Research on Supply Chain Contract of Omni-channel Retail with BOPS Mode

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Abstract. T With the rapid development of e-commerce, consumers' consumption patterns have changed, more and more consumers are used to shopping online. Therefore, in order to better connect with the consumer market and save total cost of the supply chain, more and more manufacturers have expanded their online channels, which undoubtedly has an impact on the operation of brick-and-mortar shops. Therefore, under the background of new-retail development, BOPS mode is introduced to integrate online and offline. It protects offline physical resources, and brings profit gains to different subjects of omni-channel supply chain. This paper focuses on the omni-channel supply chain members who jointly manage BOPS mode, but as different profit subjects, what kind of supply chain contracts should manufacturers and retailers adopt to guarantee their respective interests, improve the enthusiasm of each subject, and make omni-channel supply chain operate continuously. In this paper, the Stackelberg game model is used to study the influence of traditional wholesale price contract, quantity discount contract, sales rebate contract and revenue sharing contract on the profit and return of manufacturers and retailers in omni-channel supply chain, so as to effectively encourage the main body of omni-channel supply chain to actively provide BOPS, protect offline physical store resources and bring gains to omni-channel supply chain by taking advantage of online and offline advantages. It is found that sales rebate contract and revenue sharing contract can achieve Pareto optimization on the basis of traditional wholesale price contract, and the choice mainly depends on BOPS demand.

Keywords: omni-channel; BOPS; quantity-discount contract; sales-rebate contract; revenue-sharing contract.

1. Introduction

In the context of retail channel development, customer omni-channel shopping behavior occurs. Omni-channel buying means that consumers use fragmented time to shop online, and their shopping needs are indeterminate, i.e., the shopping behavior can occur at anytime, anywhere, and can be satisfied in any way [1]. Omni-channel retailing provides consumers with a continuous, complete and seamless shopping experience through the linkage and co-ordination of multiple channels, and takes advantage of omni-channel coverage to increase retail touchpoints, broaden retail surfaces, and improve merchandise retailing opportunities [2]. At the same time, omni-channel enhances the retailer's competitive advantage and channel synergy, and brings into play the resource strengths of each channel. Second, omni-channel better matches orders received by retailers with their distribution retail channels, responds to orders more economically and quickly, and realizes two-way gains for retailers and customers [3]. In the omni-channel retail model, customers use mobile electronic devices to browse and select products, then pay online, and finally choose the nearest shop to pick up the products, which is known as "Buy Online and Pick-up in Store (BOPS)".The BOPS model leverages the services of the online channel. BOPS utilizes the service resources of the online channel to reach more markets and serve more customers, while taking advantage of the front-end advantages of offline physical shops to reduce order response time and save distribution costs. Secondly, BOPS can effectively avoid the situation that offline customers go to the shop to buy the shop out of stock, and improve the fulfilment rate [4], and, the BOPS retail model increases the frequency of consumers' visits to the shop, which invariably increases the sales opportunities for the sale of goods in offline shops. Although the BOPS model can effectively integrate online and offline, bringing the convenience of omnichannel retailing to consumers and online and offline channels, the introduction

of BOPS brings more trade-offs and choices in management decisions. In an omni-channel supply chain, what kind of agreement should be reached between manufacturers and retailers, and what kind of contract should be adopted to effectively mobilize offline retailers to actively provide BOPS services, protect the resources of offline retail physical shops, and give full play to the advantages of online and offline retailing. Secondly, BOPS integrates online and offline, so there are two main bodies in the distribution of sales profit, so in the profit mechanism, what kind of supply chain contract can reasonably allocate the sales gain brought by omni-channel retailing, and realize the global supply chain optimization, i.e. win-win situation for online and offline channels. Finally, can several supply chain contracts commonly used in traditional dual-channel or multi-channel supply chains continue to play their roles after the introduction of the BOPS model? And under what circumstances can they be better used to exert their positive influence and maximize the coordination between manufacturers and retailers.

In summary, this paper focuses on the impact on offline retail shops after the rapid development of e-commerce, stimulated by online dividends, and the expansion of online channels by manufacturers, and the erosion of offline demand by online channels, and the introduction of the BOPS model of retailing in such a context, and what kind of contractual contracts can be adopted by suppliers and retailers in the omni-channel supply chain to better enjoy the profit dividends brought by omni-channels, as well as what circumstances can the different contracts better play a role in the relationship between manufacturers and retailers? What kind of circumstances can the different contracts better play their positive roles. This provides valuable management suggestions and references for future omni-channel research to better co-ordinate the supply chain in the context of omni-channel and to enhance the willingness of supply chain members to co-operate and their service capability.

2. Literature review

Regarding the research on the BOPS model, there is not much research literature on the BOPS model, even though the BOPS model is the omni-channel order fulfilment model and retailing model that is chosen and accepted by most retailers and consumers in reality [5]. Existing studies on BOPS can be broadly classified into two directions, firstly, the impact of BOPS strategy on channel demand allocation and profit distribution, and secondly, the retailer operational issues arising after the implementation of BOPS strategy [6].Gao and Yin studied the issues of new market demand and increased channel operational costs with the squeeze on other channel demand after the introduction of BOPS [7]. Lin and Hou investigated the impact of the BOPS model on pricing, service quality, and profit gains for manufacturers and retailers in an omni-channel supply chain [8]. In the study of operational issues of BOPS, Kushal et al, on the other hand, focused on the issue of the impact of delayed pickup of BOPS orders on the inventory and replenishment strategies of physical shops after the implementation of BOPS strategy in offline brick-and-mortar shops, and it proposed two inventory control strategies to help brick-and-mortar retail shops maximize their profitability [6].Lu and Xin, on the other hand, investigated whether, under the BOPS strategy, the decision problem of whether a brick-and-mortar retailer needs fused inventory as well as ordering, and their study established four models: a baseline model, a stand-alone inventory and ordering model, a fused inventory stand-alone ordering model, and a fused inventory ordering model, and they compared and analyzed the most advantageous inventory model and ordering quantity for brick-and-mortar retailers [9]. Regarding the research on manufacturer-retailer contract, its research history is long, and the supply chain background of most studies is the traditional single-channel and dual-channel supply chain, and the research literature on the contract mechanism of manufacturer-retailer contract in the omni-channel supply chain is less due to the late emergence of the omni-channel. In the contract mechanism as early as 1985, Pasternack proposed the wholesale price contract and the surplus repurchase contract [10]. Pasternack found that although the wholesale price contract can be used as a coordination mechanism to coordinate the profits between the manufacturer and the retailer, when it works, the optimal wholesale price is equal to the manufacturer's production cost, so the manufacturer's profit is zero.

profits are zero, so the contract is unenforceable. Instead, Pasternack proposed a surplus buyback contract, where the manufacturer buys back the remaining goods sold by the retailer at a fixed price at the end of the selling season in order to reduce the retailer's profit loss and to coordinate the profits of the supply chain members. Pasternack's study found that setting a reasonable wholesale price and buyback price can coordinate the whole supply chain effectively. In addition, Tsay proposed the quantity elasticity contract, under which the manufacturer wholesales the product to the retailer at a fixed price and fully compensates the retailer for the sale of the remaining product [11]. It was found that the quantity elasticity contract can achieve the same effect as the surplus buyback contract under certain conditions. Amrouche et al. found that both omni-channel price parity and price discounts can be effective in coordinating the distribution between online and offline channels [12]. Taleizadeh et al. found that alliance strategy cannot be effective in all cases when considering the return strategy of the retailer, goods production costs, and the impact of return rates, the retailer-dominated supply chain environment is insensitive to alliance strategies [13]. Yan et al. on the other hand, find that revenue sharing and rebate policies can work together to mitigate inter-channel rivalry and bring benefits to supply chain members [14].

3. Model assumptions and descriptions

In this paper, we assume that the number of manufacturers in the supply chain is one and the number of retailers is also one, so there is no competition or game between suppliers as well as between retailers. The manufacturer owns its online channel to place marketing and provide online services directly to online consumers, and the retailer owns its offline channel to provide in-store face-to-face communication services directly to offline in-store consumers; the online manufacturer and the offline retailer jointly provide BOPS services and jointly operate the BOPS channel. The manufacturer determines the wholesale price w and enters into an indenture contract with the retailer, and then the offline retail shop determines the retail price p of the goods.

It is assumed that consumers' shopping behaviors are rational decision making and they will choose the channel with the largest value of consumer utility function. In this paper, consumers are classified into two types: online preference consumers and offline preference consumers, online preference consumers shopping behaviors occurs in pure online channels and BOPS channels. Offline preference consumers choose to shop in shops as long as the value of the consumer utility function of the offline channel is greater than 0, completely disregarding the online and BOPS channels. Assuming that the total consumer market is 1, the market size for online preference consumers is α and the market for offline preference consumers is $(1-\alpha)$. Consumers will consider whether to pick up at the shop or deliver to their homes based on the actual situation of receiving the goods. If they go to the shop by themselves, consumers need to bear the transport cost and the cost of searching for goods in the shop, waiting in line to pay, etc., while delivery to home incurs the time cost of waiting for delivery and the cost of online payment, etc. Assuming that the unit transport cost of going to the shop by themselves is b_s, the transport distance is s, and $s \in [0,1]$.the transport cost of going to the shop by themselves for the consumer is $\mathrm{sb_s}$, and the cost of searching for and paying for the goods in the shop is l_s, and the waiting cost of delivery-to-home is $\mathrm{b_e}$, the online search and payment cost is $\mathrm{l_e}$, and $\mathrm{l_e} < \mathrm{l_s}$. In the shopping session, consumers perceive the value of the goods in terms of the degree of demand matching, assuming that the consumer-perceived price of the goods is v. At the same time, because of the difference between the online goods display and the goods that arrive in the store, there is a commodity differential discount on the value of the goods for the consumer after placing an order online coefficient $\theta, \theta \in [0,1]$. This leads to the consumer utility functions U_e, U_s, U_b for the three channels with the following expressions:

$$U_e = \theta v - p - l_e - b_e \tag{1}$$

$$U_s = v - p - l_s - sb_s \tag{2}$$

$$U_b = \theta v - p - l_e - sb_s \tag{3}$$

The online preference consumer chooses delivery-to-home service and BOPS service depending on the delivery cost sb_s, b_e. Let s^f be the no-difference-curve dividing value of the pure online channel and BOPS channel, when $s < s^f$, the online preference consumer chooses BOPS; when $s > s^f$, the online preference consumer chooses the pure online channel. Let s^h be the distance critical value of offline channel consumer utility function greater than 0. Since offline channel consumers' shopping occurs only offline, their shopping behaviour occurs when the value of consumer utility function of offline channel is greater than 0 for this kind of consumers, i.e. $U_s > 0$. In summary, the channel demand functions of pure online channel, BOPS channel consumers and offline preference consumers can be obtained:

$$D_e = \alpha(1 - s^f) = \alpha(1 - \frac{b_e}{b_s}) \tag{4}$$

$$D_b = \alpha s^f = \alpha \frac{b_e}{b_s} \tag{5}$$

$$D_s = (1 - \alpha)s^h = (1 - \alpha)\frac{v - p - l_s}{b_s} \tag{6}$$

The description of the model parameters in this paper is shown in Table 1.

Table 1. Parameter description table

Model Variable	Description
U_i	Consumer utility function, $i = e, s, b$(e- online consumer, s- offline consumer, b- BOPS consumer)
D_i	Consumer demand function, $i = e, s, b$(e- online consumer, s- offline consumer, b- BOPS consumer)
M_i	Optimal profit function for online manufacturers, $i = 1,2,3,4$(1- Wholesale price contract, 2- Quantity-discount contract, 3- Sales-rebate contract, 4- Revenue-sharing contract)
R_i	Optimal profit function for offline retailers,$i = 1,2,3,4$(1- Wholesale price contract, 2- Quantity-discount contract, 3- Sales-rebate contract, 4- Revenue-sharing contract)
Z_i	Optimal profit margin for manufacturer-retailer, $i = 1,2,3,4$(1- Wholesale price contract, 2- Quantity-discount contract, 3- Sales-rebate contract, 4- Revenue-sharing contract)
s	Distance to shop for consumers on their own
v	Consumer Perceived Price
θ	Discount factor for discrepancies in merchandise ordered online and deliver to home
p	Retail prices of goods (online and offline at the same price)
l_i	Commodity search and payment costs, $i = e, s$(e- online, s- offline)
b_i	Distribution waiting costs and unit transport costs, $i = e, s$ ((e -waiting, s -transport)
w	Maximum wholesale price from manufacturer to retailer
W	Manufacturer's wholesale price under a volume discount covenant
α	Online preference consumer market size
β	Unit discount factor for volume discount covenants
γ	Profit-sharing coefficient of the revenue-sharing contract
r	Sales rebate value of sales rebate deeds
t	Effectiveness thresholds for sales rebate covenants

4. Modelling and analysis

4.1. Wholesale price contract

From the above analysis, the profit functions M_1 and R_1 are obtained for the online manufacturer and the offline retailer.

$$M_1 = wD_S + p(D_e + D_b) \tag{7}$$

$$R_1 = (p - w)D_S \tag{8}$$

The optimal solution to the manufacturer's wholesale price and the retailer's retail price when all BOPS sales are accounted for online in the case of online-offline integration after the introduction of BOPS is $w_1^* = \frac{\alpha b_s}{2(1-\alpha)} + \frac{v-l_s}{2}$, $p_1^* = \frac{\alpha b_s}{4(1-\alpha)} + \frac{3(v-l_s)}{4}$, manufacturers' and retailers' profits are $R_1^* = \frac{[(1-\alpha)(v-l_s)-\alpha b_s]^2}{16(1-\alpha)b_s}$, $M_1^* = \frac{3\alpha b_s(v-l_s)+(1-\alpha)(v-l_s)^2}{4b_s}$.It follows that under the traditional wholesale price contract, the optimal wholesale price and the optimal retail price for online manufacturers and offline retailers are proportional, i.e., there exists a specific value of λ , let $\lambda p_1^* = w_1^*$.

Let $\mathrm{Z_i^*} = \mathrm{M_i^*} - \mathrm{R_i^*}$ $(i = 1, 2, 3, 4)$. Therefore, $Z_1^* = \frac{(1-\alpha)(v-l_s)[14\alpha b_s - 3(1-\alpha)(v-l_s)] - \alpha^2 b_s^2}{16(1-\alpha)b_s}$, under the wholesale price contract. The analysis shows that when $(1-\alpha)(v-l_s)[14\alpha b_s - 3(1-\alpha)(v-l_s)] - \alpha^2 b_s^2 > 0$, $Z_1^* > 0$, $M_1^* > R_1^*$, the optimal profit of the online manufacturer is larger than that of the offline retailer. when $(1-\alpha)(v-l_s)[14\alpha b_s - 3(1-\alpha)(v-l_s)] - \alpha^2 b_s^2 < 0$, $Z_1^* < 0$, $M_1^* < R_1^*$, the optimal profit of the online manufacturer is less than that of the offline retailer; when $(1-\alpha)(v-l_s)[14\alpha b_s - 3(1-\alpha)(v-l_s)] - \alpha^2 b_s^2 = 0, Z_1^* = 0$, $M_1^* = R_1^*$ and the optimal profit of the online manufacturer is equal to that of the offline retailer. Thus, it seems that under the traditional wholesale price contract, supply chain cost is the key variable controlling the profit situation of online manufacturers and offline retailers. And the offline retailer can be the main body of supply chain profit when and only when$(1-\alpha)(v-l_s)[14\alpha b_s - 3(1-\alpha)(v-l_s)] - \alpha^2 b_s^2 < 0$, which is a more demanding condition compared to the condition that the online manufacturer is the main body of supply chain profit. Therefore, under the traditional wholesale price contract, the retailer operates omni-channel retailing with the manufacturer, but in terms of profit distribution, it is the main body of supply chain profit. Under the traditional wholesale price contract, although the retailer operates omni-channel retailing with the manufacturer, the retailer is the squeezed party in terms of profit distribution and does not get the equivalent profit return of its investment.

The optimal profit of the manufacturer M_1^* and the optimal profit of the retailer R_1^* under the wholesale price contract are related to the pay-at-store costs l_s, b_sIt can be seen that under the wholesale price contract, the supply chain cost is the key influencing factor of the contract, and the optimal profits of online manufacturers and offline retailers are concavely correlated with the supply chain cost l_s, b_s.Therefore, under the traditional wholesale price contract, the reasonable control of supply chain cost can make online manufacturers and offline retailers to balance their respective profit relationships in omni-channel retailing.

4.2. Quantity-discount contract

In the traditional O2O supply chain, the manufacturer provides a certain percentage of wholesale price discount, which is quantity discount, in order to the retailer based on the quantity of goods ordered by the retailer in order to coordinate the profit situation of both in the supply chain. According to Pei et al. the manufacturer's wholesale price under quantity discount, W, is usually denoted as $W = w - \beta \mathrm{D}$ [15], where w is the manufacturer's maximum wholesale price, which is the original wholesale price in the absence of a quantity discount, and β is the wholesale price discount coefficient under the quantity discount, which the manufacturer provides a discount of the wholesale price to the retailer based on the quantity of goods ordered by the offline retailer at a sloping gradient β. In the quantity discount contract studied in this paper, the manufacturer offers the retailer a

wholesale price based on the demanded quantity in the BOPS channel and the offline channel, $W = w - \beta(D_S + D_b)$. So the profit function of online manufacturer and offline retailer at this point is expressed as:

$$M_2 = WD_S + p(D_e + D_b) \tag{9}$$

$$R_2 = (p - W)D_S \tag{10}$$

The optimal value of the profit situation for both in the supply chain:

$w_2^* = \frac{\alpha {b_s}^3 - \alpha\beta {b_s}^2(1-\alpha) + {b_s}^2(1-\alpha)(v-l_s) + 2\alpha\beta b_e b_s(1-\alpha) - \alpha\beta^2 b_e(1-\alpha)^2}{b_s(1-\alpha)[2b_s - \beta(1-\alpha)]}$, $p_2^* = \frac{\alpha {b_s}^2 + (v-l_s)[3b_s(1-\alpha) - 2\beta(1-\alpha)^2]}{2(1-\alpha)[2b_s - \beta(1-\alpha)]}$,

$M_2^* = \frac{(1-\alpha)^2(v-l_s)^2 + 6\alpha b_s(1-\alpha)(v-l_s) - 4\alpha\beta(1-\alpha)^2(v-l_s) + \alpha^2 {b_s}^2}{4(1-\alpha)[2b_s - \beta(1-\alpha)]}$, $R_2^* = \frac{[b_s - \beta(1-\alpha)][\alpha b_s - (1-\alpha)(v-l_s)]^2}{4(1-\alpha)[2b_s - \beta(1-\alpha)]^2}$

($\frac{2\beta(1-\alpha)^2[b_s(v-l_s) - \alpha\beta b_e] + 2\alpha\beta b_s(1-\alpha)(2b_e - b_s) + {b_s}^2[\alpha b_s - (1-\alpha)(v-l_s)]}{2b_s(1-\alpha)[\beta(1-\alpha) - 2b_s]} > 0$)

It can be seen that the optimal wholesale price w_2^* of the online manufacturer is related to α, b_s, β, b_e, l_s. And the retailer's optimal retail price p_2^* is related to α, b_s, β, l_s. The two different influencing variables are b_e.

Let Z_2^*=$M_2^* - R_2^*$, Z_2^* increases with the increase of β, and Z_2^* is constantly greater than 0. It means that the difference between the optimal profit of the manufacturer and the retailer increases with the increase of the coefficient of quantity discount, and Z_2^* is constantly greater than 0. That is to say, the optimal profit of the manufacturer is always greater than the optimal profit of the retailer. The explanation of this paper is that under the quantity discount contract, there exists a maximum wholesale price w. When the quantity discount coefficient $\beta = 0$, which is equivalent to $W = w$, the manufacturer's profit is larger than the retailer's. The manufacturer's profit is always larger than the retailer's. And as β increases, the profit difference increases and $M_2^* - R_2^*$ is constantly greater than 0.

Under the quantity discount contract, M_2^* and R_2^* are related to l_s , b_s. And under the quantity discount contract, M_2^* is a monotonically increasing function of β, and R_2^* is a monotonically decreasing function of β. It follows that in a real management problem, the cost of the omni-channel retail supply chain can be reduced by the joint efforts of the online manufacturer and the offline retailer so that both b_s and l_s are at a reasonable level, thus minimizing the difference between their optimal profits. In this way, the manufacturer's optimal profit is always greater than the retailer's optimal profit, which coordinates the profit distribution of the supply chain to a certain extent, and also protects the retailer's offline brick-and-mortar shop resources to a certain extent. As for the quantity discount coefficient, the above analysis shows that M_2^* and R_2^* have the opposite relationship with β. Therefore, under the quantity discount contract, a good negotiation of the quantity discount coefficient between the manufacturer and the retailer is the key for the quantity discount contract to play the role of protecting the resources of the physical shops.

4.3. Sales-rebate contract

In a supply chain, a sales rebate contract is an agreement between a manufacturer and a retailer whereby the manufacturer grants a rebate r for each unit over the contractually agreed target value t when the retailer's sales exceed it [16].In this paper, the demand of BOPS channel is counted in the conditional judgement of the sales rebate contract taking effect, i.e., the critical condition for the retailer to get the sales rebate is $D_S + D_b > t$, in which case the profit function of the manufacturer and the retailer is shown as follows. In order to ensure that the retailer's profit is not negative, $p > w > r$, and $D_S + D_b > t$, the retailer receives a sales rebate from the manufacturer.

$$M_3 = wD_S + p(D_e + D_b) - r(D_s + D_b - t) \tag{11}$$

$$R_3 = (p - w)D_S + r(D_s + D_b - t) \tag{12}$$

Optimal pricing and profitability for manufacturers and retailers: $w_3^* = \frac{2r(1-\alpha) + (v-l_s)(1-\alpha) + \alpha b_s}{2(1-\alpha)}$,

$p_3^* = \frac{\alpha b_s}{4(1-\alpha)} + \frac{3(v-l_s)}{4}$,
$M_3{}^* = \frac{[(1-\alpha)(v-l_s)+\alpha b_s]^2 - 8r(1-\alpha)(\alpha b_e - t b_s) + 4\alpha b_s(1-\alpha)(v-l_s)}{8b_s(1-\alpha)}$,
$R_3{}^* = \frac{[(1-\alpha)(v-l_s)-\alpha b_s]^2 + 16r(1-\alpha)(\alpha b_e - t b_s)}{16 b_s(1-\alpha)}$,
$(1-\alpha)(v-l_s) - \alpha b_s > 4r(1-\alpha)$ and $\frac{(1-\alpha)(v-l_s)+\alpha(4b_e-b_s)}{4b_s} > t$

Under the sales rebate contract, the retailer sets the same optimal retail price p_3^* as the wholesale price contract p_1^*. It appears that the sales rebate retains the relatively small retail price of the goods under the original wholesale price contract, when the manufacturer's optimal wholesale price, w_3^*, has increased from the original wholesale price contract, which is $w_3^* > w_1^*$. This paper explains that, under the sales rebate contract, under the premise of the contract is in force, the manufacturer in order to make up for the loss of profit given to the retailer sales rebate, in the formulation of its optimal wholesale price, according to its own profit maximisation principle, will be appropriate to increase the optimal wholesale price, in order to maintain omni-channel retailing, the supply chain of the coordination of the relationship between profit.

Let $Z_3^* = \frac{32r(1-\alpha)(tb_s-\alpha b_e)+[\alpha b_s+(1-\alpha)(v-l_s)]^2+12\alpha b_s(1-\alpha)(v-l_s)}{16b_s(1-\alpha)}$, when $D_b = \alpha b_e/b_s > t$, $Z_3^* > 0, M_3^* > R_3^*$, when $D_b = \alpha b_e/b_s < t$, $Z_3^* < 0$, $M_3^* < R_3^*$, when $D_b = \alpha b_e/b_s = t$, $Z_3^* = 0$, $M_3^* = R_3^*$^*. It can be seen that the role of BOPS channel demand as a key influence on the profit size of online manufacturers and offline retailers is magnified under the sales rebate contract. Since the sales rebate contract model studied in this paper assumes that the contract is in effect, the magnitude of the contract is mainly affected by the BOPS channel demand, but if in real omni-channel retailing, the conditions for the contract to be in effect are unknown, and the profit relationship between the manufacturer and retailer in the supply chain is related to the critical value of the contract to be in effect, t, and the relative magnitude of the BOPS channel demand.

The optimal profits of online manufacturersM_3^* and R_3^* offline retailers are related to supply chain costs l_s and b_s. When the supply chain costs b_s and l_s are controlled under optimal conditions, both the online manufacturer and the offline retailer can be guaranteed to have high optimal profit gains. However, in the sales rebate contract, the two profits are coordinated through the size of the sales rebate value agreed between the manufacturer and the retailer, but the rebate value r is not a variable, so it is not specifically studied in this paper. It can be easily concluded from the model that when the value of the rebate r is larger, the optimal profit of the online manufacturer is smaller and the optimal profit of the retailer is larger.

4.4. Revenue-sharing contract

In the omni-channel consumer choice BOPS service process, can be decomposed into: online shopping stage and offline goods delivery stage, from this point of view, the BOPS orders generated by the profit alone into the online or offline are not reasonable, so the assumption that the profit-sharing coefficient between the manufacturer and the retailer is γ, the size of which depends on the bargaining power of the manufacturer and the retailer. At this point, the manufacturer and retailer profit function is shown below:

$$M_4 = wD_S + pD_e + (1-\gamma)\,pD_b \tag{13}$$

$$R_4 = (p-w)D_S + \gamma p D_b \tag{14}$$

Optimal pricing and profitability for manufacturers and retailers: $w_4^* = \frac{\alpha b_s - 2\gamma\alpha b_e}{2(1-\alpha)} + \frac{v-l_s}{2}$,
$p_4^* = \frac{\alpha b_s}{4(1-\alpha)} + \frac{3(v-l_s)}{4}$,
$R_4^* = \frac{[(1-\alpha)(v-l_s)-\alpha b_s]^2 + 16\gamma\alpha b_e(1-\alpha)(v-l_s)}{16(1-\alpha)b_s}$,

$M_4^* = \frac{4\alpha(1-\alpha)(v-l_s)(b_s-2\gamma b_e)+[\alpha b_s+(1-\alpha)(v-l_s)]^2}{8b_s(1-\alpha)}$,$4\gamma\alpha b_e > \alpha b_s + (1-\alpha)(v-l_s)$

Under the revenue-sharing contract, the manufacturer's optimal wholesale price w_4^*, and the retailer's optimal retail price p_4^* decrease in w_4^* and remain unchanged in p_4^* compared to the wholesale price contract and the sales rebate contract. And w_4^* is also affected by the manufacturer's revenue sharing coefficient γ and the consumer's time cost of waiting for the delivery of goods b_e, compared to p_4^*. The interpretation of this paper is that, relative to the sales rebate contract and the original traditional wholesale price contract, under the revenue-sharing contract, the online manufacturer only shares a portion of its BOPS sales revenue (related to BOPS demand D_b) to the offline retailer, while the contractual conditions of the sales rebate and the wholesale price contract are related to the offline demand D_s and the BOPS demand D_b, at which point for the online manufacturer It makes more profit loss for the supply chain contract than the revenue sharing contract, so under the revenue sharing contract, the manufacturer has less profit loss and is more willing to reduce its wholesale price, when w_4^* is smaller. And w_4^* is related to the gain-sharing coefficient γ, because γ directly determines the size of the BOPS revenue shared by the online manufacturer to the offline retailer, which the size of the profit sacrifice made by the manufacturer for the gain-sharing contract, so w_4^* is related to γ.

Let $Z_4^* = \frac{8\alpha b_s(1-\alpha)(v-l_s)+[\alpha b_s+(1-\alpha)(v-l_s)]^2+4\alpha(v-l_s)(1-\alpha)(b_s-8\gamma b_e)}{16(1-\alpha)b_s}$,when $\frac{8\alpha b_s(1-\alpha)(v-l_s)+[\alpha b_s+(1-\alpha)(v-l_s)]^2}{4\alpha(v-l_s)(1-\alpha)(b_s-8\gamma b_e)} > 1$, $Z_4^* > 0, M_4^* > R_4^*$, when $\frac{8\alpha b_s(1-\alpha)(v-l_s)+[\alpha b_s+(1-\alpha)(v-l_s)]^2}{4\alpha(v-l_s)(1-\alpha)(b_s-8\gamma b_e)} < 1$, $Z_4^* < 0, M_4^* < R_4^*$, when $\frac{8\alpha b_s(1-\alpha)(v-l_s)+[\alpha b_s+(1-\alpha)(v-l_s)]^2}{4\alpha(v-l_s)(1-\alpha)(b_s-8\gamma b_e)} = 1$, $Z_4^* = 0, M_4^* = R_4^*$. As a result, under the revenue sharing contract, the size of the revenue sharing coefficient γ becomes an influential factor affecting the optimal profit size of the online manufacturer and the offline retailer. When $0 < \gamma < \frac{(\alpha b_s)^2+[(1-\alpha)(v-l_s)]^2+14\alpha b_s(1-\alpha)(v-l_s)}{32\alpha b_e(1-\alpha)(v-l_s)}$, $M_4^* > R_4^*$. When $\frac{(\alpha b_s)^2+[(1-\alpha)(v-l_s)]^2+14\alpha b_s(1-\alpha)(v-l_s)}{32\alpha b_e(1-\alpha)(v-l_s)} < \gamma < 1$, $M_4^* < R_4^*$. Thus the revenue sharing contract works so that the online manufacturer still retains most of the profits in the supply chain when γ is less than a specific value. Therefore, the revenue sharing coefficient in the contract negotiated between the online manufacturer and the offline retailer can ensure the incentive effect on the offline retailer, protect the resources of the offline physical shop, and at the same time, make the manufacturer and the retailer's profits and benefits comparable, reduce the possibility of channel competition, and ensure the harmonious development of the omni-channel supply chain.

M_4^* is a concave function with respect to l_s and b_s, and the rate of change of M_4^* varies faster with larger l_s and b_s. Similarly, R_4^* is a concave function with respect to l_s and b_s and changes more rapidly with larger l_s and b_s. It can be concluded that under the revenue sharing contract, the supply chain cost makes the profits among the supply chain members all at a better level under specific conditions, which leads to the profit efficiency and coordination between the manufacturer and the retailer. However, in reality, the control of supply chain cost is not easy, which requires effective co-operation between supply chain members.

5. Summary and outlook

This paper investigates the profit relationship between online manufacturers and offline retailers under the original traditional wholesale price contract, quantity discount contract, sales rebate contract and revenue sharing contract, as well as the impact of different contracts on supply chain profits. The findings of this paper are mainly as follows:

(1) Compared with the traditional wholesale price pact, the impact of the quantity discount pact, sales rebate pact, and revenue sharing pact on the profit gains of supply chain members under the omni-channel model depends largely on the size of the demand for the BOPS channel, and the role of the BOPS channel is amplified. The ability to play the positive role of the covenants depends on the

ability to operate the BOPS channel reasonably, and at the same time, playing the role of the covenants well is inversely affected by the operation of the BOPS channel. Therefore, in the real management, the role of the contract and the active management of each member of the BOPS channel are important factors for the sustainable development of the omni-channel model.

(2) Under the traditional wholesale price contract, the profit difference between online manufacturers and offline retailers is large. Moreover, under the omni-channel model, the online manufacturer is the main body of profit gain, and the offline retailer participates in the operation of the BOPS model, but does not get a considerable return in profit gain. Thus, under the omni-channel model, when the sales volume of BOPS channel is all counted online, the online manufacturer becomes the first beneficiary of the omni-channel, and there is a gap in the coordination of profits in the supply chain, and it is necessary to look for the most suitable contract that can increase the optimal profits of the online manufacturer and the offline retailers, and also coordinate the benefits of the manufacturing and retailing companies in the distribution of profits in the supply chain on the basis of the traditional wholesale price contract.

(3) Under the quantity discount contract, the online manufacturer is still the main body of profit distribution revenue, so the positive effect of the quantity discount contract on incentivizing offline retailers to actively participate in the BOPS model is limited. Moreover, compared with the original traditional wholesale price contract, the conditions under the omni-channel model for the quantity discount to enable online manufacturers to gain more profits are very strict, and the quantity discount contract can only bring online manufacturers more profit gains than the wholesale price contract if the cost of the consumer's visit to the shop is very small. At the same time, the retailer's profit is not greater than the traditional wholesale price contract due to the quantity discount contract, but rather decreases. To sum up, the quantity discount contract cannot guarantee the profit gain of online manufacturers and offline retailers under the omni-channel model, and it has limited effect on the profit maintenance and incentive of offline retailers.

(4) The sales rebate contract is the most convenient supply chain contract in reality, and the supply chain members only need to agree on the value of the conditions for the contract to take effect and the size of the rebate, and then they can co-ordinate the profit relationship of the supply chain members in terms of profit. In this paper, the sales rebate contract in the omni-channel model enables the retailer to maintain a small retail price so as to ensure that the demand of consumers in the original channel will not be reduced, while the manufacturer makes up for the reduced profit due to the sales rebate by raising the wholesale price. So the coordination of the profit relationship between the online manufacturer and the offline retailer depends mainly on the size of the demand in the BOPS channel relative to the value of the conditions under which the pact takes effect. Compared to the original traditional wholesale price contract, the sales rebate contract increases the optimal profit of the online manufacturer, while the optimal profit of the offline retailer depends on the magnitude of the demand for sales in the BOPS channel. Thus, in the choice of the contract mechanism of the real omni-channel supply chain, the reasonable formulation of the sales volume threshold for the contract to take effect can make the profits of online manufacturers and offline retailers increase on the basis of the wholesale price contract at the same time, and make offline retailers become the main body of the supply chain's profits, which is not only a "win-win strategy" for online and offline gains, but also effectively incentivizes the offline retailers to make more profits. This is not only a "win-win strategy" for both online and offline gains, but also effectively incentivizes offline retailers to participate in the operation of the omni-channel model.

(5) Under the revenue-sharing contract, in the omni-channel model, the optimal wholesale price of the online manufacturer decreases while the optimal retail price of the offline retailer remains unchanged, which ensures that the optimal profit of the offline retailer increases in the traditional wholesale price contract, while the optimal profit of the online manufacturer is smaller than the traditional wholesale price contract. However, in the revenue-sharing contract, the profit relationship of supply chain members depends on the size of the revenue-sharing coefficient in the revenue-sharing contract, and when the revenue-sharing coefficient is greater than a specific value, the offline retailer

becomes the main profit gainer. From this point of view, the supply chain members can negotiate a reasonable revenue sharing coefficient that can have positive incentives for the offline retailers in the omni-channel model.

(6) Both the sales rebate contract and the revenue sharing contract can achieve Pareto optimization on the basis of the original wholesale price contract and, under certain conditions, effectively incentivize offline retailers to increase their participation in the BOPS channel. However, in the case of low demand for the BOPS channel, the online manufacturer of the sales rebate contract is able to earn more profit, while the revenue sharing contract enables the offline retailer to earn more profit; on the contrary, in the case of high demand for the BOPS channel, the revenue sharing contract enables the online manufacturer to earn more profit, while the sales rebate contract enables the offline retailer to earn more profit. Thus, it seems that the choice between a sales rebate contract or a revenue sharing contract on top of the original wholesale price contract depends on the demand volume of the BOPS channel. In this regard, in the selection of supply chain pacts under the realistic omni-channel model, improving the forecasting accuracy of channel demand can ensure that supply chain pacts play a better and positive role.

About future research. First, in this paper, the manufacturer is the leader of the omni-channel supply chain, so the manufacturer has more supply chain power. But after the omni-channel integration of online and offline, the synergistic development of both online and offline channels plays an important role in omni-channel operation, so in the later research, the research of omni-channel supply chain synergistic mechanism can be investigated in the case of comparable supply chain status of the manufacturer and the retailer. Secondly, the online manufacturer and offline retailer cost factors are not considered in the expression of the profit function in this paper, so in the later synergistic mechanism, the impact costs affecting the implementation of the omni-channel supply chain synergistic mechanism can be considered in a comprehensive way, such as the shop costs after the retailer provides BOPS services and the transport costs of the manufacturer to deliver the BOPS channel goods to the shop, as well as the process of waiting for the pick-up of BOPS orders. retailer inventory costs incurred in the process of waiting for BOPS orders to be picked up.

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