Research and Application of Multiwave Number Line measurement Model based on Geometric Algebraic Plane Number Equation

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Abstract. In order to solve the problem of designing the shortest line of multi-beam sounding in the specified sea area, a multi-parallel sounding model based on the plane beam equation is constructed for the first time. At the same time, in order to find the shortest exploration line in the unknown sea area, we divide the optional directions of the survey line into three different cases that can cover all the optional directions, namely, the Angle between the direction of the survey line and the projection of the normal direction of the seabed slope on the horizontal plane β = 0°, 90°, 180°, 270°. Compared with other papers that solve similar problems, we can make a comparison in different situations by quantitative analysis of the plane beam equation established in different directions, considering that the overlap rate of multi-beam sounding under the redefinition of three-dimensional model should not be too high, and the need to minimize the area redundancy caused by the exploration of unnecessary sea areas. The direction corresponding to the shortest measurement line is obtained. After determining the direction, the specific shortest line and its shortest length can be obtained according to the relevant knowledge of spatial analytic geometry and algebra. In the problem studied in this paper, space analytic geometry and the use of algebraic related knowledge with refined steps can finally get more accurate results, and usually better than the performance of the applied algorithm, so we adopt this method for modeling and solving.

Keywords: Multiple-beam; Space Analytic Geometry; Plane Beam Equation; Overlapping Ratio;Shortest Line Design.

1. Introduction

In recent years, China has attached great importance to the exploration and protection of this mysterious area of the ocean. In July and August 2023, the team of Sun Zhen from the South China Sea Institute of Oceanology, Chinese Academy of Sciences, conducted and successfully completed China's first deep-sea transoceanic mid-ridge artificial source electromagnetic and magnetotelluric detection profile experiment [1]. In addition, single beam sounding and multi-beam sounding are also two important technical means of ocean exploration. Under this background, this paper establishes a multi-beam submarine depth model, which calculates the seabed depth of a certain sea area through the set parameters and draws the bottom of the seabed for model application.

Single beam sounding uses the principle that sound waves propagate uniformly in a straight line in a uniform medium, and obtains the depth of the sea water according to its propagation speed and propagation time. Because it adopts a one-way continuous measurement method, it can only complete the task of project scale side map, and the test result data is very dense, and there will be data missing between the measurement lines [2].

Multi-beam sounding is based on the single beam sounding. Through the transducer [3] composed of multiple groups of piezoelectric ceramics, different piezoelectric elements are combined in a certain way, and the combined wires are connected to form the transducer array, and the electrical and acoustic signals are converted, which becomes an important part of the multi-beam sounding system. It can reflect the changes of terrain and geomorphology in detail, and the subsequent data processing will be more complicated and cumbersome [4]. The data model formed by combining the data obtained from multi-beam sounding and single-wave velocity sounding has also been widely

studied and used, and the two complement each other to improve the overall accuracy of multi-beam sounding [5].

At present, most of the articles available on the Internet are the introduction of the single beam or multi-beam sounding technology and the introduction of the working principle. Very little research has been done on the design of the shortest line of a ship in the process of sounding. Most of the papers focus on how to make the multi-beam sounding more closely match the irregular sea area topography, and how to reduce the error caused by the approximate results of the irregular surface of the seabed. So it is very necessary to study the design of the shortest line.

In this paper, the shortest line design of idealized submarine slope is studied in depth, and a multi-beam shortest line design model based on geometric algebraic plane beam equation is established. Assuming that the bottom of the seabed is a plane with a certain slope, a three-dimensional space rectangular coordinate system is established with the center point of the water surface of the sea level as the origin and the direction of the slope falling fastest as the positive direction of the X-axis. A three-dimensional slope beam coverage model is established by determining the seabed depth of the center point of the sea area, the opening Angle of the transducer of the measuring ship, the Angle between the measurement route and the X-axis, and the inclination Angle of the seabed slope. Each measurement line will generate a measurement strip, and then the covering strip will be extended to the entire sea area by the method of parallel plane beam. In this paper, the relevant knowledge of spatial analytic geometry and algebra is used to solve the problem. There is a certain overlap rate between adjacent measurement strips. Under normal circumstances, in order to ensure the simplification of the sounding process and the rigor of the data, the overlap rate should be kept between 10% and 20% [6], and the measurement line should be shortest when the task of sea depth exploration is completed.

2. Construction of slope beam coverage model

2.1. Determination of overlap rate

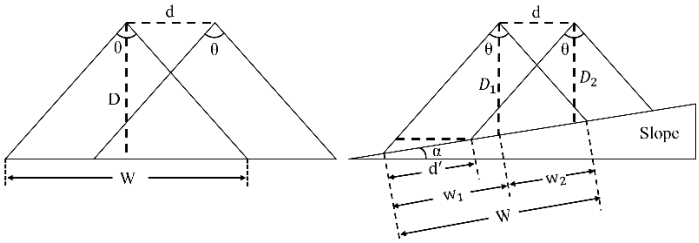


Fig.1 Schematic diagram of strip width

The overlap rate of adjacent strips is η, the opening Angle is θ, the seabed depth of single-beam depth is D, the width of multi-beam coverage strip is W, and d is the spacing of two adjacent lateral lines. As shown in Fig.1. The overlap rate of adjacent strips is calculated as follows:

$$\eta = \frac{W-d}{W} = 1 - \frac{d}{W} \tag{1}$$

2.2. Determination of the coverage width

It is assumed that the Angle between the plane S where the measurement line is located and the X-O-Z plane is β, and the calculation of the coverage width W is based on different Angle β :

1. $\beta = 90°$ or $270°$

$$\begin{cases} \frac{D}{sin\left(\frac{\pi-\theta}{2}-\alpha\right)} = \frac{w_1}{sin\left(\frac{\theta}{2}\right)} \\ \frac{D}{sin\left(\frac{\pi-\theta}{2}+\alpha\right)} = \frac{w_2}{sin\left(\frac{\theta}{2}\right)} \\ W = w_1 + w_2 \end{cases} \tag{2}$$

Using the sine theorem, the equations are obtained simultaneously:

$$W = Dtan\left(\frac{\theta}{2}\right)\left[1 + \frac{cos\left(\frac{\theta}{2}+\alpha\right)}{cos\left(\frac{\theta}{2}-\alpha\right)}\right] \tag{3}$$

2. $\beta = 0°$ or $180°$

Let l be the distance of the ship from the Y-axis, and obtain the depth of the sea water

$$D' = \tan(\alpha)\, l + D \tag{4}$$

The coverage width W increases with the increase of seawater depth, and D, W and plane S are independent of the distance from the center point of the sea area, so the width W can be obtained:

$$W = 2\tan\left(\frac{\theta}{2}\right)[\tan(\alpha)\, l + D] \tag{5}$$

$$(\beta = 0°)$$

3. $\beta \neq 0°, 90°, 180°, 270°$

The plane where the multi-beam detector is located is S', the plane where the slope is located is SP, and the slope Angle is α. Figure 2.6; The left and right planes intersecting the plane S' are S1 and S2, which can be obtained by the calculation of spatial analytic geometry

Slope equation:

$$\tan(\alpha)\, x + z - D = 0 \tag{6}$$

The equation for the plane S' is:

$$(x - lcos(\beta)) + tan(\beta)(y - lsin(\beta)) = 0 \tag{7}$$

The equations of the two planes S1 and S2 are respectively:

$$-sin(\beta)x + cos(\beta)y + tan\left(\frac{\theta}{2}\right)z = 0 \tag{8}$$

$$-sin(\beta)x + cos(\beta)y - tan\left(\frac{\theta}{2}\right)z = 0 \tag{9}$$

Through simultaneous formulas (4), (5), (6) and (4), (5) and (7), the coordinates of the left and right endpoints covering the strip at a certain time can be obtained respectively [7]. The left endpoint coordinates are set as: P1 (x_1,y_1, z_1), and the right endpoint coordinates are set as: P2 (x_2,y_2, z_2), which are expressed by matrix expression as follows:

$$\begin{bmatrix} -\sin(\beta) & \cos(\beta) & \tan\left(\frac{\theta}{2}\right) \\ \tan(\alpha) & 0 & 1 \\ 1 & \tan(\beta) & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 0 \\ D \\ \frac{l}{\cos(\beta)} \end{bmatrix}$$

$$\begin{bmatrix} -\sin(\beta) & \cos(\beta) & -\tan\left(\frac{\theta}{2}\right) \\ \tan(\alpha) & 0 & 1 \\ 1 & \tan(\beta) & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 0 \\ D \\ \frac{l}{\cos(\beta)} \end{bmatrix}$$

Let the above matrix be expressed as:

$$A_1X_1 = b \quad A_2X_2 = b$$

After a simple matrix transformation of the linear equations, we can obtain:

$$X_1 = A_1^{-1}b \quad X_2 = A_2^{-1}b$$

The 2-norm of the chosen vector $X_1' - X_2'$ is the final coverage width W:

$$W = \|X_1' - X_2'\|_2 = \sqrt{|x_1 - x_2|^2 + |y_1 - y_2|^2 + |z_1 - z_2|^2} \tag{10}$$

By synthesizing formulas (3), (4) and (10), the expression of the final coverage width W can be obtained. Due to the complexity of the expression, MATLAB software is directly used to calculate the relevant data. Let the final expression be:

$$W = f(\alpha, \beta, \theta, l, D) \tag{11}$$

2.3. Coverage of local sea areas

In the seabed depth detection of a certain sea area, the survey line will not always pass through the center point of the sea area. In the established slope coverage width model, in order to facilitate the calculation of the multi-beam coverage width W under the circumstances of different positions, different distance l from the center point of the sea area, and different Angle β between the survey line and the X-axis, the model shown in Fig.2 was established by using the method of parallel plane number:

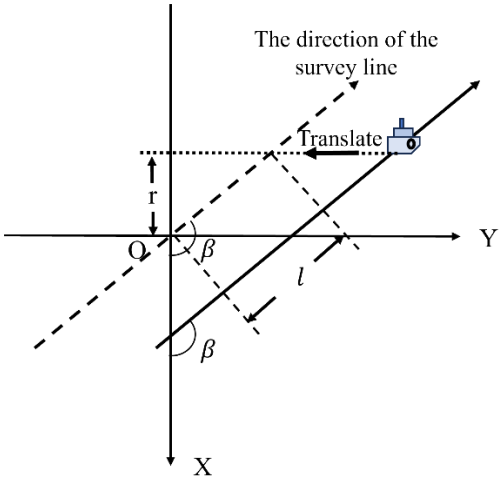


Fig.2 Schematic diagram of survey line translation

The principle of the model is to cover the entire local sea area through multiple parallel surveys [8] and draw multiple parallel survey lines. Where r is the distance of the measuring ship from the Y axis, that is, the absolute value of the X-axis coordinate; l is the distance between the measuring ship and the center point of the sea area after translation. Through calculation, it can be obtained:

$$l = \frac{r}{\cos(\beta)}, \qquad \beta \neq \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$$

1.$\beta = 90°, 270°$

(i) The solution of the plane beam equation

Suppose a total of n survey lines are required for depth exploration, and the parallel plane beam equation of the survey lines [9] is expressed as:

$$y^{(i)} = r_i, \qquad i = 1,2,\dots,n$$

The case of the first measurement line is more special, when i=1:

$$\begin{cases} D_1 = D - r_1 \times \tan(\alpha) \\ \frac{D_1}{sin\left(\frac{\pi-\theta}{2}+\alpha\right)} = \frac{w_2^{(1)}}{sin\left(\frac{\theta}{2}\right)} \\ w_2^{(1)}\cos(\alpha) + r_1 = \frac{m}{2} \\ y^{(1)} = r_1 \end{cases} \tag{12}$$

Where, $w_2^{(1)}$is the coverage width of the right half beam of the first measurement line on the slope. After simplification, it is obtained:

$$y^{(1)} = \frac{Dcos(\alpha)sin\left(\frac{\theta}{2}\right) - \frac{m}{2}cos\left(\frac{\theta}{2} - \alpha\right)}{\sin(\alpha)sin\left(\frac{\theta}{2}\right) - cos\left(\frac{\theta}{2} - \alpha\right)} \tag{13}$$

According to formula (2), the class can be deduced when i=2,3,... Line equation $y^{(i)}$ at n:

$$\begin{cases} D_i = D - r_i \times \tan(\alpha) \\ \frac{D_i}{sin\left(\frac{\pi-\theta}{2}+\alpha\right)} = \frac{w_2^{(i)}}{sin\left(\frac{\theta}{2}\right)} \\ w_2^{(i)}\cos(\alpha) + r_i = \frac{m}{2} - (1-\eta)W_{i-1}\cos(\alpha) \\ y^{(i)} = r_i \end{cases} \tag{14}$$

Since the subsequent measurement line equation takes into account the coverage width of the previous measurement line, it is impossible to give a simple formula, and this paper will calculate and map through MATLAB software in the future.

(ii) Problems solved by ordinary iterative methods:

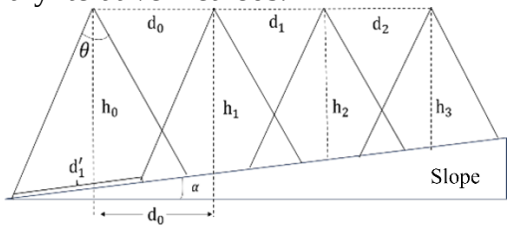


Fig.3. Cross-sectional view of routes at different β angles

Where θ, α, h 0 are known. w n coverage width, the definition of η coverage can be seen in 2, (1). As shown in Fig.3.

$$\begin{cases} h_{n+1} = h_n - d_n tan\alpha & (15) \\ d'_n = \frac{d_n cos\frac{\theta}{2}}{\cos\left(\frac{\theta}{2}+\alpha\right)} & (16) \\ d'_n = w_n - \eta w_{n+1} & (17) \\ w_n = \frac{h_n sin\frac{\theta}{2}}{\cos\left(\frac{\theta}{2}-\alpha\right)} + \frac{h_n sin\frac{\theta}{2}}{\cos\left(\frac{\theta}{2}+\alpha\right)} & (18) \end{cases}$$

By substituting formula (16), (17), (18) into formula (15), we can obtain h 1 according to the known h 0, and then continue to iteratively obtain h 2, h 3, h 4... h n. Then, by substituting (17) (18) and the obtained h n into equation (16), the spacing d n of each survey line can be obtained, the number of survey lines required for the required exploration sea area can be obtained, and the shortest survey map can be drawn. This solution requires more iterations and is cumbersome, and this ordinary iterative algorithm can only calculate the special case of β=90° or 270°, while other directions can only be excluded by qualitative analysis. This problem can be solved in our method.

2.$\beta = 0°, 180°$

At this time, the equation of measuring line is:

$$x = \lambda_i, \qquad i = 1,2,\dots,n$$

Where λ_i represents the intercept between the measured line and the X-axis. According to the formula (3) for calculating the coverage width, we can know that under different depths D_i, the coverage width W i will have a large difference, for example, in the sea area to be measured at 2×4 (unit: nautical miles) :

$$\begin{cases} W_1 = 0.0430, & l_1 = -2 \\ W_n = 0.4059, & l_n = 2 \end{cases}$$

If the overlap rate of 10% is maintained at the first end of the line in the sea area to be measured, then the overlap rate at the end of the line is:

$$\eta_n = \frac{W_n - W_1(1-\eta_1)}{W_n} \times 100\% = 90.47\%$$

It can be seen that $\eta_n \gg 20\%$, the coverage width W of the upper and lower ends of the sea area to be measured is nearly ten times different, in the case of ensuring complete coverage of the sea area to be measured, it is impossible to keep the overlap rate in the range of 10%≤η≤20%. Therefore, the case that the included Angle β=0° or 180° is excluded.

3. $\beta \neq 0°, 90°, 180°, 270°$

According to the equation of line of survey (2.5), we can derive the equation of the plane beam where the line of survey is located in the sea area:

$$-\tan(\beta)\, x + y + r_i = 0,\ i = 1,2,\dots,n \tag{19}$$

Where n is the number of measured lines in the sea area, and other variables are the same as before. The inclined survey line is used to measure the sea area to be measured, and the inclined survey line model is established covering the entire area to be measured. The effect diagram is as follows:

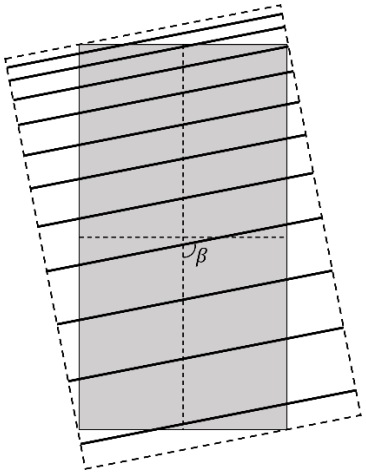


Fig.4. Renderings of the oblique survey line model

As shown in Fig.4, the shaded part is the sea area to be measured. In order to ensure that the overlap rate of the measured sea area in the whole piece is always kept within the range of 10%≤η≤20% and the length of the total measurement line is the shortest, let the $\eta_i^{(1)}$ of the i measurement line from the farthest end of the central sea area =10%, and let its coverage width be $W_i^{(1)}{}_i$; At the same time, ensure that the $\eta_i^{(2)} \leq 20\%$ at the nearest end of the measurement line from the central sea area is less than 20%, so that its coverage width is d_i;

$$\begin{cases} r_1^{(1)} = \frac{m}{2} + t\cos\left(\beta - \frac{\pi}{2}\right) \sin\left(\beta - \frac{\pi}{2}\right) \\ r_1^{(2)} = \frac{m}{2} - m\left[\sin\left(\beta - \frac{\pi}{2}\right)\right]^2 \\ r_{i+1}^{(1)} = r_i^{(1)} - d_i \cos\left(\beta - \frac{\pi}{2}\right) \\ r_{i+1}^{(2)} = r_i^{(2)} - d_i \cos\left(\beta - \frac{\pi}{2}\right) \end{cases} \tag{20}$$

Where d_i is the distance between the i measuring line and the i+1 measuring line, and the coverage width only considers the distance r from the Y axis of the measuring ship and the included Angle β. For the overlap rate:

$$\eta_i^{(1)} = \frac{\frac{W_i^{(1)} + W_{i+1}^{(1)}}{2} - d_i}{W_i^{(1)}} \times 100\%$$

$$= \left(\frac{1}{2} + \frac{W_{i+1}^{(1)}}{2W_i^{(1)}} - \frac{r_{i+1}^{(1)} - r_i^{(1)}}{\sin(\beta) W_i^{(1)}} \right) \times 100\%$$

$$= \left[\frac{1}{2} + \frac{f\left(r_{i+1}^{(1)}, \beta\right)}{2f\left(r_i^{(1)}, \beta\right)} - \frac{r_{i+1}^{(1)} - r_i^{(1)}}{\sin(\beta) f\left(r_i^{(1)}, \beta\right)} \right]$$

$$\times 100\% = 10\%$$

By the same token, constraints can be obtained:

$$\eta_i^{(2)} = \left[\frac{1}{2} + \frac{f\left(r_{i+1}^{(2)}, \beta\right)}{2f\left(r_i^{(2)}, \beta\right)} - \frac{r_{i+1}^{(2)} - r_i^{(2)}}{\sin(\beta) f\left(r_i^{(2)}, \beta\right)} \right]$$

$$\times 100\% \leq 20\%$$

By determining the other factors and the Angle β, all the $r\ i^{(1)}$ can be obtained recursively, and all the line positions can be determined. Then MATLAB software was used to analyze and solve the total length of the measuring line.

3. Case analysis

3.1. Parameter settings

Table 1 Parameter Settings

Description	Setting
Slope of the seabed	α
Multi-beam transducer opening angle	θ
The depth of the seabed in the central sea	D
The angle between the direction of the measurement line	β
Coverage width	W
Overlap rate	η
The width of the local sea area	t
The length of the local sea area	m

3.2. Data Selection

Set Table 1 according to the parameters. For descriptive analysis, this paper selects each survey line passing through the center point of the sea area with an Angle β ranging from 0° to 360° and an interval of 45° from the fastest direction of slope descent, and calculates that the distance d of each side line from the center point of the sea area is the data of the coverage width W at a distance of 0.2 nautical miles from 0 to 2.1, where the depth of the center point of the sea area is 110m. The submarine slope slope α is set to 1.5°, and the Angle θ of the transducer opening Angle is 120°.

For the calculation of the local sea area, the area of the local sea area to be detected is 2×4 (unit: nautical miles), that is, m=4,t=2; The topography of the sea bottom is deep in the east and shallow in the west, and the slope, the depth of the center point of the sea area and the opening Angle of the transducer remain unchanged.

3.3. Result Analysis

For a line β=180° passing through the center of the sea, the overlap rate along the lateral line will vary as follows:

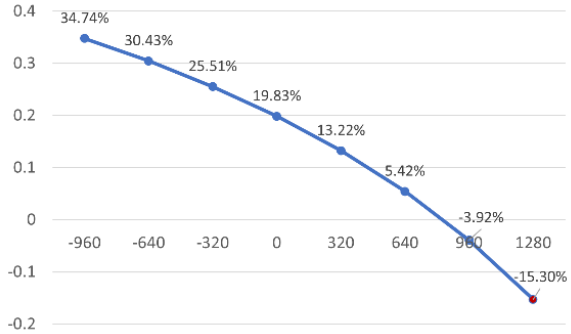


Fig.5 Schematic diagram of the change of overlap rate

As shown in Fig.5, it can be seen that when the sea water becomes shallower and shallower, the coverage width W and the overlap rate η gradually decrease, and the overlap rate ηwill occur, that is, part of the seabed has not been surveyed.

For any survey line passing through the center point of the sea area, according to the established slope beam coverage model, with the increase of the distance d between the survey ship and the center point of the sea area at different angles β, the calculation results of the coverage width W are as follows:

Table 2 Data results

d / β	0	0.3	0.6	0.9	1.2	1.5	1.8	2.1
0	416	466	516	567	617	668	718	768
45	416	452	488	523	559	595	630	666
90	416	416	416	416	416	416	416	416
135	416	381	345	309	273	238	202	166
180	416	365	315	265	214	164	113	63
225	416	381	345	309	273	238	202	166
270	416	416	416	416	416	416	416	416
315	416	452	488	523	559	595	630	666
360	416	466	516	567	617	668	718	768

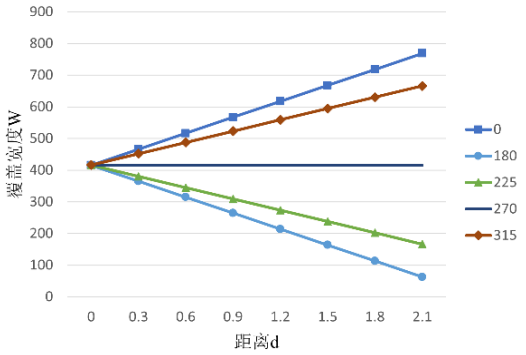


Fig.6 Line chart of coverage width

From the data in Table 2 and Fig.6, it can be seen that |β-π|π/2 the coverage width W widens with the increase of the distance d. When β=π/2,3π/2, the coverage width W does not change with the increase of distance d.

For the survey of the whole sea area, the overlap rate η=10% is kept constant according to the side line equation when β=90° and 270°

$$y^{(i)} = r_i, \qquad i = 1,2,\dots,n$$

MATLAB software was used to draw all the survey lines in the same local sea area, as shown in the figure below

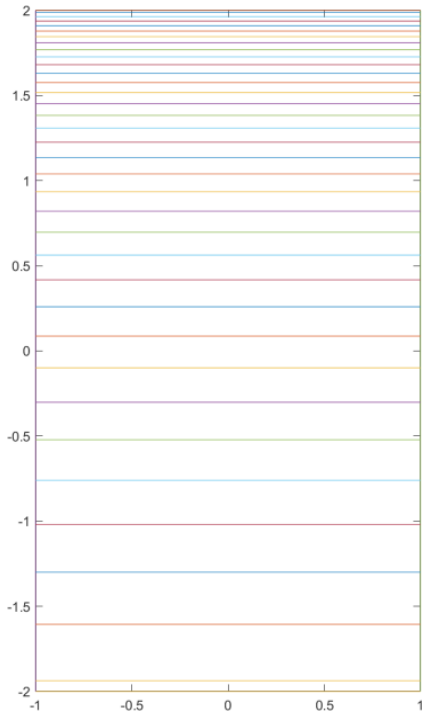


Fig.7 Line map of local sea area survey

From the above Fig.7, it can be calculated that the total length of the measurement line is 68 nautical miles when the required measurement area is 2×4 (unit: nautical miles).

4. Conclusions

In addressing the issue of designing the shortest survey line for multibeam bathymetry, determining the direction of the detection route is the primary entry point. This can be represented using the plane bundle equation method, which allows for the expression of any plane passing through a certain straight line, meaning that any direction can be taken into account. By combining this with the specific constraints and requirements of the problem, the exact direction of the survey line can be determined, and a greedy algorithm can be used to calculate the length of the shortest survey line.

This paper uses the plane bundle equation to establish a model to solve the problem of designing the shortest survey line for multibeam bathymetric surveys in idealized sea areas. Compared to other articles addressing multibeam seabed detection issues, this paper is the first to apply the plane bundle equation from analytical geometry to the problem-solving process. Specifically, in cases where $\beta \neq 0°, 90°, 180°, 270°$, not only can this paper provide qualitative analysis, but it can also perform quantitative analysis to demonstrate that the directions $\beta = 90°$ or $270°$ correspond to the shortest survey lines. Other articles can only qualitatively state that the survey lines are shortest at $\beta = 90°$ or $270°$. Compared to other articles, the method used in this paper is more scientific, rigorous, and persuasive.

In real-world problems, no ideal conditions exist in unknown sea areas, and the variations in seabed depth are even more complex. Consequently, the unavoidable issue of overlap rates in multibeam bathymetry becomes even harder to satisfy. Additionally, various sources of error can affect the multibeam bathymetric process [10]. The simplified model in this paper lacks some flexibility; future work could incorporate heuristic optimization algorithms to improve the plane bundle equation and enhance its flexibility, as well as to compensate for its inapplicability to complex seabed terrain errors.

In conclusion, this paper has derived the optimal survey line plan and shortest survey line length for idealized seabed slopes. For regular idealized seabed slopes, the horizontal bathymetric survey

line plan is superior to other scenarios. Further research could combine the advantages of single-beam single-point bathymetry's accuracy with heuristic algorithms to optimize the model, thereby further enhancing the model's utility and significance.

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