

# Modeling the Impact of Lamprey Population Dynamics on Aquatic Food Webs: A Leslie and Lotka-Volterra Approach

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**Abstract.** In recent years, the widely distributed invasive parasitic species of marine lampreys have had a profound impact on host populations and the entire aquatic ecosystem through their unique survival and reproduction strategies, highlighting the importance of species interactions and ecological balance in ecosystems. In response to this, this study establishes a species growth model for lampreys based on the Leslie model from the perspective of gender ratio changes. The idea of the Lotka-Volterra predator-prey model is used to establish the food web centered on lamprey and the fluctuation mode model of various groups in it. Based on the model results, we conclude that the growth trend of the lamprey itself is difficult to be affected by other populations, but the lamprey population will have serious adverse effects on other species. The results of this model are in good agreement with the actual situation, and have profound significance in laying the foundation for the study of the community characteristics of lampreys and their related ecosystems.

**Keywords:** Leslie model; Logistic growth model; Lotka-Volterra predator-prey model; sex ratio.

## 1. Introduction

The adaptive sex ratio variation of animals is one of the eye-catching phenomena in the field of biology. The sex ratio of marine lampreys may vary depending on the external environment. Sea lampreys become male or female depending on how quickly they grow during the larval stage<sup>[1]</sup>. This adaptive sex ratio variation may have a profound impact on the ecological role of lampreys and interactions with other species. For example, the open waterway economy in the United States has led to the invasion of lampreys into the Great Lakes, which has had a major impact on the original ecology<sup>[2][3]</sup>. Therefore, it is of unique significance to explore the ecological effects of lampreys adjusting their sex ratio under different resource availability conditions through model development.

To describe the change in the sex ratio of lampreys, we constructed a sex ratio change model based on the Leslie Matrix, a mathematical method introduced by scientist Leslie PH<sup>[4]</sup> in 1945 that uses the age distribution of a population at a certain initial moment to dynamically predict the evolution of the age distribution and population over time<sup>[5]</sup>. In addition, we used the Logistic model and Lotka-Volterra predator-prey model to explore its more profound impact on the ecological environment. The former is often used to predict population density changes, such as JiaYong Pan et al. used it to study the growth of fungal communities<sup>[6]</sup>, while the latter is used to describe the relationship between predators and prey, and then to study more complex ecological problems such as biological invasion<sup>[7]</sup> and population growth in special environments<sup>[8]</sup>.

The data on the sex ratio of Lampreys in the Great Lakes in this paper are derived from the studies of Johnson N S, Swink WD, Brenden TO, et al.<sup>[1]</sup>, and Docker M F's *Lampreys: Biology, Conservation and Control Volume*<sup>[9][10]</sup>. In this study, we first established the correlation between the sex ratio of the lamprey population and population density, and based on this, we established the growth model of the lamprey population. The model calculates the number of females in a population and then reflects the fertility rate through the sex ratio. Then, we established the model of other species in the ecosystem where lampreys live, and the population evolution of fish, parasites, and other species was analyzed with this model. Finally, we model the lamprey-centered food web and the interactions between various groups within it. Based on this, models can be built to analyze its impact on the larger ecosystem.

2. Lamprey Sex Ratios and Food Chain Dynamics

Due to the unpredictability of environmental changes and human activities, and considering the normal range of activity throughout the life of the lamprey as the research object, in this article, we ignored the effects of environmental changes and human intervention on the reproduction of lampreys and did not consider the migration and migration of the lampreys population. We took the normal range of lampreys' life as the research object, and the change in population number only depended on fertility and mortality. We first established the relationship between the sex ratio and population density of lampreys and then constructed an improved Leslie population evolution model. Based on the Lotka Volterra predator-prey model, we considered the interactions between different species in the food web centered on lampreys.

2.1. Gender ratio and population density model of lampreys population

Firstly, data from a study, *Field study suggests that sex determination in sea lamprey is directly influenced by the larval growth rate*, done by Johnson N S, Swink WD, Brenden TO et al <sup>[1]</sup>, the population density and male proportion of lampreys in Great Lakes are as shown on table 1;

Table 1 Population density and male proportion of lampreys in small streams

|                    |       |       |       |       |       |       |       |       |       |       |       |
|--------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Population density | 0.462 | 0.251 | 0.072 | 0.148 | 0.078 | 0.044 | 0.003 | 0.009 | 0.003 | 0.005 | 0.125 |
| Male proportion    | 0.71  | 0.62  | 0.46  | 0.48  | 0.4   | 0.38  | 0.36  | 0.42  | 0.32  | 0.38  | 0.53  |

We found that there is a certain correlation between the sex ratio of lampreys and the population density. The reason is that the population size is negatively correlated with the availability of food, while the availability of food is negatively correlated with the male ratio, and Docker M F et al have done some research on the correlation between the two <sup>[10]</sup>. We set the male ratio of the lamprey population to  $m$ . Consider using the following Function fitting:  $m = 1 - e^{-ax_{rou}+b}$ . where  $x_{rou}$  groups the population density, and  $a$  and  $b$  are the coefficients related to population density. This function can ensure that the male proportion  $m$  will not exceed 1. We performed empirical fitting and repeated iterations, taking the abscissa as the population density and the ordinate as the proportion of male individuals in the entire population. It was finally determined that  $a = 2.0, b = -0.4$ , the fitting results are shown in figure 1. So, the relationship between population density and sex ratio was obtained:

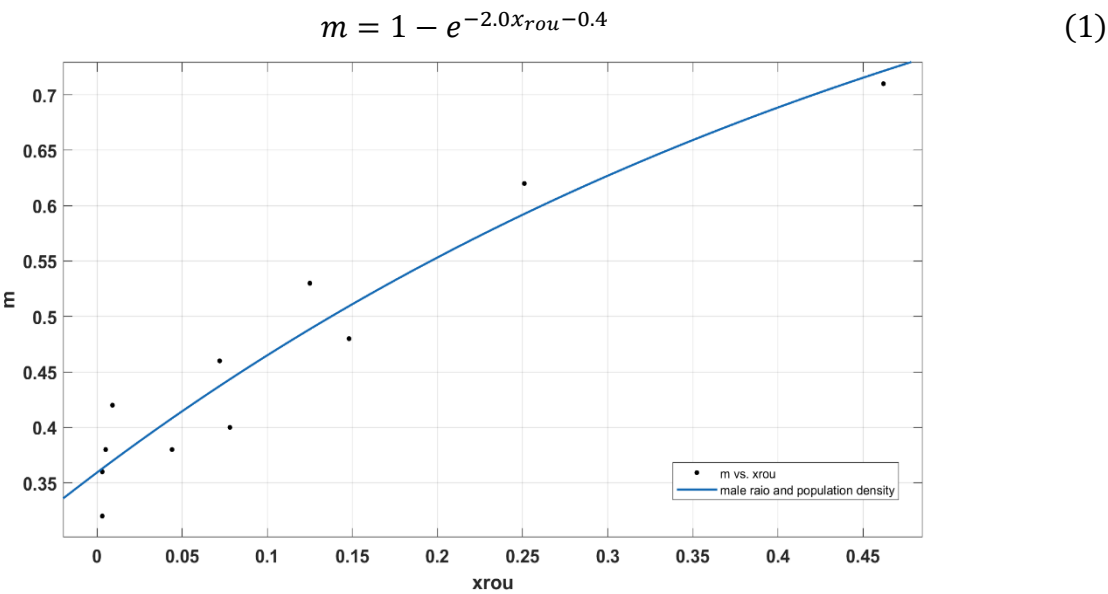


Figure 1 Fitting curve between sex ratio and population density of lampreys

## 2.2. Establishment of Improved Leslie Model

Based on the statistical results of the lifespan of lampreys, we set the life span of lampreys  $L_t$  as 7 years and divide  $[0, L_t]$  into  $n = 7$  age groups, each with a length of 1 year. Therefore, these  $n$  age groups are:

$$\left[0, \frac{L_t}{n}\right), \left[\frac{L_t}{n}, \frac{2L_t}{n}\right), \dots, \left[\frac{(n-1)L_t}{n}, L_t\right) \quad (2)$$

Because of the short incubation time of lamprey's eggs<sup>[1]</sup>, lampreys can be ignored as the egg stage. In addition, according to the life span of lampreys is about seven years, and the larval stage is about four years according to its growth and development law<sup>[9]</sup>, so this paper assumes that years 1-4 are the larval stage and years 5-7 are the adult stage.

The number of species in each age group in year  $k$  is recorded with the vector  $x^{(k)}$ :

$$x^{(k)} = [x_1^{(k)}, x_2^{(k)}, \dots, x_7^{(k)}]^T, k = 0, 1, 2 \dots \quad (3)$$

Among them, the superscript  $(k)$  represents the  $k$ -th year of population evolution, and the subscript number represents the corresponding  $i$ -th age range.

The Leslie matrix is:

$$L = \begin{bmatrix} a_1 & a_2 & \cdots & a_{n-1} & a_n \\ b_1 & 0 & \cdots & 0 & 0 \\ 0 & b_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & b_{n-1} & 0 \end{bmatrix} \quad (4)$$

Among them,  $a_i$  is the fertility rate for the  $i$ -th age group,  $b_i$  is the survival rate of the  $i$ -th age group ( $i = 1, 2, \dots, n$ ). When considering the evolution of the lamprey species alone,  $b_i$  is a constant.

Docker MF also pointed out that female lampreys died within one week after laying eggs, and there was a polygamous system. When the proportion of males was too high, the fertility rate would be limited by the number of females<sup>[9][10]</sup>. Therefore, if the population of lampreys had fertility in the last year of life, the fertility rate of female lampreys would be negatively correlated with the proportion of males.

we let  $a_1 = a_2 = \dots = a_6 = 0$ ,  $a_7 > 0$ , and it is a constant value, in addition, due to the negative correlation between female fertility rate and gender ratio  $m$ , specifically  $h - jm$ . Males do not have a fertility rate, therefore the total fertility rate is  $a_7 = (1 - m)(h - jm)$ , where  $h$  and  $j$  are linear fitting coefficients, and their values are related to resource availability. Based on the population size of the previous year, we determine  $m$  as

Based on the population size of the previous year, we determine  $m$  as follows:

$$m = 1 - e^{-2.0x^{(k-1)} - 0.4} \quad (5)$$

To represent the impact of population density in the previous year on the gender ratio in that year. According to the Leslie model, there are:

$$x^{(k)} = Lx^{(k-1)}, k = 0, 1, 2 \dots \quad (6)$$

Based on this model, we only need to provide the number of species  $x^{(0)}$  for each age group in the initial (year 0) to obtain the number of species for each age group in the  $k$ -th year, that is:

$$x^{(k)} = L^k x^{(0)}, k = 0, 1, 2 \dots \quad (7)$$

The number of species of lampreys  $M$  is obtained by adding the items in  $x^{(k)}$ :

$$M = \sum_{i=1}^n x_i^{(k)} \quad (8)$$

### 2.3. Build a food chain model

In this section, we modifications to the above model and construct a growth model for newly added species based on the Lotka Volterra predator-prey model.

We introduce a fish species related to the lamprey into the environment of the model, which is referred to as fish in the following text. On the one hand, the survival rate of lamprey's eggs mainly depends on the probability of the eggs leaving the nest<sup>[9]</sup>. To simplify the model, the effect of fish on the reproduction rate of lampreys is ignored. On the other hand, the main source of nutrition for adult lampreys is parasitic fish, and the survival rate of parasitized fish is extremely low. Therefore, it is considered to describe the relationship between adult lampreys and fish using the relationship between predation and prey.

Considering that the survival rate of adult lampreys is related to the number of fish:

$$b_5 = b_6 = p + qy^{(k-1)}, k = 0, 1, 2 \dots \quad (9)$$

Among them,  $b_5$ 、 $b_6$  represents the survival rate of adult lampreys, and  $y^{(k)}$  represents the population size of fish in the  $k$ -th year, here  $p$  and  $q$  are linear fitting coefficients. The values of the two are influenced by environmental resources and jointly regulate the survival rate of lampreys. However, we consider that the survival rate and fertility rate in childhood are still not affected by the number of fish.

(1)The growth model of fish

We made adaptive modifications based on the differential form of the logistic model and established the natural evolution model of the fish population as follows:

$$y^{(k+1)} - y^{(k)} = r \left( 1 - \frac{y^{(k)}}{Y} \right) y^{(k)}, k = 0, 1, 2 \dots \quad (10)$$

In the formula,  $r$  is the natural growth rate of the fish,  $Y$  is the environmental capacity of the fish, and  $y^{(k)}$  is the population size of the fish in the  $k$ -th year.

From the definition of the population of lampreys mentioned above, we can conclude that the total number of adult lampreys  $x_a^{(k)}$  is:

$$x_a^{(k)} = \frac{x_5^{(k)} + x_6^{(k)} + x_7^{(k)}}{1 - m} \quad (11)$$

Define the effective contact number between fish and adult lampreys as  $d$ , and represent the decrease in population growth due to the influence of lampreys in the form of the product. The population of fish should:

$$y^{(k+1)} - y^{(k)} = r \left( 1 - \frac{y^{(k)}}{Y} \right) y^{(k)} - dx_a^{(k)} y^{(k)}, k = 0, 1, 2 \dots \quad (12)$$

Namely

$$y^{(k+1)} = \left[ r \left( 1 - \frac{y^{(k)}}{Y} \right) - dx_a^{(k)} + 1 \right] y^{(k)}, k = 0, 1, 2 \dots \quad (13)$$

Combining  $x^{(k)} = Lx^{(k-1)}$ , if  $x^{(0)}$  and  $y^{(0)}$  are given, the population evolution relationship between lampreys and fish in the  $k$ -th year can be determined.

(2)Population model after adding parasites

Based on the model of lampreys and fish mentioned above, we added parasitic species. It is assumed that the parasite is a type that only parasitizes fish. Therefore, it can be assumed that there is no direct connection between the parasite and the lamprey and only a relationship between the parasite and the fish needs to be established.

In the model, we introduce the parasitic coefficient  $p_{zy}$ , to represent the contribution of parasites to fish mortality rate; And the parasitic coefficient  $p_{yz}$  represents the contribution of fish to the growth rate of parasites. And with  $r_z$  represents the natural mortality rate of parasites in the absence of a host. We express the number of parasites in the k-th year as  $z^{(k)}$ , The improved model is:

$$y^{(k+1)} = \left[ r_y \left( 1 - \frac{y^{(k)}}{Y} \right) - d_2 x_a^{(k)} + 1 \right] y^{(k)} - p_{zy} y^{(k)} z^{(k)}, k = 0, 1, 2 \dots \quad (14)$$

$$z^{(k+1)} = (1 - r_z) z^{(k)} + p_{yz} y^{(k)} z^{(k)}, k = 0, 1, 2 \dots \quad (15)$$

At this point, the population model containing three species: lampreys, fish, and parasites has been established. By changing the expression of male proportion  $m$  in the model to a constant value of 0.5, the evolution of populations without adaptive changes in gender proportion corresponding to lampreys can be simulated. For the convenience of description, the following text uses ratio change and ratio unchanged to represent the two situations of lampreys and other conventional species.

### 3. Lamprey Dynamics and Ecosystem Health

This chapter calculates the evolution of the population of lampreys when their sex ratio adapts to changes based on established models, the evolution when lampreys and fish coexist, and the evolution when lampreys, fish, and parasites coexist. Based on this, analyze its significance.

#### 3.1. Consider the situation of a single species separately

When considering the lamprey separately and controlling the initial population capacity to be the same, the calculation results of population density over time are shown in figure 2:

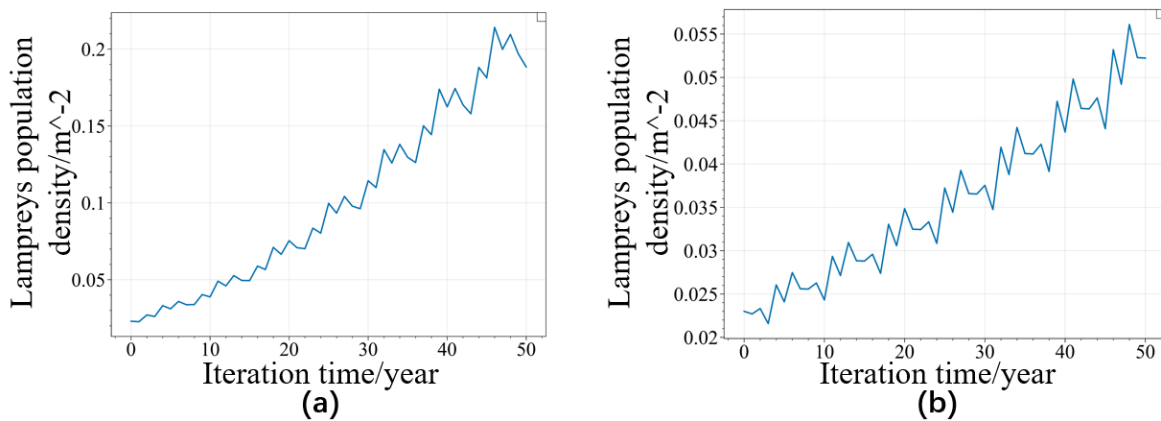


**Figure 2** The graph of the population density of lampreys over time. Where (a) the situation when the sex ratio changes, and (b) the situation when the sex ratio does not change

It can be observed that in actual situations, when the male ratio changes, the growth rate is faster and it is easier to reach a steady state, with better stability. It is not difficult to understand the practical significance of the adaptive changes in the sex ratio of lampreys in nature.

The population density evolution curve of fish when they exist alone is shown in figure 3:

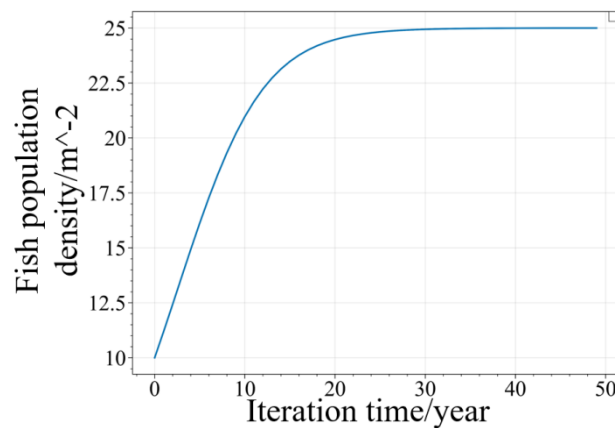


**Figure 3** The temporal variation of fish population density

It can be seen that the number of fish reaches the environmental capacity after some time. The differential evolution of the Logistic model by the natural state.

### 3.2. Simultaneously considering the situation of lampreys and fish

When fish and lampreys coexist, adult lampreys prey/parasitize the fish. The calculation results are shown in figure 4 and 5.

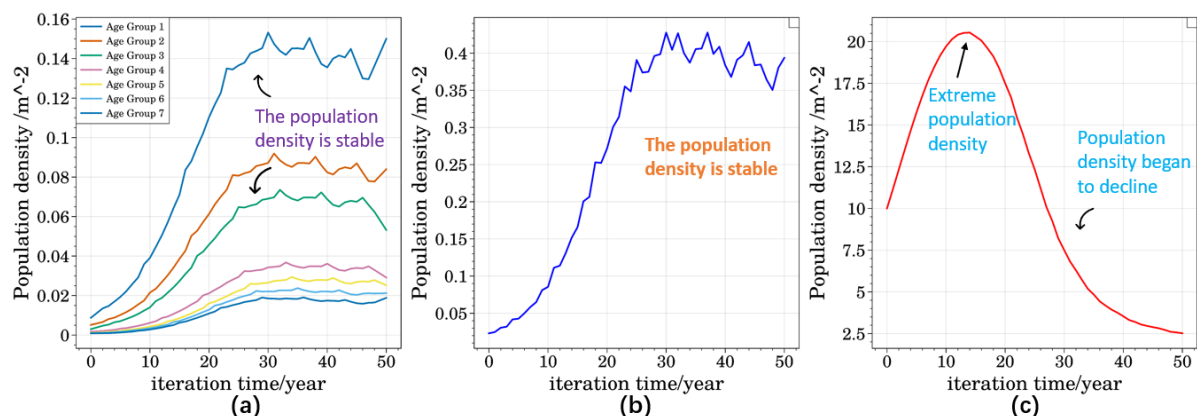


**Figure 4** The situation when the sex ratio of lampreys changes. Where (a) is the population density of lampreys in different age groups, (b) is the overall population density of lampreys, (c) is the population density of fish

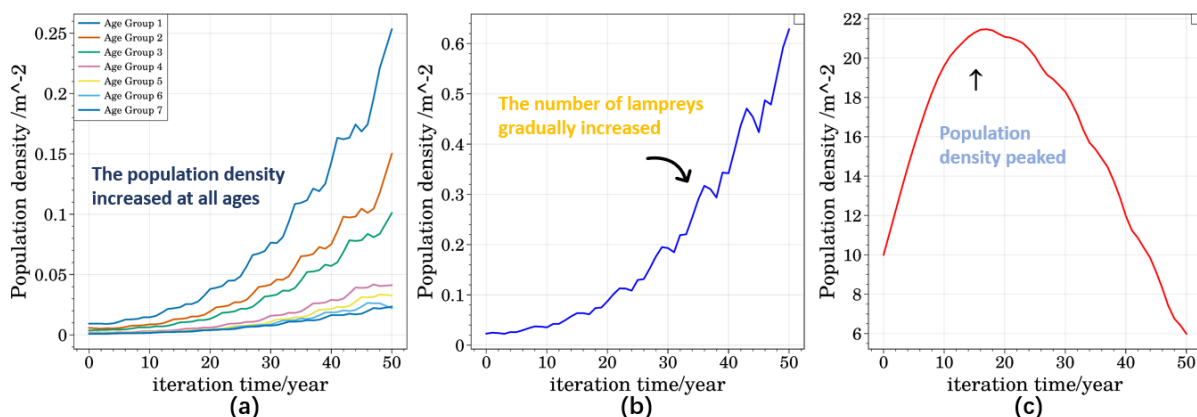


**Figure 5** The situation when the sex ratio of lampreys remains unchanged. Where (a) is the population density of lampreys in different age groups, (b) is the overall population density of lampreys, (c) is the population density of fish

Based on the above calculation results, it can be seen that when the number of lampreys is low, the fish can grow normally; When the number of lampreys increases to a certain extent, the number

of fish rapidly decreases but does not become extinct. This can also reflect the significant impact that the lampreys, as an invasive species in the Five Great Lakes, have on the local fishing industry.

The change in gender ratio also affects the speed at which the ecosystem reaches a steady state, and when the gender ratio changes, the ecosystem can reach a steady state faster. However, for the lamprey itself, the change in gender ratio reduces its steady-state population density.

### 3.3. Simultaneously consider lampreys, fish, and parasites

Community evolution when considering only fish and parasites. The calculation results are shown in figure 6:

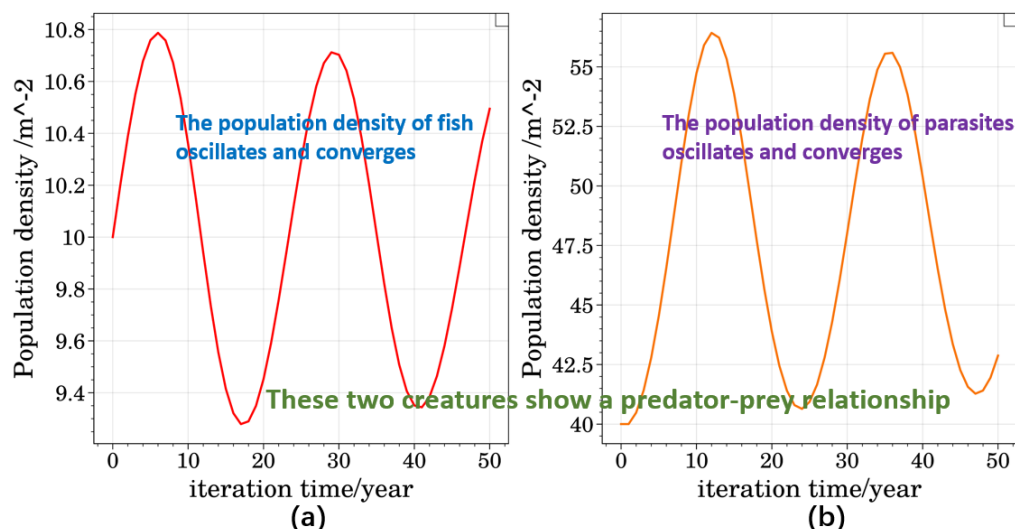


**Figure 6** Population density of fish and parasites over time. Where (a) is the population density of fish and (b) is the population density of parasites

From Figure 6, it can be seen that parasites and fish approximately satisfy the relationship between predator and prey.

In the presence of three species simultaneously. The calculation results are shown in figure 7 and 8:

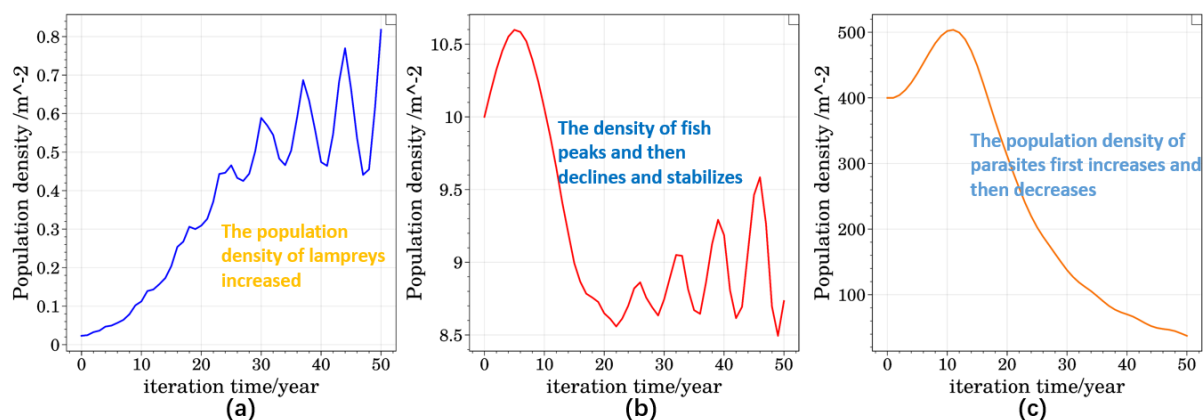


**Figure 7** The situation when the sex ratio of lampreys changes. Where (a) is the population density of lampreys, (b) is the population density of fish, (c) is the population density of parasites

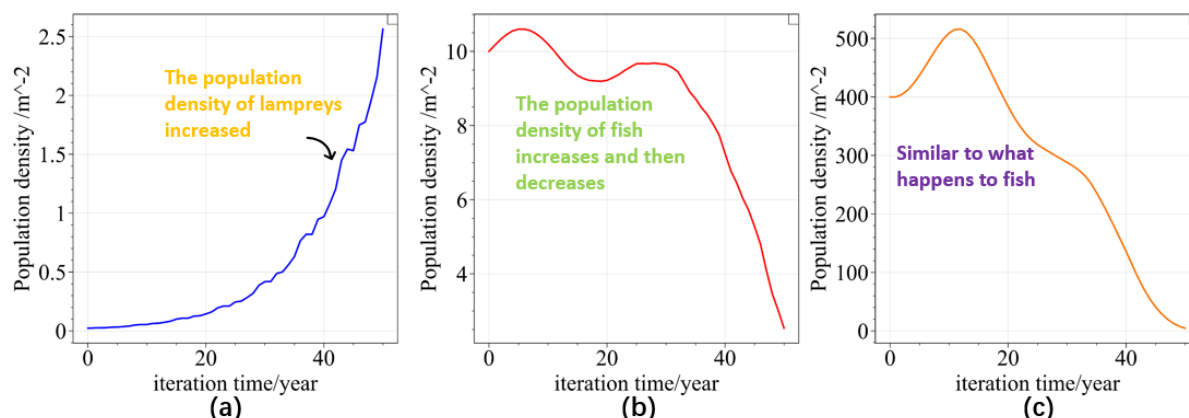


**Figure 8** The situation when the sex ratio of lampreys remains unchanged

Where (a) is the population density of lampreys, (b) is the population density of fish, (c) is the population density of parasites

It can be found that the lamprey disrupted the original dynamic balance of fish and parasites, causing a sharp decline in the numbers of both species, once again confirming the ecological identity of the lamprey as an invader.

The results obtained from changes in the sex ratio of lampreys vary greatly. Although a change in gender ratio may lead to a decrease in the number of fish and lampreys, it becomes stable after some time; However, under steady state conditions, the number of each species fluctuates quite dramatically. If the gender ratio remains unchanged, although the impact on the other two species is relatively small in the initial years, it will almost lead to the extinction of the other two species after some time. However, for the lampreys themselves, maintaining a constant gender ratio gives them an advantage.

## 4. Conclusions

If we consider the population of lampreys separately, when the gender ratio changes, the growth rate of lampreys is faster and it is easier to reach a steady state, with better stability. If multiple species including lampreys are considered, the changing sex ratio will reduce the steady-state population of lampreys, but for the entire environment, its stability will be better. However, regardless of gender ratio changes, lampreys can cause serious harm to other species, even if their initial population is small.

This study reviews that the Leslie population growth model and the Lotka Volterra predator-prey model can be combined to more accurately simulate biological growth dynamics, which can help solve the problem of multiple population interactions when considering factors such as age and gender.

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