Calculation of the Optical Efficiency of a Fixed-sun Mirror Field Based on a Stationary Circular Model

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Abstract. As the global demand for electricity continues to surge, traditional power generation methods are confronted with the challenges of resource depletion and ecological degradation. Solar energy, due to its widespread availability and non-polluting nature, has emerged as a promising alternative in the realm of power generation. Among solar energy technologies, tower-type photovoltaic power generation stands out with its stability, high rate, and maintainability, offering significant potential for development in regions rich in solar resources. This paper begins by delineating the calculation of shadow shading effect, the classification of projection types, and the methodologies for addressing shadow shading efficiency. Leveraging this foundation, the study proceeds to determine the optical efficiency of the heliostat, taking into account shadow shading efficiency, cosine efficiency, atmospheric transmittance, collector truncation efficiency, and mirror reflectivity. Ultimately, by computing the output thermal power of the heliostat mirror field on the 21st day of each month, the study arrives at the annual average optical efficiency, output power, and the annual average output thermal power per unit area of the mirror surface for the Gansu region.

Keywords: Heliostat Field, Shade Shading Efficiency, Optical Efficiency, Average Annual Output Power.

1. Introduction

With the improvement of people's living standards, the demand for electricity is increasing, but traditional power generation methods consume a large amount of non-renewable resources and cause damage to the environment. New energy sources such as solar energy, wind energy and water energy are gradually being emphasized[1]. Among them, solar energy has become an effective way to solve the problem of new energy generation due to its widely distributed and non-polluting characteristics. Solar power generation includes photovoltaic power generation and solar thermal power generation, in which solar thermal power generation adopts thermal energy storage system, which can supplement power generation when there is insufficient light to ensure the normal operation of the power station[2]. Tower-type solar thermal power station is one of the forms, which focuses the sun's rays to the tower top receiver through reflective panels, generating high-temperature steam to drive the turbine to generate electricity. This type of power generation is environmentally friendly, sustainable and makes efficient use of solar energy resources to achieve continuous and stable power output[3]. The heliostat field is an important part of the tower solar thermal power station, which can focus the sun's rays into a small area to generate high temperatures, improve the efficiency of solar energy utilization and reduce environmental pollution[4]. With the advancement of technology, tower solar thermal power plants are expected to play an important role in the future energy field and make greater contributions to sustainable development.

Gao Bo et al. established the optical efficiency model of solar thermal power plant fixed-sun mirror field based on the unobstructed (EB) layout model, and proved the accuracy of the optical efficiency modeling by calculating the efficiency of the fixed-sun mirror field with SolarPILOT for the same coordinate data[5]. Based on the analysis and research on the design scheme of molten salt storage tanks of constructed projects, Xu Liguo et al. proposed a variety of design optimization schemes from the perspective of reducing the redundant design under the dead level of molten salt storage tanks, which can effectively reduce the construction cost of such projects[6].

At present, scholars have less research on the optical efficiency and the output power of heliostat field under the consideration of shadow shading efficiency, based on this paper, according to the simplified assumption conditions, under the condition of considering the shadow shading efficiency, the optical efficiency and the output power of heliostat are calculated by calculating the solar altitude angle, azimuth angle and other parameters.

2. Modeling and solving the heliostat field

2.1. Modeling of the heliostat field

In order to simplify the modeling process for calculating the mirror field efficiency of tower-type photovoltaic power plants, the following basic assumptions are used in this paper:

The sunlight is parallel light;

The sun setting mirror surface is a whole mirror

The tracking of the heliostat mirror is error-free;

The heliostat is a plane mirror;

Sunny days throughout the year with no clouds covering the heliostat field.

In this paper, it is assumed that a stationary circular heliostat mirror field model is constructed in a circular area centered at 98.5° east longitude and 39.4° north latitude, with an elevation of 3000 m and a radius of 350 m. The center of the circular area is the origin of the mirror field, and the z-axis is positive in the east direction. The mirror field coordinate system is established with the center of the circular area as the origin, the east direction as the x-axis positive, the north direction as the y-axis positive, and the upward direction perpendicular to the ground as the z-axis positive. The height of the absorber tower is 80 m, and the collector is a cylindrical exterior light receiving collector with a height of 8 m and a diameter of 7 m. The area around the absorber tower is 100 m. No heliostat will be installed within 100 m around the tower, and the open space will be reserved for the construction of the plant, which will be used for the installation of power generation, energy storage, control and other equipment. The shape of the mirror is a planar rectangle with the top and bottom edges always parallel to the ground, the distance between these two edges is the height of the mirror, and the distance between the left and right edges of the mirror is the width of the mirror. The absorber tower is built in the center of the circular heliostat field, the dimensions of the heliostats are 6 m×6 m, the installation height is 4 m, and the location of the centers of all the heliostats is given (data from: www.mcm.edu.cn).

2.2. Model solving

A heliostat is an optical device that utilizes the principle of specular reflection to gather and utilize sunlight. The optical efficiency of a heliostat is the proportion of sunlight energy incident on its surface that is converted by the mirror into energy that can be effectively utilized. The optical efficiency of each mirror in a heliostat field consists of five main components: shadow blocking efficiency η_{sb} , cosine efficiency $\eta_{\cos}$, atmospheric transmittance η_{at} , collector cut-off efficiency η_{trunc} and specular reflectivity η_{ref} [7]. The optical efficiency of a heliostat mirror at any point in time is calculated as follows:

$$\eta = \eta_{\mathrm{sb}}\eta_{\cos}\eta_{\mathrm{at}}\eta_{\mathrm{trunc}}\eta_{\mathrm{ref}} \tag{1}$$

Calculate the solar altitude angle α_s and solar azimuth angle γ_s with the following formulas[8]

$$\sin\alpha_{\mathrm{s}} = \cos\delta\cos\varphi\cos\omega + \sin\delta\sin\varphi \tag{2}$$

$$\cos\gamma_{\mathrm{s}} = \frac{\sin\delta - \sin\alpha_{\mathrm{s}}\sin\varphi}{\cos\alpha_{\mathrm{s}}\cos_{\varphi}} \tag{3}$$

Where φ is the latitude of the heliostat field, ω is the solar hour angle, δ is the solar declination angle, and ω and $\sin\delta$ are calculated as follows:

$$\begin{cases} \omega = \dfrac{\pi}{12}(ST-12) \\ \sin\delta = \sin\dfrac{2\pi D}{365}\sin\left(\dfrac{2\pi}{360}23.45\right) \end{cases} \tag{4}$$

Where ST is the local time and D is the number of days counted from the equinox as day 0.

In the optical design of heliostat mirrors, the cosine efficiency (Cosine Efficiency) is one of the evaluation indexes to measure the effective utilization of incident sunlight by the mirror. Cosine Efficiency takes into account the angle between the angle of incidence of light and the normal to the mirror, as well as the distribution of incident light, reflecting the mirror's ability to receive sunlight. The value of cosine efficiency ranges from 0 to 1, and the higher the value, the more efficient the mirror is in utilizing the sunlight. When the incident angle is close to perpendicular to the mirror surface, the cosine efficiency is the largest, while when the incident angle is close to horizontal, the cosine efficiency is lower[9]. In the optical design of heliostat mirrors, the cosine efficiency can be improved by optimizing parameters such as mirror shape, mirror material and reflective coating to achieve higher light energy capture and focusing effect. The cosine efficiency directly affects the optical performance and energy utilization efficiency of the heliostat. The cosine efficiency formula is derived below.

A coordinate system is established with the center of the heliostat as the origin, and the X , Y and Z axes point to the south, east and sky respectively, where s is the unit vector parallel to the sun's rays, r is the unit vector parallel to the rays from the heliostat to the receiver, and n is the unit vector normal to the mirror. For unit vectors in any direction in space, they can be expressed in terms of their azimuth and altitude angles:

$$V = \left(V_x, V_y, V_z\right) = (\cos\alpha\cos\delta, \cos\alpha\sin\delta, \sin\alpha) \tag{5}$$

Set the coordinates of the neighboring heliostat P point as (X0 , Y0 , Z0), the coordinates of the P point and the unit vector s (take the unit vector s when calculating the shadowing efficiency, and take the unit vector r when calculating the occlusion efficiency) can be obtained from the equation of the light ray which can be obtained from the P point and is parallel to s (or r). Normal unit vector n perpendicular to the surface of the mirror and over the X Y plane origin, can get the mirror equation. Plane equation can be obtained from the projected point C coordinate value, and then the C point coordinate value will be transformed to the center of the mirror surface as the origin, parallel to the mirror side of the X and Y axes of the coordinate system, expressed as follows

$$x_e = \{[V_{nx}(V_xV_{ny} - V_yV_{nx})/\Omega - V_{ny}]X_0 + [V_{ny}(V_xV_{ny} - V_yV_{nx})/\Omega + V_{nx}]Y_0 + V_{nz}(V_nV_{ny} - V_yV_{nx})Z_0/\Omega\}/\Phi$$

$$y_e = [-V_zV_{nx}X_0 - V_zV_{ny}Y_0 + (V_xV_{nx} + V_yV_{ny})Z_0]/(\Omega\Phi)$$

$$z_e = 0$$

where $\Phi = \left(V_{nx}^2 + V_{ny}^2\right)^{1/2}, \Omega = V_xV_{nx} + V_yV_{ny} + V_{zz}V_{nz}\Phi = \left(V_{nx}^2 + V_{ny}^2\right)^{1/2}, \Omega = V_xV_{nx} + V_yV_{ny} + V_{zz}V_{nz}$. The centers of adjacent fixed-sun mirrors P produce shadows and occlusions on the current mirror if and only if the two conditions y_e and z_e are satisfied, then

$$\begin{cases} |x_e| < L_x \\ |y_e| < L_y \end{cases} \tag{6}$$

It is assumed that the adjacent fixed-sun mirrors have the same geometry as the fixed-sun mirror to be analyzed as well as the same azimuthal angle, and that the projections to the mirrors are all

rectangular. For each projection, depending on the coordinate values of the projection points, it can be classified into four projection types, a, b, c, and d, which are expressed by the equation

$$\begin{cases} a \text{ if } x_e < 0 \text{ and } y_e > 0 \\ b \text{ if } x_e > 0 \text{ and } y_e > 0 \\ c \text{ if } x_e < 0 \text{ and } y_e < 0 \\ d \text{ if } x_e > 0 \text{ and } y_e < 0 \end{cases} \tag{7}$$

In each case, the shaded region is long $u = L_x - |x_e|$ and wide $v = L_y - |y_e|$. Finally, each side of the mirror is divided into n equal parts, so that each neighboring fixed-sun mirror projects onto it, and for each equal part there is:

$$F(i,j) = v \begin{cases} \text{for } 1 \text{ 到 } u/s & \text{case } a \\ \text{for } \frac{L_x - u}{s} \text{ 到 } n & \text{case } b \\ \text{for } n+1 \text{ 到 } n+1+u/s & \text{case } c \\ \text{for } n+1+\frac{L_x - u}{s} \text{ 到 } 2n & \text{case } d \end{cases} \tag{8}$$

Where $s = L_x / n$ is the width of each aliquot, i represents the i-th aliquot, and j represents the j-th neighboring heliographic mirror. If the aliquot has a shadow mask then the value is v , if not then it is 0. Then take the maximum value of the projection of one of the N mirrors at the aliquot, i.e., $R(i) = \max_{j=1}^{N}(F(i,j))$. Finally, summing up all the shadowed regions to find the shadow occlusion efficiency is

$$\eta_{sb} = \left[\sum_{i=1}^{2n} R(i) \right] / nL_y \tag{9}$$

Let the incident light shine at the point O_n on the mirror surface, and let BA_nD_nO be a rectangle, where O is the origin of the coordinate system xyz , and the coordinates of the point D_n are $(x_n^{`}, y_n^{`}, 0)$, $|O_nB_n| = |A_nO_n| = d_n, A_nD_n = h_n, O_nD_n = l_n$. The triangle A_nO_nB is an isosceles triangle, and O_nC_n vertically bisect A_nB , $OB = H_n$. Then we have.

$$\begin{cases} h_n = d_n \cdot \sin\alpha_s \\ l_n = d_n \cdot \cos\alpha_s \\ \dfrac{y_n - y_n^{'}}{l_n} = \cos\gamma_s \\ \dfrac{x_n - x_n^{'}}{l_n} = \sin\gamma_s \\ \dfrac{\sqrt{\left(x_n^{'}\right)^2 + \left(y_n^{'}\right)^2 + \left(h_n - H_n\right)^2}}{2} = d_n \cdot \sin\Theta_s \end{cases} \tag{10}$$

Where γ_s , α_s is the azimuth and altitude angle of the sun; θ_n is a fixed sun mirror n mirror reflection in the angle of incidence (or reflection angle). Associative formula (6) to formula (10), get:

$$\cos\Theta_n = \left(\frac{\sin\alpha_s \cdot H_n + x_n \cos\alpha_s \cos\gamma_s + y_n \cos\alpha_s \cos\gamma_s + d_n}{2 \cdot d_n} \right)^{1/2} \tag{11}$$

This gives the cosine efficiency:

$$\eta_{\cos} = \cos\Theta_n \tag{12}$$

The atmospheric transmittance is calculated as follows:

$$\eta_{at} = 0.99321 - 0.0001176 d_{HR} + 1.97 \times 10^{-8} \times d_{HR}^2 \left(d_{HR},, 1000 \right) \tag{13}$$

Specular reflectivity can be taken as a constant 0.92, where θ is the angle of incidence (or reflection) in specular reflection, and d_{HR} indicates the distance from the center of a mirror to the center of the collector (unit: m).

The truncation efficiency is the percentage of the total converged energy in the mirror field that is truncated by the heat absorber. It is calculated using the ray tracing method[10]. The usual step in the ray tracing method for calculating the truncation efficiency is to determine the target plane in the optical system, which is usually where the output image of the optical system is located. A beam of light is emitted. Then for each light ray, it is traced through the optical system, the path of the light ray is tracked, and the intersection of the light ray with the optical element is recorded. Then for each light ray, it is judged whether the light ray is truncated in the target plane. Usually a threshold judgment is used, if the distance between the light ray and the target plane is less than the set truncation radius, the light ray is considered to be successfully truncated. Finally, the number of successfully intercepted rays is counted and divided by the total number of rays emitted to obtain the interception rate. The truncation rate is the value used to calculate the truncation efficiency, which indicates the percentage of light rays that are successfully focused on the target plane in the optical system.

Let the coordinates of the center position of a mirror be $M\left(x_M, y_M, z_M\right)$, and its focus point is the center position of the heat absorber $Q(0,0,H)$,, by the altitude angle and azimuth angle of the sun, according to the positional relationship of the coordinate system, the direction vector of the sun's incident light is calculated as

$$\text{in } = \left(\sin\alpha_s \cos\gamma_s, \cos\alpha_s \cos\gamma_s, -\sin\gamma_s\right) \tag{14}$$

From the coordinates of the reflection point and the focusing point, the direction vector of the reflected rays is given as

$$re = \frac{Q-M}{|Q-M|} = \frac{\left(-x_M, -y_M, H-z_M\right)}{\sqrt{x_M^2 + y_M^2 + \left(H-z_M\right)^2}} \tag{15}$$

According to the law of reflection, then the normal vector of the mirror of the heliostat is

$$nor = \frac{re-in}{|re-in|} \tag{16}$$

Let the effective number of points of intersection of the incident ray with the heliostat be n_1 and the effective number of points of intersection of the reflected ray with the heat absorber be n_2 . Then the truncation efficiency is

$$\eta_{\text{trunc}} = \frac{n_2}{n_1} \tag{17}$$

Normal direct radiant irradiance DNI (in units of kW/m^2) is the solar radiant energy received per unit area of a plane on the earth perpendicular to the sun's rays, per unit time, on the 21st day of each month DNI can be approximated according to the following equation

$$\begin{cases} DNI = G_0\left[a + b\exp\left(-\dfrac{c}{\sin\alpha_s}\right)\right] \\ a = 0.4237 - 0.00821(6-H)^2 \\ b = 0.5055 + 0.00595(6.5-H)^2 \\ c = 0.2711 + 0.01858(2.5-H)^2 \end{cases} \tag{18}$$

Where G_0 is the solar constant whose value is taken as $1.366\,kW/m^2$, H is the altitude (in km).

A heliostat field is an optical system for solar energy focusing, consisting of multiple curved mirrors that can focus the energy of solar radiation onto a small area, thereby increasing the efficiency of light energy utilization. In applications such as solar power generation, the output thermal power of a heliostat field is an important parameter to measure how well it actually works.

The output thermal power of a heliostat field depends on a number of factors, including the angle of incidence of the sun, the geometry of the heliostat field, the quality and material of the mirrors, and the focusing distance. Theoretically, the output thermal power of a heliostat field can be determined by calculating the solar radiation flux in the focusing region and the focusing efficiency.

Calculate the output thermal power of the fixed-sun mirror field on the 21st day of each month under the assumptions of the model E_{field} with the following equation

$$E_{\text{field}} = DNI \cdot \sum^{N} A_i \eta_i \tag{19}$$

Where N is the total number of heliostat mirrors (unit: surface); A_i is the i surface area of heliostat mirrors (unit: m2); η_i is the i surface optical efficiency of the mirrors. The output thermal power from the fixed-sun mirror field divided by the product of individual mirror area and the number of mirrors can be obtained from the average output thermal power per unit area of the mirror surface.

3. Results

Above, the shadow shading efficiency, the cosine efficiency and the collector truncation efficiency were first calculated to determine the optical efficiency of a particular mirror on the 21st day of each month. Then, by combining these monthly average optical efficiencies with the normal direct radiant irradiance obtained by calculating the solar altitude angle on the 21st day of each month, the output thermal power of a fixed-sun mirror field on the 21st day of each month can be calculated, and the annual average output thermal power and the annual average output thermal power per unit of mirror surface area are further obtained for that fixed-sun mirror field.

According to the above method, the average optical efficiency, the average output thermal power, and the average output thermal power per unit area of mirror surface for 12 months are calculated to obtain the annual average optical efficiency, the annual average output thermal power, and the annual average output thermal power per unit area of mirror surface for the fixed-heaven mirror field. The results are shown in Tables 1 and 2

Table 1 Average optical efficiency and output power on the 21st day of each month

dates	on average optical efficiency	on average cosine efficiency (math.)	average shadow Sheltering efficiency	on average truncation efficiency	Average thermal power output per unit area of mirror (kW/m^2)
January 21	0.33	0.73	0.83	0.55	0.35
February 21	0.34	0.74	0.84	0.55	0.37
March 21	0.33	0.73	0.84	0.55	0.37
April 21	0.35	0.75	0.85	0.55	0.39
May 21	0.34	0.74	0.86	0.55	0.40
June 21	0.35	0.75	0.87	0.55	0.43
July 21	0.37	0.78	0.87	0.55	0.45
August 21	0.39	0.78	0.88	0.55	0.48
September 21	0.38	0.79	0.89	0.55	0.46
October 21	0.35	0.78	0.87	0.55	0.43
November 21	0.34	0.76	0.85	0.55	0.40
December 21	0.34	0.75	0.83	0.55	0.38

Table 2 Table of annual average optical efficiency and output power

annual average optical efficiency	annual average cosine efficiency (math.)	annual average Shadow Occlusion Efficiency	annual average truncation efficiency	Average annual thermal power output (MW)	Average annual thermal power output per unit area of mirror (kW/m^2)
0.351	0.757	0.857	0.55	26.1	0.409

4. Conclusions

This study focuses on the output power of fixed-sun mirror fields in photovoltaic power generation. We have derived the optical efficiency of these mirror fields by considering multiple factors, including shadow shading efficiency, cosine efficiency, atmospheric transmittance, collector truncation efficiency, and specular reflectance. Additionally, this paper have established a circular fixed-sun mirror field model to evaluate its optical efficiency and annual average output power, specific to hypothetical conditions representative of the Gansu region. The model's optical efficiency was computed to be 0.351, with an annual average cosine efficiency of 0.757, shadow blocking efficiency of 0.857, truncation efficiency of 0.55, and an annual average output thermal power of 26.1 MW. The annual average output thermal power per unit area of the mirror surface was found to be 0.409 KW/m2.

While recognizing that real-world heliostat fields are influenced by numerous factors, the inclusion of more variables in the model would increase its complexity but also its realism. This study assumes uniform heliostat mirror size and installation height, with the absorption tower situated at the center of the circular heliostat field. Future research should aim to enhance the model's variable indices and explore how to establish and calculate heliostat field models under more variable conditions.

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