The study on the force analysis model of wave energy devices in heaving motion

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Abstract. This study focuses on the heaving motion of a float in waves, where the oscillator moves reciprocally along the central axis. Assuming that the damping force of the linear damper is directly proportional to the relative velocity of the float and the oscillator, a motion model for the float and oscillator is established. By applying the principles of mechanics and dynamics, a mathematical analysis model for wave energy devices under heaving motion conditions is developed. Utilizing planning concepts and multi-threading technology, Java is employed to calculate the relevant data, and Python is used for data visualization to achieve simulation effects. In the process of solving and analyzing the related models, the masses of the central axis, base, and separator, as well as various frictional forces, are neglected. A mathematical model for wave energy devices undergoing only heaving motion is established. Based on the force analysis model, force equations for the float and oscillator are derived, and Newton's second law is applied to solve for the relevant accelerations. Differential equations are then used to further determine the associated velocities. Finally, displacement equations are established to solve for the trajectories of displacement versus time and velocity versus time.

Keywords: Wave Energy Device, Simulation, Planning Concept.

1. Introduction

Energy is the cornerstone of human societal development and progress. Driven by rapid global economic growth and a surge in population, the demand for energy has been continuously rising. This not only intensifies energy consumption but also causes severe environmental damage. In response to this challenge, the global community has reached a consensus on promoting the use of green, energy-efficient, and environmentally friendly energy sources [1]. Countries around the world have made significant efforts to actively explore and develop clean energy solutions [2]. Against this backdrop, wave energy stands out as one of the most promising marine energy sources due to its widespread distribution, vast reserves, and ease of access [3].

Although the concept of wave energy generation was proposed by scientists as early as the 18th century, the technology remains at the forefront of research and development [4]. The key challenge in harnessing wave energy lies in designing a device (WEC) that can effectively convert wave energy [5]. In reality, many models face complex deployment processes, high costs, and low returns on investment. For instance, the Levelized Cost of Energy (LCOE) achieved by some WECs [6], and the "Zhoushan" hydraulic wave energy generator model developed by the Guangzhou Institute of Energy Conversion, Chinese Academy of Sciences [7], have not been widely promoted due to cost issues. These challenges highlight the difficulties that need to be overcome in the commercialization of wave energy technology.

In this paper, based on Newtonian mechanics principles, an energy output system (PTO) that is both efficient and simple to deploy has been designed. The notable features of this system are its straightforward deployment process and low construction cost, while also ensuring a relatively considerable power output. Compared to other wave energy conversion devices, our design places a stronger emphasis on practicality and economy, aiming to drive the commercialization of wave energy technology and make it an emerging force in the field of sustainable energy.

The data from: http://www.mcm.edu.cn/html_cn/node/388239ded4b057d37b7b8e51e33fe903.html

2. Analysis of Forces and Model Establishment for Wave Energy Devices Under Heaving Motion Only

To conduct a mechanical analysis of the wave energy device, the mechanical model of the entire system is shown in Figure 1:

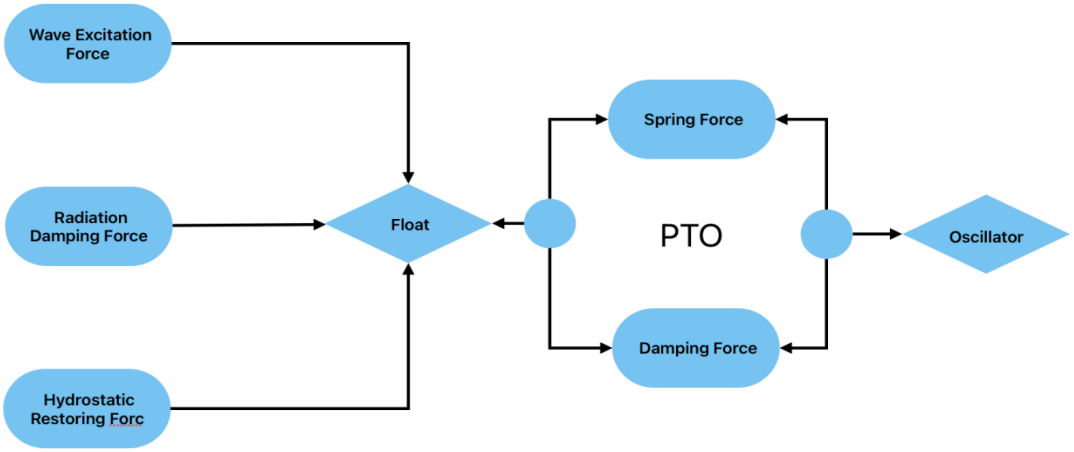


Figure 1. Force Analysis diagram of the wave energy device

Based on the mechanical analysis model shown in Figure 1, the forces acting on the float and the oscillator are analyzed, and the system force analysis equations are established.

$$F_f = F_\omega + F_w + F_{ww} - F_{dc} - f_s \tag{1}$$

$$F_o = F_{dc} + F_s \tag{2}$$

Where F_f is the force on the float under heaving motion only, F_o is the force on the oscillator under heaving motion only, $F_\omega$is the wave excitation force, F_w is the wave-making damping force, $F_{ww}$is the static water restoring force, $F_{dc}$is the force exerted by the damper, and F_s is the spring force.

Wave excitation:

$$F_\omega = f * \cos(\omega t) \tag{3}$$

Dangling wave damping force:

$$F_w = -d_{hwmdc} * v_f \tag{4}$$

Hydrostatic recovery force:

$$F_{ww} = -mf * S * \rho * g \tag{5}$$

Damper force:

$$F_{dc} = (v_f - v_O) * d_{cc} \tag{6}$$

spring force:

$$F_w = (m_f - m_o) * K \tag{7}$$

In the equation, ω represents the incident wave frequency, f represents the amplitude of the heaving wave excitation force, trepresents time, d_{hwmdc} represents the heaving wave-making damping coefficient, S represents the bottom area of the cylindrical part of the float, ρ represents the density of seawater-$1025 kg/m^3$, g represents the acceleration due to gravity-$9.8 m/s^2$, K represents the stiffness of the spring-80000N/m.Based on Newton's second law, the acceleration equations for the float and the oscillator in heaving motion are established [8]:

$$a_f = \frac{F_f}{M_f + a_q} \tag{8}$$

$$a_o = \frac{F_o}{M_o} \tag{9}$$

In the equation, M_f represents the mass of the float-4866kg, M_o represents the mass of the oscillator-2433kg, and a_q represents the heaving added mass.

Based on the kinematic principles, the velocity equations for the float and the oscillator in heaving motion are established [9]:

$$V_f = \int a_f dt = a_f * t \tag{10}$$

$$V_o = \int a_o dt = a_o * t \tag{11}$$

In the equation, V_f represents the velocity of the float in heaving motion, and V_o represents the velocity of the oscillator in heaving motion. Based on the kinematic principles, the displacement equations for the float and the oscillator in heaving motion are established [10]:

$$X_f = v_f * \Delta t + \frac{1}{2} * a_f * (\Delta t)^2 \tag{12}$$

$$X_o = v_o * \Delta t + \frac{1}{2} * a_o * (\Delta t)^2 \tag{13}$$

The displacement and velocity update equations for the float and the oscillator are derived from the given time interval:

$$v_f = v_f + a_f * \Delta t \tag{14}$$

$$v_o = v_o + a_o * \Delta t \tag{15}$$

$$m_o = m_o + X_o \tag{16}$$

$$m_f = m_f + X_f \tag{17}$$

3. Result

This dataset contains the incident wave frequency (s^{-1}), additional heave mass (kg), additional pitch moment of inertia (kg· x^2), heave wave damping coefficient ($N \cdot s/M$), pitch wave damping coefficient ($N \cdot m \cdot s$), heave excitation force amplitude (N), and pitch excitation torque amplitude (N ·m).

Direct translation: Use Java to solve the model equations for simulation, and provide the heave displacement and velocity of the float and the oscillator at 10s, 20s, 40s, 60s, and 100s, respectively. The results obtained are visualized using Python for data presentation.

3.1. Solution of Displacement and Velocity for a Buoy and an Oscillator under Determined Damping Coefficient

The displacement and velocity of the float and the oscillator were calculated using the damping coefficient of the linear damper, and the results are shown in Figures 2 and 3.

"The biomimetic model in Figure 2 simulates the motion trajectory of a buoy under certain conditions of time and displacement. The motion model of the buoy is established under the premise of ($p = 10000N \cdot s/m$). As shown in Figure 2, during the oscillation motion of the buoy, the time period for reaching the maximum displacement is ($0s - 10s$), and the displacement reaches ($\pm 0.7s$). The time period for reaching the minimum displacement is ($10s - 20s$), with a displacement around ($\pm 0.4m$); before ($50s$), the oscillation motion of the buoy is more intense in the back-and-forth process, and there is a significant difference in displacement between different times; after ($50s$), the displacement frequency becomes stable, and the displacement basically stays between ($\pm 0.4m - - \pm 0.6m$).

From Figure 3, it can be observed that when ($p = 1000N \cdot s/m$), at time ($10s$), the buoy reaches its maximum speed during oscillation motion, which is ($-0.641m/s$); during the time period of ($10s - 20s$), it achieves the lowest speed between ($0.4m/s - 0.5m/s$). The speed fluctuates significantly before ($40s$), and after ($40s$), with the increasing time, the speed remains relatively stable.

Combining the data from Figure 2 and Figure 3, and based on the damping coefficient of the buoy in a linear damper being $(10000\mathrm{N}\cdot\mathrm{s/m})$, using Python solving tools, the data for buoy displacement and speed under $(10\mathrm{s}, 20\mathrm{s}, 40\mathrm{s}, 60\mathrm{s}, 100\mathrm{s})$ can be obtained. The data is shown in Table 1:

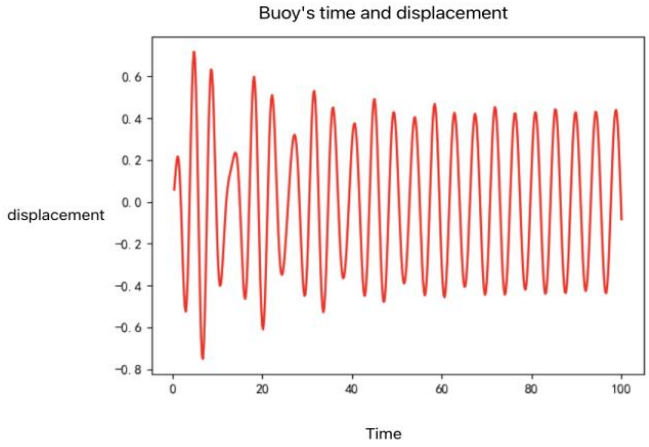


Figure 2. Graph depicting the relationship between time and displacement of the buoy

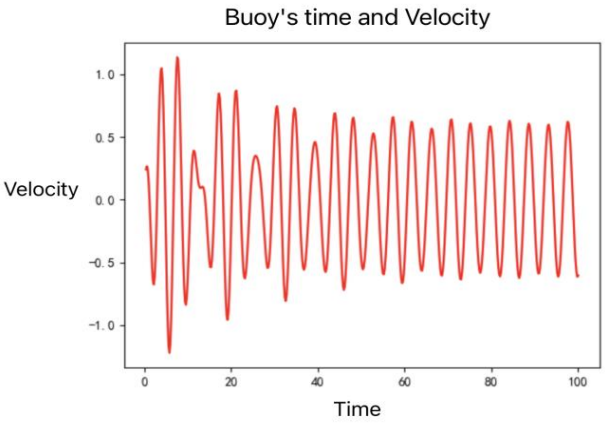


Figure 3. Graph illustrating the relationship between time and velocity of the buoy

Table 1. Requested results for buoy displacement and velocity at various time points

Time (s)	Buoy	
	Displacement (m)	Velocity (m/s)
10	-0.191	-0.641
20	-0.591	-0.241
40	0.285	0.313
60	-0.315	-0.479
100	-0.084	-0.604

From the document, it is evident that the pendulum undergoes oscillatory motion around the central axis propelled by the motion of the buoy in the waves. Under identical conditions, Python is still employed to create visual graphs. This allows for a more intuitive and clear analysis of the oscillatory motion trajectory of the pendulum after establishing the pendulum motion model.

As can be seen from Figure 4, during the time period from 10s to 20s, the displacement value reaches its peak, exceeding -0.8m. However, the displacement value of the float is less than -0.8m during the same time. Under condition 3.1, the motion trajectory between the oscillator and the float is roughly similar, but there is also a significant size difference between the float and the oscillator.

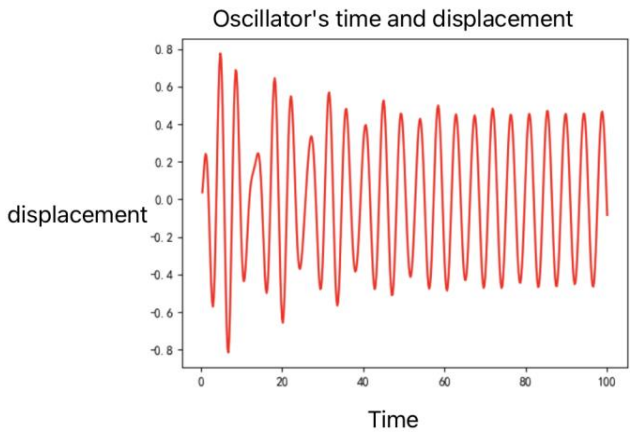


Figure 4. Oscillator's Time-Displacement Relationship Graph

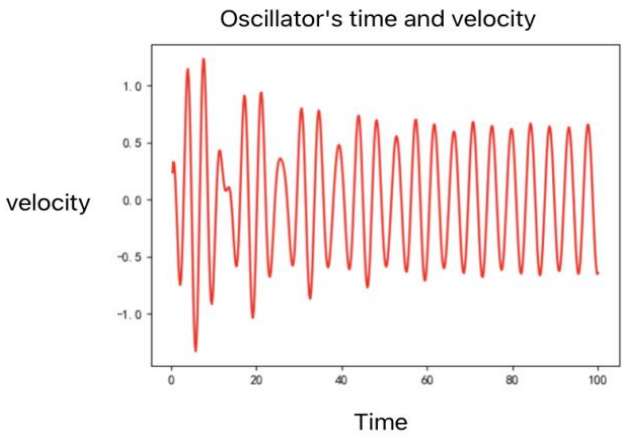


Figure 5. Oscillator's Time-Velocity Relationship Graph

Table 2. Results of Oscillator's Displacement and Velocity at Various Time Points

Time(s)	Oscillator	
	Displacement (m)	Velocity (m/s)
10	-0.212	-0.694
20	-0.635	-0.273
40	0.196	0.333
60	-0.332	-0.516
100	-0.084	-0.64

Under the same conditions, we conducted a comparative analysis of Figure 2, Figure 3, Figure 4, Figure 5, as well as Table 1 and 2, and found that at certain times $p = 10000\mathrm{N} \cdot \mathrm{s/m}$, the velocity of the oscillator is slightly greater than that of the float.

3.2. The displacement and velocity of the float and oscillator are solved under the condition that the damping coefficient is determined.

The damping coefficient of the linear damper is directly proportional to the absolute value of the power of the relative velocity between the float and the oscillator, with the proportionality coefficient set to 10000 and the power exponent set to 0.5.

Figures 6 to 9 show the relationship between the velocity and time, as well as the displacement and time, for the float and the oscillator under condition 3.2.

Table 3 and Table 4 show the results of displacement and velocity of the float and the oscillator at various time points.

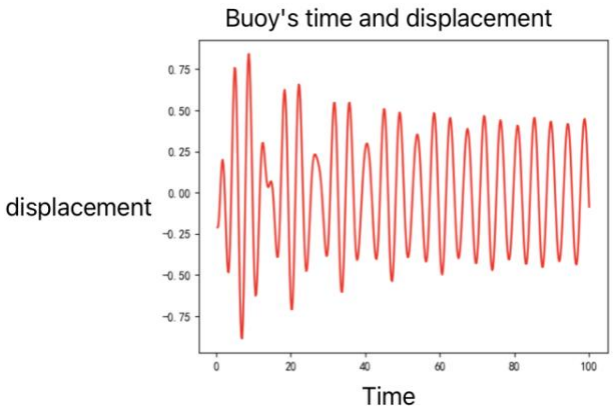


Figure 6. Diagram of the float's time versus displacement relationship

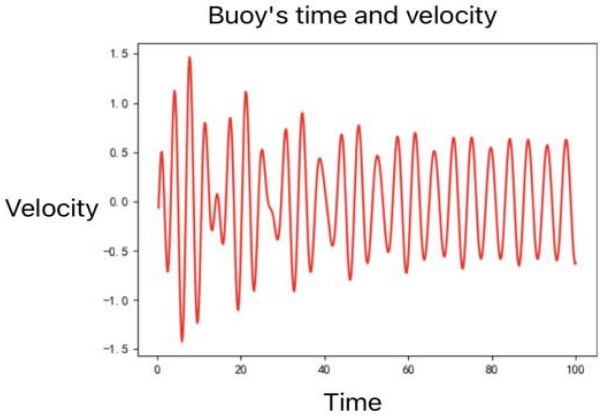


Figure 7. Diagram of the float's time versus velocity relationship

Table 3. Results of the float's displacement and velocity at various time points

Time (s)	Float	
	Displacement (m)	Velocity (m/s)
10	-0.25	-1.062
20	-0.633	-0.539
40	0.266	0.158
60	-0.325	-0.557
100	-0.084	-0.624

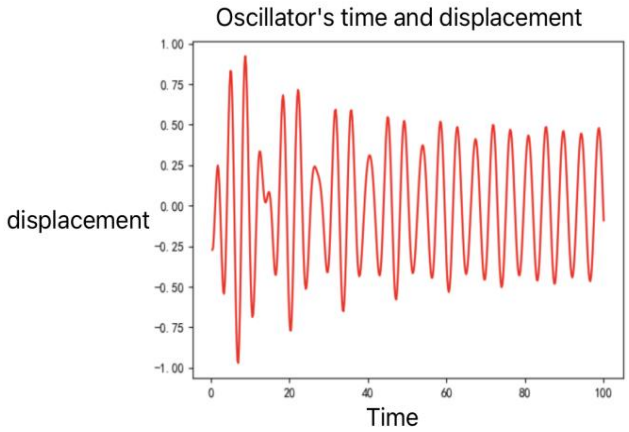


Figure 8. Diagram of the oscillator's time versus displacement relationship

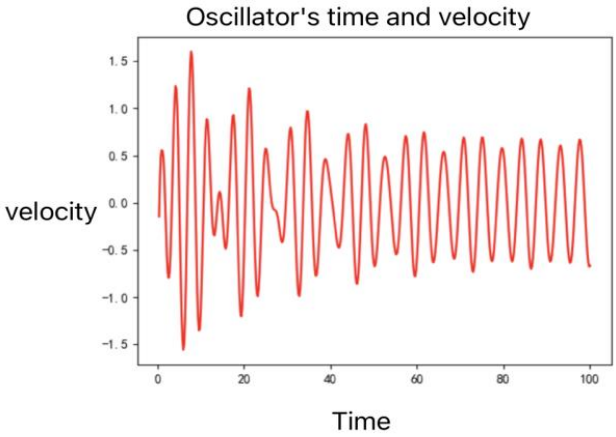


Figure 9. Diagram of the oscillator's time versus velocity relationship

Table 4. Results of the oscillator's displacement and velocity at various time points.

Time (s)	Oscillator	
	Displacement (m)	Velocity (m/s)
10	-0.282	-1.164
20	-0.685	-0.599
40	0.277	0.159
60	-0.347	0.599
100	-0.091	0.666

4. Conclusions

In this section, the research focused on establishing a mathematical model for a wave energy device undergoing heaving motion. The study began with a force analysis, establishing the force equations for both the float and the oscillator. Subsequently, Newton's second law was applied to solve for the relevant accelerations. Through differential equations, the velocity was further derived, and displacement equations were established, thereby solving for the trajectories of displacement over time and velocity over time. This part of the research provides a foundation for the performance analysis of wave energy devices under heaving motion. Future research can build upon this model to further explore and optimize the design of wave energy devices, enhancing their efficiency and stability in practical applications.

Although this model provides a mathematical basis for the heaving motion of wave energy devices, there may be limitations and shortcomings in practical applications. For instance, the model may not have considered all possible real-world environmental factors, such as ocean currents and wind speeds, which could affect the actual performance of the device. Additionally, the model overlooked the mass of the shaft, base, intermediate layers, and Power Take Off (PTO), as well as various types of friction when it was established, which might lead to deviations between predicted results and actual conditions in certain cases. Future research can further optimize the model by including these factors.

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