Comparative Analysis of Viscosity Measurement Techniques Poiseuille Method versus Falling Ball Method

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Abstract. In fluid dynamics and related fields, accurately measuring liquid viscosity is vital. The traditional falling ball method, common in education and research, faces challenges in ensuring uniform fall and correcting for phenomena like eddy currents. This thesis introduces a new device based on the Poiseuille method, measuring viscosity through pressure difference and adjustable flow rates. Comparing it with the falling ball method, we found the latter has an average error of 5.21%, while the Poiseuille method has only 1.23%. The Poiseuille method avoids issues like Reynolds coefficient correction and inaccuracies from high ball speeds, offering stability and practical value.

Keywords: Poiseuille method, falling ball method, viscous coefficient, flow rate control, inaccuracy analysis.

1. Introduction

In fluid dynamics, accurately measuring liquid viscosity is crucial for various industrial and scientific uses, as well as for understanding basic fluid properties. Traditional methods like the falling ball technique, though common, face practical challenges. This method requires precise control of the ball's falling speed and complex adjustments for issues like eddy currents, which are difficult in laboratory settings. Therefore, finding simpler and more precise methods for viscosity measurement is a key research focus in this field.

Previous research indicates the falling ball method for measuring viscosity has limitations, necessitating complex corrections due to vortex effects and the interaction between the tube wall and ball, as highlighted by Dan Cai et al [1]. Although modifications like Yangguang Zhang et al.'s multi-tube method [2] and Shilei Zheng et al.'s floating ball method [3] have been proposed, these adaptations still face issues like time-consuming procedures and complex corrections. The fundamental principle of the falling ball method contributes to these drawbacks. Alternative approaches include Liangke Wang et al.'s use of the least squares method [4], which enhances accuracy but still relies on the falling ball principle, and Xingting Pu et al.'s application of the Poiseuille method [5], which lacks flow rate control and data processing solutions. However, the Poiseuille method's broader potential in fluid mechanics is supported by studies like Shugu Song et al. on pulsating micro-flow [6], Renjia Qin's work on Poiseuille flow velocity [7], Liping Lu et al.'s research on positive polygonal flow tubes [8], and Yingxun Xu et al.'s analysis and experimental validation of the method [9]. Therefore, employing the Poiseuille method for viscosity measurement is considered feasible in principle.

Our research group is exploring a new method to measure liquid viscosity using the Poiseuille method, comparing it with the traditional falling ball method. We derive the viscosity coefficient using Newton's and Poiseuille's formulas, involving easily measurable physical quantities. Our experimental setup uses height differences to indicate pressure variations, controls flow rate via the height of the orifice, and measures flow rate with an electronic balance. We also consider temperature effects and apply the least squares method for error analysis. This study diversifies viscosity measurement techniques and enhances the accuracy of our method.

2. Materials and Methods

2.1. Fundamental Physical Principle

2.1.1. Newton's Formula of Viscous Lag

Consider a fluid that is between two large flat plates of equal area, which are separated everywhere by a small distance, and the system was originally at rest. Suppose that the upper plate is allowed to move in the x-direction with a constant velocity v and that a thin layer of fluid immediately below the moving plate moves with the same velocity. When v is not too large, the fluid between the plates will form a steady laminar flow. The fluid near the moving plate has a greater velocity than the fluid farther away from the plate, and the farther the thin layer is from the plate, the smaller the velocity becomes, dropping to zero at the stationary plate. This velocity changes continuously according to a certain curvilinear law, and this change in velocity along the distance is called the velocity distribution.

Let a stream layer velocity for v, with its neighboring stream layer velocity for v + dv, dv is the value of its flow rate change, set the difference between the stream layer along the y-axis distance for dy, if the distance between the two boards is very small, then the flow rate between the two boards is infinitely close to the linearity of flow rate change, which is flow rate gradient.

Then let F be the internal friction between the layers of the fluid, and the contact area between the fluids is S. A large number of experiments have proved that the magnitude of the internal friction of the fluid is related to the nature of the fluid, and is directly proportional to the gradient of the change in the velocity of the fluid and the contact area, which is Eq.1.

$$F_\eta = \eta \frac{dv}{dr} \mathrm{S} \tag{1}$$

2.1.2. Poiseuille's Formula

Take a section of the liquid column, for which there is Eq. 2.

$$F_\eta = -(p_1 - p_2)\pi r^2 \tag{2}$$

For S in Newton's law of viscosity, which is the contact area between different liquid layers, in a section of a liquid column, it is the lateral area of its cylinder, which is $2\pi rl$. Combining Eq. 1 and Eq. 2, we can obtain Eq.3.

$$(p_1 - p_2)\pi r^2 = -2\pi rl\eta \frac{dv}{dr} \tag{3}$$

After shifting the term and integrating it to get the fluid in the equal cross-section of the horizontal thin circular pipe in the stable flow, we can get the relationship of the flow velocity v and the radius r in Eq.4.

$$v = \frac{(R^2 - r^2)}{4\eta l}(p_1 - p_2) \tag{4}$$

This gives the differential expression for the flow rate as Eq.5.

$$dQ = vds = v2\pi rdr = \frac{p_1 - p_2}{4\eta l}(R^2 - r^2)2\pi rdr \tag{5}$$

Integration leads to Eq.6.

$$Q = \frac{\pi R^4 (p_1 - p_2)}{8\eta l} \tag{6}$$

2.1.3. Application of Poiseuille's Formula

Because of the difficulty of determining flow and pressure differences and the need for more complex instrumentation, some deformation of Poiseuille's formula is required to enable it to be characterized by simpler instrumental. From this, we can think that although p is not easy to measure, the height of the liquid column caused by p can be measured. Although Q between the tubes is not easy to measure, we can convert the flow to mass difference, and thus conveniently measure by an electronic balance.

The deformation of the Poiseuille formula yields Eq.7.

$$\frac{\Delta m}{\rho \Delta t} = \frac{\pi R^4 \rho g \Delta h}{8 \eta l} \tag{7}$$

Then the expression for the coefficient of viscosity can be obtained as Eq.8.

$$\eta = \frac{\pi R^4 \rho^2 g \Delta h \Delta t}{8 l \Delta m} \tag{8}$$

2.2. Formation of Experimental Instrumentation

Concerning Poiseuille's formula, to measure the coefficient of viscosity of a liquid, we need to simultaneously measure two parameters, the flow rate of the liquid (characterized by the mass difference divided by the time multiplied by a constant) and the difference in height of the liquids separated by a certain length in a flow environment. To create the flow environment, we use a certain height of castor oil and use the pressure of the liquid to push the liquid to flow. Also, By measuring the flow rate at the end, the flow rate of the entire section can be replaced. Then, the flow rate can be measured by the difference in mass over a period when the liquid is received at the end. At the same time, a thin tube is established at a certain distance, through the average of the difference in height of the liquid level of each tube into the formula, we can obtain the viscosity coefficient.

Based on the above conceptualization, my group set up the experimental setup on our own. The setup consists of three main sections: Flow Control Module, Pressure Measuring Module, and Current Measurement Module.The setup overview is shown in Figure 1.

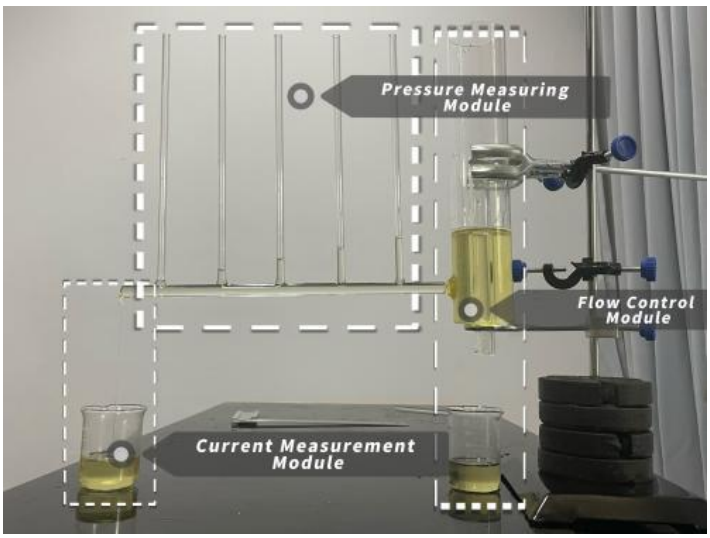

Figure 1. Overview of instrumentation

The flow control module (shown in Figure 2) utilizes the pressure generated by the height of the liquid to make the liquid flow to the right, which facilitates the measurement of each parameter. At the same time, after controlling the constant height of the liquid level of this device, then the power brought about by the liquid pressure is constant (P=ρgh), to achieve the effect of a constant flow rate.

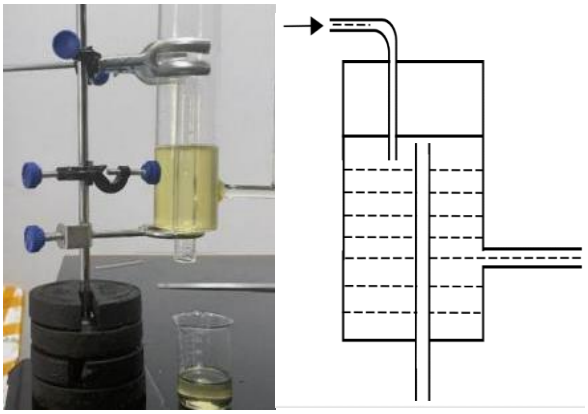

Figure 2. Flow control module

The pressure measuring module (shown in Figure 3) uses the liquid level heights of the tubules to characterize the pressure at various locations in the main side tube. The average height difference between neighboring tubes is obtained by measuring the liquid surface height of each tube, and then combined with the distance between the tubes, the corresponding data in the formula can be obtained. The advantage of this module is to minimize random errors by measuring multiple sets of height difference data in a single pass.

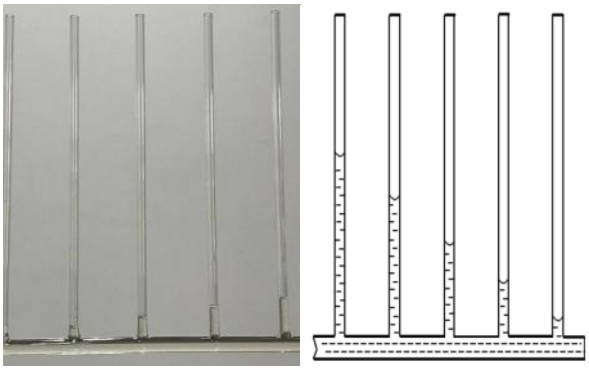

Figure 3. Pressure measuring module

The current measurement module (shown in Figure 4) utilizes the measurement of the mass of the liquid flowing from the right-hand side of the tube within a certain period, combined with a series of weighing operations of the electronic balance, to obtain accurate data.

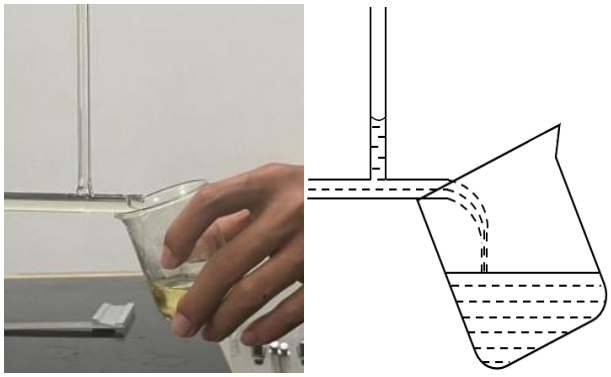

Figure 4. Current measurement module

2.3. Experimental Procedure

First, the apparatus such as the beaker and customized glass tubes were cleaned using standard alcohol solution and the glass unit was fixed on the iron frame table through condenser tube clamps so that its position fit with the small iron ring at the lower end, and the position was adjusted so that the main side tubes were horizontal. The distance between the tubes and the inner diameter of the

main side tube was measured several times with vernier calipers and recorded as instrument parameters. Following this, the castor oil reagent was removed from the sealed and light-protected reagent, the customized glass tube outlet was plugged with a stopper, and the thicker storage tube was filled with castor oil so that the liquid level reached the height of the flow control tube. Keep adding castor oil to the storage tube to maintain the level at the height of the flow control tube. Preheat the electronic balance place the beaker on the electronic balance for tare operation and measure the temperature of the castor oil reagent at this point. Remove the stopper and when the difference in liquid level of the different glass columns has stabilized, start collecting the effluent castor oil in a beaker and time it, while reading the liquid level height of each glass tubule with a steel straightedge. At the same time, the liquid level in the storage tube was maintained at N1 by replenishing the liquid in the storage tube, and the collection of castor oil was stopped after a certain period. The interval was read and the beaker was placed on the tared electronic balance and read out. The castor oil reagent temperature was again measured and the average value was calculated by combining it with the initial temperature. Finally, the height position of the flow control tube in the initial coarser storage tube was changed and the above experiment was repeated. The experimental data was recorded and analyzed.

3. Viscosity Comparison of Poiseuille Method and Falling Ball Method

3.1. Falling Ball Method for Measuring Liquid Viscosity

To compare the falling ball method with Poiseuille's method, our group first carried out the experiments of the falling ball method to measure the viscous coefficient of liquid, including the measurement of instrumental parameters, measurement of experimental parameters, and post-correction.

Table 1. Instrument parameters

Experiment Number	Radius (ball 1)	Radius (ball 2)	Pipe wall radius
1	1.9775	1.2955	149.550
2	1.9775	1.2950	149.550
3	1.9750	1.2955	149.550
4	1.9755	1.2950	149.500
average value	1.9764	1.2952	149.540

Table 1 is measured by using vernier calipers to first measure the diameter of the pipe wall and the ball, and then get the corresponding radius. The zero reading of the vernier calipers for the above measurements is 0.001mm, and the units are all in mm.

Table 2. Experimental parametersn (falling ball methods)

Experiment Number	drop time (Ball 1)	drop time (Ball 2)
1	2.4525	4.2900
2	2.4925	4.2950
3	2.5075	4.1700
4	2.4800	4.2750
5	2.5250	4.3650
average value	2.4915	4.2790

The falling length corresponding to the falling time in Table 2 is 0.175m, which is selected in the middle region of the falling in the experiment where the ball has been moving at a uniform speed.

Combined with the above data, we can use the falling ball method to get the theoretical value of the viscous coefficient and the correction value, where the correction method is shown in Eq.9, Eq.10, and Eq.11.

$$\eta_1 = \frac{(m-\frac{4}{3}\pi r^3\rho)g}{6\pi r v_0(1+2.4\frac{r}{R})(1+3.3\frac{r}{h})} \tag{9}$$

$$\eta_2 = \eta_1(1+\frac{3}{16}Re-\frac{19}{1280}Re^2)^{-1} \tag{10}$$

$$Re = \frac{2rv_0\rho}{\eta_1} \tag{11}$$

Multiple measurements of the experimental data were taken and the experimental results obtained are shown in Table 3.

Table 3. Falling ball method measurement results

Experiment Number	η	η_1	Re	η_2	T	η (standard) [10]
1	0.835924	0.679686	0.392203	0.634476	25.1	0.58-0.6
2	0.821201	0.654433	0.401132	0.635243	25.1	0.58-0.6
3	0.825032	0.664372	0.384323	0.637642	25.1	0.58-0.6
4	0.827893	0.667385	0.385738	0.638275	25.1	0.58-0.6
5	0.822423	0.700064	0.167687	0.678999	24.8	0.6-0.64
6	0.822331	0.701124	0.167872	0.678322	24.8	0.6-0.64
7	0.823574	0.702243	0.170324	0.658274	24.8	0.6-0.64
8	0.823472	0.705466	0.198324	0.668925	24.8	0.6-0.64
9	0.827384	0.702948	0.184245	0.673526	24.8	0.6-0.64

It can be seen that the final results of the liquid viscosity coefficient measured by the falling ball method have some deviation from the confidence range, and the average error is 5.21%. Meanwhile, the post-processing of the experimental data of the falling ball method involves two corrections, which have high requirements for the processing of complex calculations and the accuracy of instrumental measurements.

3.2. Poiseuille Method for Measuring Liquid Viscosity

The instrumental parameters measured using the Poisson Leaf method are shown in Table 4. where R denotes the inner diameter of the main side tube and d denotes the distance between each pressure measuring tube. The units are mm.

Table 4. Instrument parameters(poiseuille method)

Experiment Number	R	d1	d2	d3	d4
1	0.375	59.75	61.21	55.5	58.71
2	0.378	59.71	61.19	55.28	58.83
3	0.379	59.79	61.2	55.88	58.63
4	0.378	59.74	61.1	55.34	58.62
5	0.377	59.77	61.13	55.36	58.71
average value	0.3774	59.752	61.166	55.472	58.7

The data measured in the experiment are shown in Table 5, where H is the height of the storage tube, which is used to control the flow rate, and h is the height of the liquid level in each manometer tube. The units of the following data are all in meters.

Table 5. Liquid level measurement

Experiment Number	H	h1	h2	h3	h4	h5
1	0.1200	0.1100	0.0960	0.0835	0.0720	0.0600
2	0.1300	0.1170	0.1030	0.0890	0.0760	0.0615
3	0.1400	0.1265	0.1110	0.0940	0.0800	0.0640
4	0.1500	0.1340	0.1170	0.0990	0.0830	0.0650
5	0.1600	0.1435	0.1235	0.1035	0.0850	0.0660
6	0.1700	0.1520	0.1310	0.1080	0.0885	0.0670
7	0.1800	0.1590	0.1360	0.1130	0.0920	0.0690
8	0.1900	0.1680	0.1440	0.1180	0.0950	0.0710
9	0.2000	0.1770	0.1515	0.1235	0.0995	0.0735

At the same time, the mass of the outflowing liquid is measured at any interval of time (which is shown in Table 6), and multiple measurements are taken to visually determine the stability of the flow rate in conjunction with the residuals of the least-squares method.

Table 6. Flow measurement

Experiment Number	m1/kg	t1/s	m2/kg	t2/s	m3/kg	t3/s
1	0.01542	51.38	0.01406	46.73	0.01859	61.94
2	0.01539	51.09	0.01323	43.15	0.01311	42.24
3	0.02126	55.51	0.02197	58.38	0.01765	48.08
4	0.01384	33.57	0.01937	49.19	0.01860	46.46
5	0.03490	75.78	0.02651	56.02	0.02189	46.17
6	0.02339	48.30	0.02680	45.97	0.02439	48.31
7	0.02208	42.47	0.01975	37.50	0.02350	44.95
8	0.02874	54.81	0.02390	44.82	0.02475	46.12
9	0.03853	63.51	0.02633	43.09	0.02926	46.67

The next step is to process the experimental data. Our group used the least squares method to fit △h-l, △m-△t linearly, which can get the data in Eq.8. and calculate the viscous coefficient.

As an example, for experiment number 1, the results of the least-squares fitting obtained by using the Python program are shown in Figure 5 (where the horizontal coordinate is determined by d in the instrument parameters, and the horizontal coordinate of the manometer tube closest to the flow-control tube is noted as zero; the vertical coordinate is determined by the height of the liquid level).

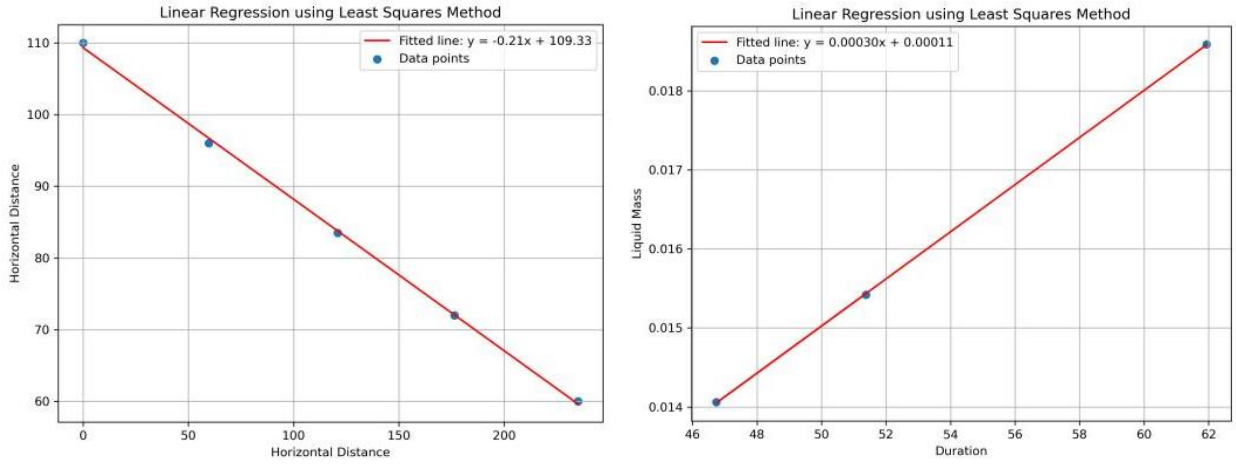

Figure 5. Least squares fitting plot

The Python program can be used to calculate that the mean square error of △h-l is 0.23 and that of △m-△t is 0.0001, so it can be seen that the straight line fitted by the least squares method can reflect the characteristics of the original data accurately.

Using the above method, the results of the liquid viscosity coefficient calculated are shown in Table 7.

Table 7. Poiseuille method measurement results

Experiment number	T/℃	△h/l	△m/△t	η	η (standard) [10]
1	27.9	0.211356	0.0003004339	0.485305	0.49-0.53
2	27.1	0.235141	0.0003084603	0.525869	0.49-0.53
3	27.8	0.265874	0.0003723056	0.492636	0.49-0.53
4	27.6	0.293087	0.0003969369	0.509359	0.49-0.53
5	28.1	0.329747	0.0004736134	0.480293	0.47-0.49
6	28.5	0.362148	0.0005426342	0.460393	0.47-0.49
7	27.5	0.381710	0.0005245028	0.502036	0.49-0.53
8	27.1	0.414101	0.0005349762	0.533975	0.49-0.53
9	27.9	0.441385	0.0006194237	0.491563	0.49-0.53

From the above data, the relative uncertainty of η is obtained to be 1.32% and thus the experiment is quite accurate.

4. Advantageous Analysis of Poiseuille Method over Falling Ball Method

During the experiment, we found that the falling ball method has the following disadvantages:

·inaccurate time measurement due to excessive ball speed.

·The ball cannot be guaranteed to fall from the center of the tube.

·Measuring the diameter of the ball and removing the ball after the test increased the difficulty of the experiment.

In addition, the biggest advantage of the Poiseuille method over the ball drop method is that the measured data is more accurate and does not require the Reynolds coefficient and other cumbersome corrections. Results of the viscosity coefficient measured by the falling ball method and the viscosity coefficient measured by the Poiseuille method are compared in Figure 6:

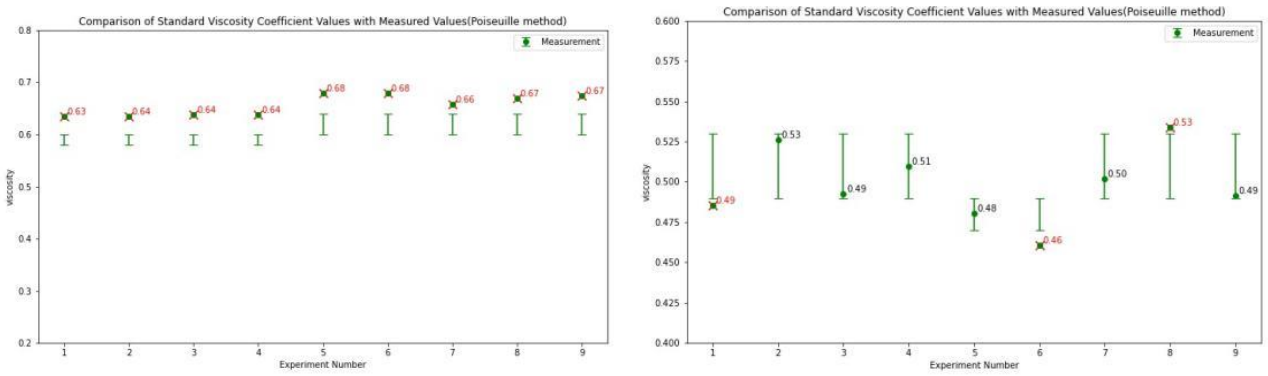

Figure 6. Comparison of the results of the Poiseuille method and the falling ball method

It can be seen that the viscous coefficient value measured by the falling ball method deviates from the standard value interval, the measured value is large, and the average error reaches 5.21%; while the viscous coefficient value measured by the Poiseuille method falls within the standard value interval in most of the experiments, and the average error is 1.32%. Thus, the advantages of the Poiseuille method can be visualized.

5. Conclusion

In this research, a set of viscous coefficient measurement devices with strong application prospects is fabricated based on the Poiseuille method, which improves the accuracy by 3.89% compared with

the traditional falling ball method, and each parameter (e.g., flow rate) is controllable and avoids the complex correction in the drop-ball method.

The project not only carried out the principle of innovation, through the Poisson Leaf method to avoid the complexity of the Reynolds data processing links and the control of uniform velocity, in the flow environment to get each parameter; but also carried out the visualization of innovation, through the intuitive height of the liquid surface to express the flow of liquid at all places of the invisible pressure, through the intuitive mass difference per unit of time to express the flow of the liquid; at the same time, through the more innovative glass structure, the control of the height of the liquid level in the storage tube is realized, to maintain a constant flow rate and facilitate subsequent measurements. In the field of teaching, the Poiseuille method of measuring viscous coefficient instrument cost is low, the instrument operation is simple, and the experimental phenomenon is clear, at the same time, compared with the falling ball method Poiseuille method is more interesting, digging out the relevant knowledge of fluid mechanics. Also, in the research field of viscous coefficient, the liquid viscous coefficient in production, life, engineering technology, and medicine has an important use, this project through the knowledge of fluid dynamics to measure the viscous coefficient, to broaden the field of research and enrich the understanding of the viscous coefficient.

In addition, this study also reveals a research idea and framework for use in the field of fluid dynamics and demonstrates the feasibility of measuring viscous coefficients in fluid dynamics using the Poiseuille method through data comparisons, which is of value for a wide range of applications.

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