An Ecological Network Analysis Based on The Lotka-Volterra Model

Qi Huang [1, *], Jia Huang [2]

[1] BSC Mathematics, Liverpool university, Liverpool, England, L69 3BX

[2] School of AI and Advanced Computing, Xi' an Jiaotong-Liverpool University, SuZhou, China, 215000

* Corresponding Author Email: sgqhuan9@liverpool.ac.uk

Abstract. Changes in adaptive sex ratio is a phenomenon in ecology and evolutionary biology, and the growth rate of the lampreys is affected by food availability, which directly affects the sex ratio of the lampreys. In order to investigate the effects of changes in the sex ratio of lampreys on the stability of the ecosystem, this paper takes phytoplankton, lampreys and their predators as the objects of study, and assesses the stability of the ecosystem by measuring the Shannon diversity index. The results shows that an increase in male sex ratio contributed to the stability of the ecosystem under specific parameters. In addition, this paper investigated the potential effects of changes in the sex ratio of lampreys on their parasite populations and introduced the parasites into an ecological network model. In this paper, this paper found that changes in parasite abundance may give certain parasites an advantage. These findings contribute to a better understanding of the effects of sex ratio changes on ecosystem stability and provide an important reference for ecological conservation and management.

Keywords: Sex ratio change, Lotka-Volterra model, ecological network modelling.

1. Introduction

1.1. Restatement of the Problem

Adaptive sex ratio change is a phenomenon in ecology and evolutionary biology [1]. In the case of the lampreys, for example, the growth rate of lampreys is affected by food availability [2], and the growth rate of lampreys directly affects the sex ratio of lampreys [3]. The adaptive sex ratio variation of lampreys may interact with the local ecological environment [4]. Therefore, our modeling study aims to provide insights into the interactions betthis paperen lamprey sex ratios and ecosystems.

1.2. Analysis of Problem

1. Construct a prediction model for lamprey populations. Through this model, this paper will explore how changes in the sex ratio of lampreys affect changes in population size.
2. Establish a Shannon diversity index [5] prediction model. To assess the evolution of ecosystem stability by predicting changes in the Shannon diversity index

2. The RT model

2.1. The Establishment of RT model

For the question of the effect of changes in sex ratio on the stability of ecosystems, since ecosystems are relatively large y and complex, this paper modeled it by ignoring some small factors. this paper'll use the Lotka-Volterra equations To build a model involving phytophytoplankton, lampreys, the prey of lampreys, and the predators of lampreys. These equations model predator prey dynamics and can be extended to multiple trophic levels.

This paper make the following assumptions about the model:

1. Phytoplankton grows logistically, with growth limited by carrying capacity[5].

2. Lampreys feed on phytoplankton.

3. Prey of Lampreys (other than phytoplankton follow a logistic growth model, affected by lampreys predation.[6]

4. Predators of Lampreys consume lampreys, with their growth rate influenced by lampreys density.

5. Lampreys sex ratio can influence reproductive rates and hence the population dynamics of lampreys[7].

The food chains of RT model is shown in Figure 1.

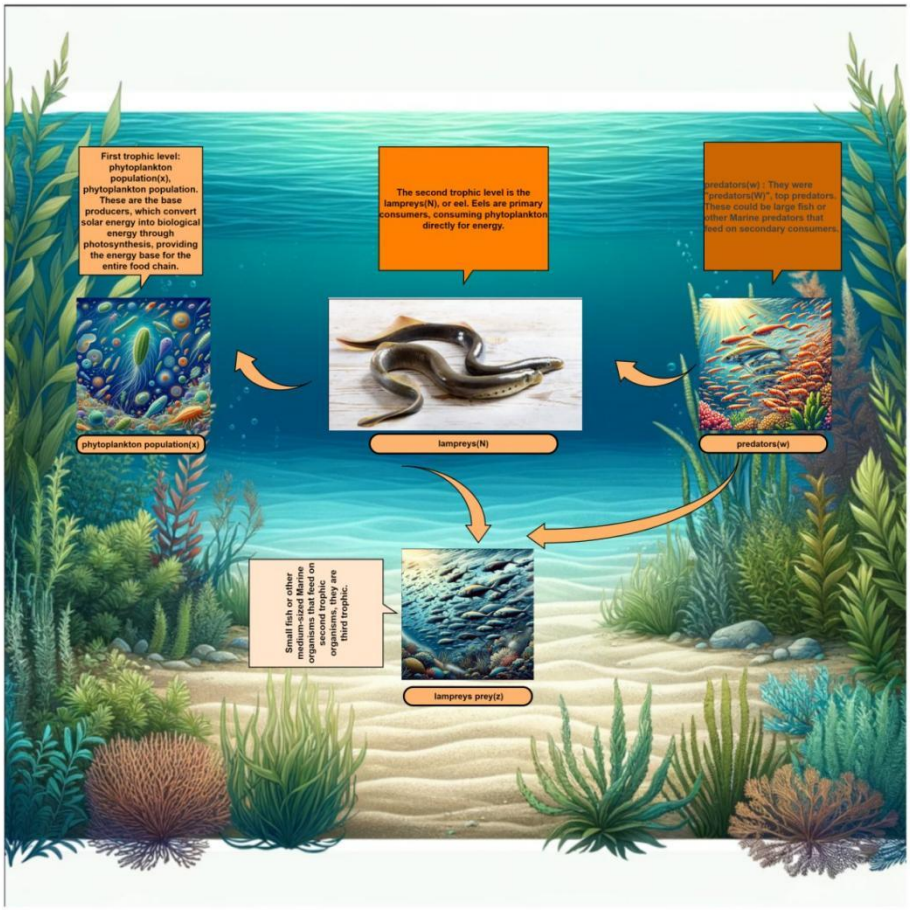


Figure 1: [8]: Food chains of RT model

Explanation of the above hypothesis: N preys on x, z, w preys on N, z. The following diagram shows the thinking path for modeling.

Firstly, according to the fact that x is predated by N, so this paper introduces the predation coefficient a_{xN} of N on x to get the optimized Lotka-Volterra model:

$$\frac{dx}{dt} = r_x \mathrm{x}\left(1 - \frac{x}{K_x}\right) - a_{xN}\mathrm{xN} \tag{1}$$

Similarly, since w is predated by N, this paper introduces the predation coefficient a_{Nw} of N on w, which gives us the optimized Lotka-Volterra model.

$$\frac{dN_m}{dt} = \mathrm{p} \cdot \left(r_N\left(\frac{x}{x+h_x} + \frac{z}{z+h_z}\right) - a_{Nw}Nw\right) \tag{2}$$

$$\frac{dN_f}{dt} = (1-p) \cdot \left(r_N N\left(\frac{x}{x+h_x} + \frac{z}{z+h_z}\right) - a_{Nw}Nw\right) \tag{3}$$

Since z is predated by N and w, this paper introduces the predation coefficients a_{zN} and a_{zw} of N on w, which leads to the optimized Lotka-Volterra model.

$$\frac{dz}{dt} = r_z \mathrm{z}\left(1 - \frac{z}{K_z}\right) - a_{zN}\mathrm{zN} - \mathrm{a_{zw}} \cdot \mathrm{zw} \tag{4}$$

Since w is the topmost layer of this ecosystem and is not predated by any organisms, the optimized Lotka-Volterra model is established.

$$\frac{dw}{dt} = r_w \mathrm{w}\left(\frac{N}{N+h_N}\right) + \mathrm{a_{zw}} \cdot \mathrm{zw} \tag{5}$$

To determine the stability of an ecosystem, this paper introduce a metric used in ecology to quantify the diversity of an ecosystem, the Shannon diversity index, which takes into account two important factors: Species evenness refers to the balance of species distribution, and species abudance refers to the number of different species. The Shannon index is usually defined as:

$$H' = -\sum(p_i \log p_i) \tag{6}$$

where $p_i$is the relative abundance of species i which is the number of individuals of that species as a proportion of the total number of individuals.

$$p_i = \frac{i}{\sum i} \tag{7}$$

$$N = N_m + N_f \tag{8}$$

Finally, all the above equations this paper integrated to obtain a model of sex ratio and trophic level, the RT model:

$$\frac{dN_m}{dt} = \mathrm{p} \cdot \left(r_N\left(\frac{x}{x+h_x} + \frac{z}{z+h_z}\right) - a_{Nw}Nw\right) \tag{9}$$

$$\frac{dN_f}{dt} = (1-p) \cdot \left(r_N N\left(\frac{x}{x+h_x} + \frac{z}{z+h_z}\right) - a_{Nw}Nw\right) \tag{10}$$

$$\frac{dx}{dt} = r_x \mathrm{x}\left(1 - \frac{x}{K_x}\right) - a_{xN}\mathrm{xN} \tag{11}$$

$$\frac{dz}{dt} = r_z \mathrm{z}\left(1 - \frac{z}{K_z}\right) - a_{zN}\mathrm{zN} - \mathrm{a_{zw}} \cdot \mathrm{zw} \tag{12}$$

$$\frac{dw}{dt} = r_w \mathrm{w}\left(\frac{N}{N+h_N}\right) + \mathrm{a_{zw}} \cdot \mathrm{zw} \tag{13}$$

$$N = N_m + N_f \tag{14}$$

$$R = \frac{N_m}{N} \tag{15}$$

$$H' = -\sum(p_i \log p_i) \tag{16}$$

$$p_i = \frac{i}{\sum i} \tag{17}$$

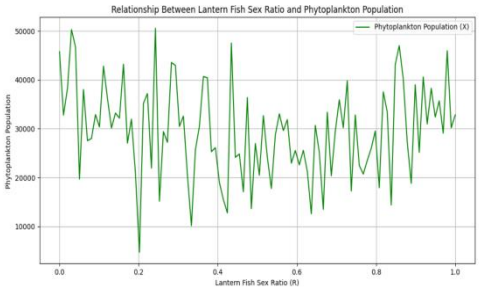
2.2. The Solution of RT model

this paper are given the parameters:

Table 1 [9]: Specific parameter table1

Symbols	Digital	Symbols	Digital
r_N	0.21	h_N	60000
r_x	0.45	K_x	10000
r_z	0.36	K_z	80000
r_w	0.09	a_{Nw}	0.37
p	0.4	a_{xN}	0.79
h_x	50000	a_{zN}	0.42
h_z	40000	a_{zw}	0.32

The relationship between sex ratio and the number of various groups is shown in Figure 2,3,4,5.



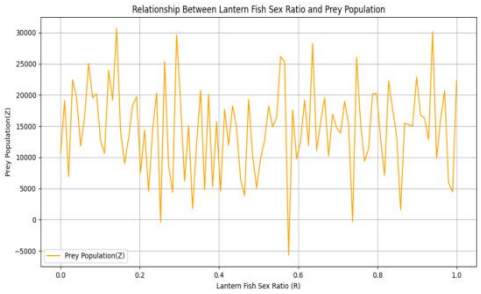
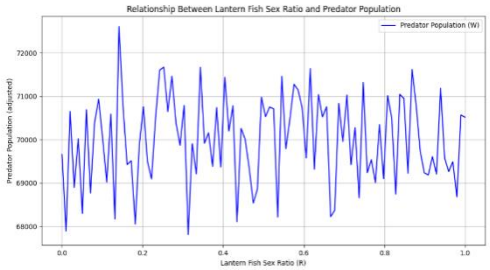


Figure 2: Ratio vs parasites **Figure 3:** Ratio vs prey



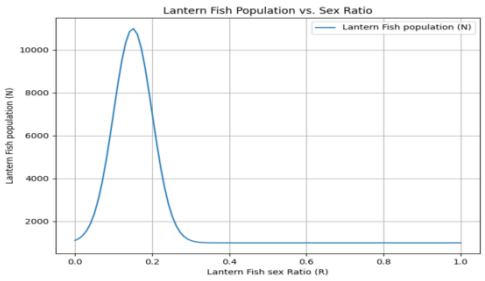


Figure 4: Ratio vs predators **Figure 5:** Ratio vs lampreys

From the changes in Figures 11, 12, and 13, this paper observed more significant population fluctuations of predators, prey, and phytoplankton. This suggests that it is difficult for us to intuitively assess the resistance and resilience of this ecosystem to external environmental disturbances through the changes in population size alone. Therefore, this paper decided to further explore the relationship betthis paperen Shannon diversity index and ecosystem resilience (R) by adjusting the parameters to gain a deeper understanding. Through this approach, this paper hope to be able to more accurately evaluate the stability of the ecosystem and its adaptability to environmental changes.
This paper are given the parameters:

Table 2: Specific parameter table2

Symbols	Digital	Symbols	Digital
r_N	0.20	h_N	60000
r_x	0.47	K_x	10000
r_z	0.35	K_z	80000
r_w	0.29	a_{Nw}	0.38
p	0.43	a_{xN}	0.44
h_x	50000	a_{zN}	0.46
h_z	40000	a_{zw}	0.33

The three-dimensional relationship among Shannon diversity index, gender ratio, and time is shown in Figure 6.

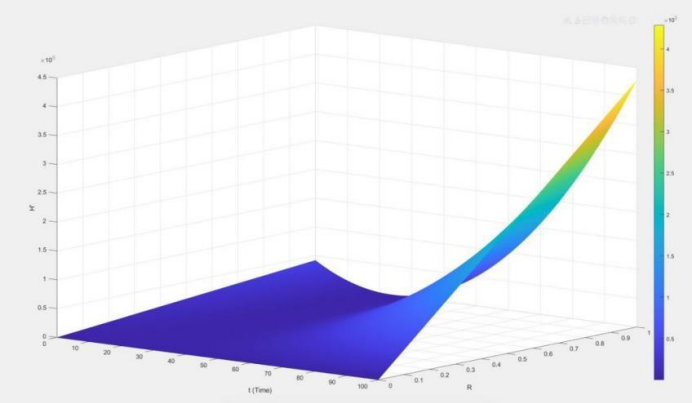


Figure 6: (H', R , t)

Based on the graphical observations, this paper found that over time, when the sex ratio was elevated, the Shannon diversity metrics also increased. This phenomenon implies an increase in both species abundance and evenness. High species abundance refers to the many different species of organisms living in a given area. High species evenness, on the other hand, indicates that the distribution of these different species within the area is relatively balanced, with no single species being overly prominent or dominant. As species diversity increases, the ecosystem's resistance to external disturbances and pressures is strengthened, and its ability to adapt to and resist environmental change is increased, thus contributing to greater environmental stability.

3. The RT* model

3.1. The Establishment of RT* model

Here are the ideas for this modeling:：

First, this paper optimize the RT model to build a larger ecosystem model by introducing the trophic level Para as a parasite.

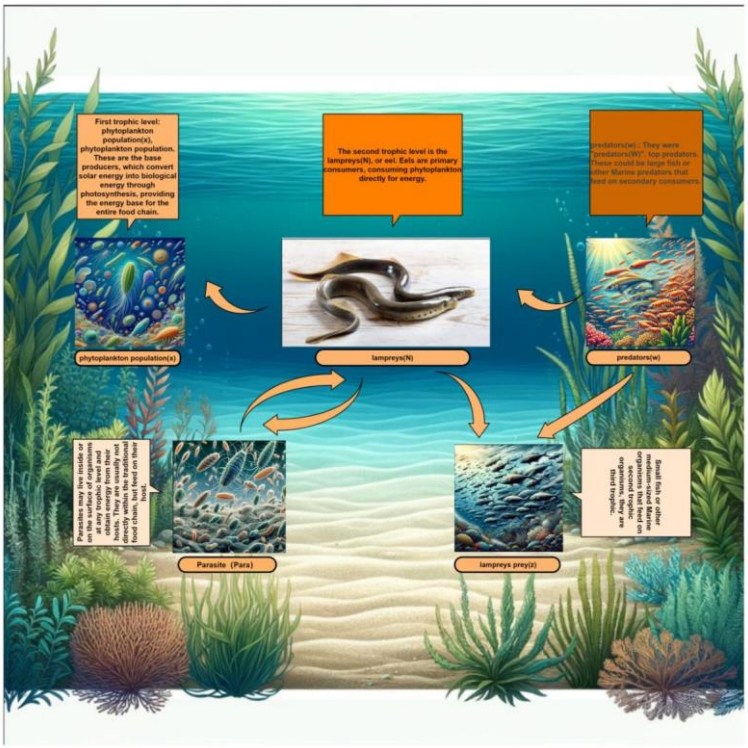


Figure 7: [10]: Food chain of RT* model

This paper can set the parasite *Para* to affect the increase rate of lampreys, while the increase rate of parasite is affected by the population size of lampreys. Here this paper optimizes the RT model:

$$\frac{dN_m}{dt} = p \cdot + + \left(r_N\left(\frac{x}{x+h_x}+\frac{z}{z+h_z}\right) - a_N w N_w - a_{NmPara} Para\right) \tag{18}$$

$$\frac{dN_f}{dt} = (1-p)\cdot\left(r_N\left(\frac{x}{x+h_x}+\frac{z}{z+h_z}\right) - a_N w N_w - a_{NfPara} Para\right) \tag{19}$$

Because this paper assume that the parasite Para directly affects only the lamprey population, the equations for the dynamics of, x and lamprey prey z, as this paperll as lamprey trappers this paper are held constant.

$$\frac{dx}{dt} = r_x x\left(1-\frac{x}{K_x}\right) - a_{xN} x N \tag{20}$$

$$\frac{dz}{dt} = r_z z\left(1-\frac{z}{K_z}\right) - a_{zN} z N - a_{zw}\cdot zw \tag{21}$$

$$\frac{dw}{dt} = r_w w\left(\frac{N}{N+h_N}\right) + a_{zw}\cdot zw \tag{22}$$

Adding the equation of variation for the parasite Para, whose rate of increase depends on the number of host lampreys.

$$\frac{dPara}{dt} = r_{Para} Para\left(1-\frac{Para}{K_{Para}}\right) + a_{ParaN} N \tag{23}$$

Since the presence of the parasite reduces the total number of lampreys, this paper introduces a conversion factor f_{Para} to indicate the effective reduction in the total number of lampreys by the parasite *Para*, and then this paper creates a new equation for the total number of lampreys.

$$N = N_m + N_f - \delta_{Para} Para \tag{24}$$

Finally, this paper integrates the above equations to obtain the new RT model and RT* model.

$$\frac{dN_m}{dt} = p \cdot \left(r_N\left(\frac{x}{x+h_x}+\frac{z}{z+h_z}\right) - a_N w N_w - a_{NmPara} Para\right) \tag{25}$$

$$\frac{dN_f}{dt} = (1-p)\cdot\left(r_N\left(\frac{x}{x+h_x}+\frac{z}{z+h_z}\right) - a_N w N_w - a_{NfPara} Para\right) \tag{26}$$

$$\frac{dx}{dt} = r_x x\left(1-\frac{x}{K_x}\right) - a_{xN} x N \tag{27}$$

$$\frac{dz}{dt} = r_z z\left(1-\frac{z}{K_z}\right) - a_{zN} z N - a_{zw}\cdot zw \tag{28}$$

$$\frac{dw}{dt} = r_w w\left(\frac{N}{N+h_N}\right) + a_{zw}\cdot zw \tag{29}$$

$$\frac{dPara}{dt} = r_{Para} Para\left(1-\frac{Para}{K_{Para}}\right) + a_{ParaN} N \tag{30}$$

$$N = N_m + N_f - \delta_{Para} Para \tag{31}$$

3.2. The Solution of RT* model

The three-dimensional relationship among phytoplankton population numbers, gender ratio, and time is shown in Figure 8.

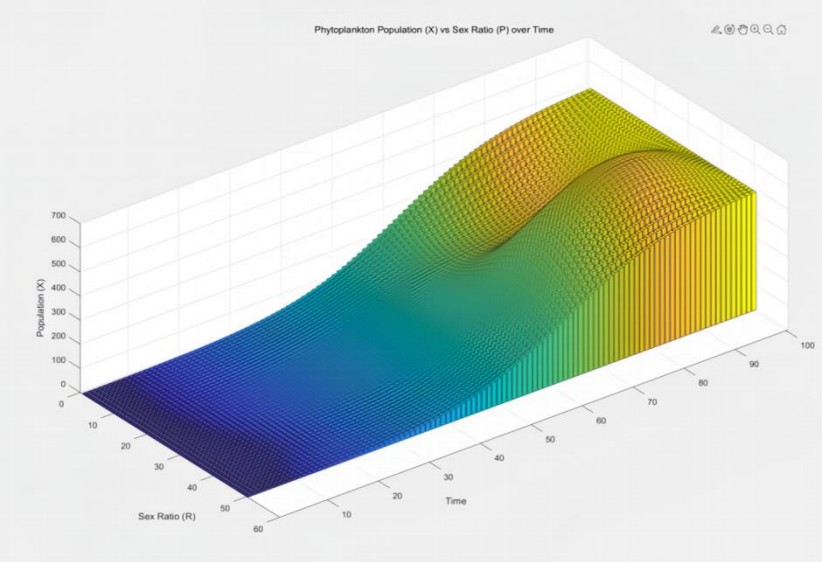


Figure 8: (x, r, t)

Through this three-dimensional image this paper can judge that, in general, when the time is 82.3 and the sex ratio is 41.8%, the population of host organisms is the largest, from the practical to start with, when the sex ratio is 41.8%, after 82.3 days of reproduction, the number of this population reaches the maximum, when after 82.3 days, probably due to the competition is too intense, resulting in the reduction of hosts, leading to the a decrease in population size. Due to the limitation of our modeling, at 100 days, our model fitting data had a significant error with the actual situation, when R=0, the lamprey population could not reproduce, resulting in the impossibility of the increase of the hosts, but the image this paper made was contrary to the fact, so this paper considered that the number of hosts increased significantly at 80 days when the sex ratio was more than 23%, giving a clear advantage to the hosts. Hothis paperver, when the species reproduced for less than 60 days, this paper looked at the images and found that the change in the sex ratio did not have a significant effect on the number of parasites, so it could not give a significant advantage to the parasites. This may be because in making the actual environment, when the reproduction days of lampreys are less than 60 days they are still immature, resulting in the male and female lampreys do not have much difference, so the parasites can not accurately recognize the male and female lampreys, so the change of the sex ratio of the lampreys has a very small effect on the parasites.

4. Conclusion

This study utilizes the Lotka-Volterra model to explore the dynamics of sex ratio changes in lamprey populations, focusing on the impact on ecosystem stability. By examining the interactions between phytoplankton, lampreys, their predators, and parasites within an ecological network, it assesses ecosystem stability through the Shannon diversity index. The findings reveal that an increased male sex ratio enhances ecosystem stability under certain conditions and that variations in parasite populations could advantage specific parasites. This research contributes to understanding the ecological implications of sex ratio changes, offering valuable insights for ecological management and conservation. The integration of parasites into the model adds depth, highlighting the complex interactions within ecosystems and their potential influence on biodiversity and stability. This comprehensive approach provides a foundation for future studies on the ecological effects of sex ratio alterations, underscoring its applicability in ecological conservation strategies and the broader understanding of ecosystem resilience.

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