixed nature of the accompaniment and the strict adherence to the pentatonic scale. This approach was chosen to ensure that the numbers corresponding to Pi's digits would align harmoniously with specific musical notes, thereby creating a pleasing melody. However, this constraint led to a lack of diversity in the musical experience. The fixed accompaniment, although carefully crafted to complement the melody, does not change in response to variations in the numerical sequence. This rigidity makes different versions of the song sound

remarkably similar, regardless of the numerical changes made to the melody. While this approach aligns with the project's goals of creating a harmonious connection between mathematics and music, it may limit the diversity and richness of the musical experience. The constraint to the pentatonic scale, although effective in creating a coherent melody, may also hinder the exploration of more complex and varied musical expressions.

The limitations identified in this research provide valuable insights and directions for future work. Building on the evaluation's findings, there is an opportunity to develop a more dynamic and adaptable algorithm that escapes the constraints of the pentatonic scale and fixed accompaniment. One promising avenue is the creation of a tool that studies existing music to uncover mathematical patterns, recommend chords and rhythms, and incorporate a machine learning component for personalized refinement. This approach could involve analyzing a vast array of musical compositions to identify underlying mathematical structures and relationships. By understanding these patterns, the algorithm could generate more varied and expressive musical pieces that still maintain a connection to mathematical concepts. Furthermore, the integration of machine learning could allow for continuous learning and adaptation. By enabling users to provide feedback on generated chords and melodies through reactions such as "like," "dislike," or "neutral," the algorithm could learn from user preferences and continuously refine its output. This iterative process would pave the way for a personalized tool that aligns with individual musical tastes and styles, enhancing the diversity and richness of the compositions. Such an approach would not only overcome the limitations identified in this research but also contribute to the broader field of algorithmic music composition. It would offer a more nuanced and expressive method for translating mathematical concepts into musical form, bridging the gap between mathematical precision and artistic creativity. The potential applications of this research extend beyond the realm of mathematical representation, opening new possibilities for personalized music creation, educational tools, and artistic exploration.

In conclusion, while the current research has achieved significant success in creating a novel perspective on mathematical representation in music, the identified limitations offer exciting opportunities for further exploration and innovation. The future outlook for this field is bright, with the potential to push the boundaries of algorithmic composition and create more personalized, expressive, and engaging musical experiences.

5. Conclusion

To sum up, the exploration of mathematical complexity through music, specifically the digits of Pi, has led to a novel and innovative approach to composition. This research has successfully bridged the gap between mathematics and music, employing both manual and algorithmic techniques to create a harmonious melody enriched by an algorithmically generated countermelody. Through careful selection of scales, rhythm, chords, and traditional Chinese instruments, the research has crafted a piece that resonates with the mathematical nature of Pi, while overcoming the limitations of purely algorithmic methods. However, the study's limitations, such as the fixed nature of the accompaniment and adherence to the pentatonic scale, have led to a lack of diversity in the musical experience. These constraints open exciting avenues for future exploration, including the development of more dynamic algorithms and the integration of machine learning for personalized refinement.

The significance of this research lies in its innovative approach to translating mathematical concepts into artistic expressions. It offers fresh insights into the creative possibilities of mathematical representation in music, contributing to the broader field of algorithmic music composition. The potential applications extend beyond mathematical representation, opening new possibilities for personalized music creation, educational tools, and artistic exploration. The research stands as a testament to the limitless possibilities of human creativity when art and science converge, enriching the musical landscape and challenging traditional notions of creativity.

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