Research on Multi-Beam Sounding Based on Local Optimal Model

Xinrui Song *,†, Zhenyu Song †, Zheng Chai †

College of electronic information and optical engineering, Taiyuan University of Technology, Taiyuan, China, 030024

* Corresponding Author Email: S13223555690@outlook.com

† These authors contributed equally.

Abstract. Multi-beam sounding technology can provide higher spatial resolution than single-beam sounding by transmitting fan-shaped acoustic signals from the transducer of a survey ship along the normal plane of the survey line to measure the sea depth. It is of great significance to study the line design scheme of multi-beam sounding for uneven seabed topography to improve the efficiency of underwater terrain mapping. This paper mainly studies the optimization of multi-beam sounding line under ideal slope. Firstly, a single angle model of ideal slope is established, and the flat seabed overlap ratio formula is extended to slope seabed to calculate the coverage width of beam sounding. Secondly, an angle model of ideal slope is established to calculate the corresponding coverage width under different positions of the slope sea area. Finally, the optimal routing model of ideal slope is established. The design scheme of the local optimal survey line is obtained through comprehensive analysis. In engineering practice, the optimal routing of real terrain can be solved by using the ideal slope optimal routing model through the idea of plane approximation, which provides a new idea for the optimal route selection of underwater terrain mapping.

Keywords: Multi-beam sounding, Local optimal model, Greedy algorithm.

1. Introduction

Single-beam sounding sends a single pulse wave vertically down, which misses the characteristic points and does not accurately reflect the real terrain [1]. Compared with the single beam sounding, the multi-beam sounding sends the fan beam, which can realize the full coverage of the underwater terrain measurement [2, 3]. In the process of ocean surveying and mapping, in order to achieve full coverage measurement as much as possible, the multi-beam sounding method is often used [4]. Based on the analysis of the relationship between the coverage width and the overlap rate between adjacent strips, this paper establishes the local optimal model of the shortest line length under the condition of full coverage measurement, and provides a solution for the route selection of underwater terrain surveying by multi-beam sounding technology.

2. Materials and methods

2.1. Symbol Declaration

The symbols are described in Table 1.

Table 1. Symbol Declaration

Symbols	Instructions	Units
D_i	Depth of sea water as measured by Line i	Meters (m)
α	Grade of the slope	Degree ($^\circ$)
β	The angle between the direction of the survey line and the projection of the normal direction of the seafloor slope on the horizontal plane	Degree ($^\circ$)
γ	The angle between the intersection line of the multi-beam scanning surface and the submarine slope and the critical wave line in the deep	Degree ($^\circ$)
θ	The opening angle of the multi-beam transducer	Degree ($^\circ$)
d	The distance between two adjacent measuring lines	Meters (m)
W	The coverage width of the multi-beam sounding strip	Meters (m)
η	Overlap rate between adjacent strips	%
ε	The plumb plane formed by multi-beam scanning	-
σ	The plumb plane where the line is located	-
α'	The line angle formed by the intersection line of the plane ε and the slope and the horizontal plane	Degree ($^\circ$)
φ	The line angle formed by the intersection line of the plane σ and the slope and the horizontal plane	Degree ($^\circ$)

2.2. Data

For the convenience of discussion, the ideal rectangular sea area is stipulated to be 2 nautical miles long from north to south and 4 nautical miles wide from east to west, with a depth of 110m at the central point of the sea area, deep in the west and shallow in the east, with a slope of 1.5° and an opening angle of 120° for the multi-beam transducer.

2.3. Method introduction

Firstly, a single angle model of ideal slope is established in two-dimensional plane. According to the definition of overlap rate, the expression of overlap rate with the previous line is obtained. Then, the angle β between the direction of the surveying line and the normal direction of the submarine slope projected on the horizontal plane is added, and the ideal slope variable angle model is established. The model is extended to the three-dimensional plane for discussion, and the corresponding coverage width of the sea area on the slope under different positions is calculated. Finally, the optimal routing model of the ideal slope is established, the local optimal solution is carried out, the survey line schemes under different β values are discussed, and the design scheme of the optimal survey line is finally obtained.

3. Model establishment and solution

3.1. Model hypothesis

(1) The sea surface is assumed to be a plane parallel to the horizontal plane without considering the curvature of the Earth.

(2) The influence [5] of yaw angle, roll and pitch on driving is not taken into account. It is assumed that the course of the survey ship overlaps with the survey line, which is strictly straight, and there is no intersection between the survey lines.

(3) Do not consider the impact [6, 7] of yaw angle, roll and pitch on the multi-beam transducer. Assume that the scanning surface of multi-beamforming is perpendicular to the survey line.

3.2. Single angle model of an ideal slope

In the two-dimensional plane discussion, the Cartesian coordinate system is established with the positive direction of x axis from due south to due north of the geographical orientation, the positive direction of y axis from due west to due east of the geographical orientation, and the positive direction

of z axis from the outer normal direction of the sea level, as shown in Figure 1. The discussion is carried out in the two-dimensional yoz plane.

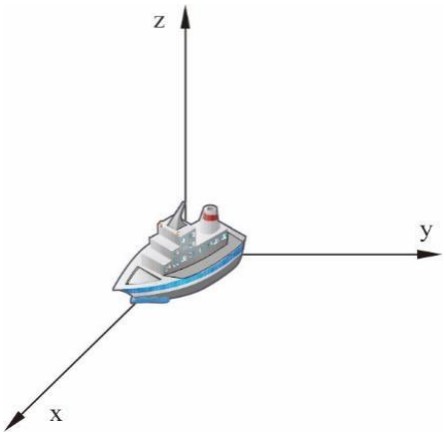


Figure 1. Cartesian coordinates

Define the variables: As shown in Figure 2, the ship is numbered Y_i according to the distance of the surveying ship from the center point of the sea area, the water depth at the corresponding position is D_i, and the ship number at the distance of 0m from the center point is Y_0. When the surveying ship is in Y_{i-1} position, the intersection line between the multi-beam scanning surface and the submarine slope is AC; when the ship is in Y_i position, the intersection line between the multi-beam scanning surface and the submarine slope is PQ. Make a horizontal parallel line through point P and intersect A at point E. The length of EP is the horizontal distance between the ship Y_i and Y_{i-1}. Let the angle between the intersection line of the multi-beam scanning surface and the seafloor slope and the critical wave line at depth be γ, i.e., $\angle CAE = \gamma$.

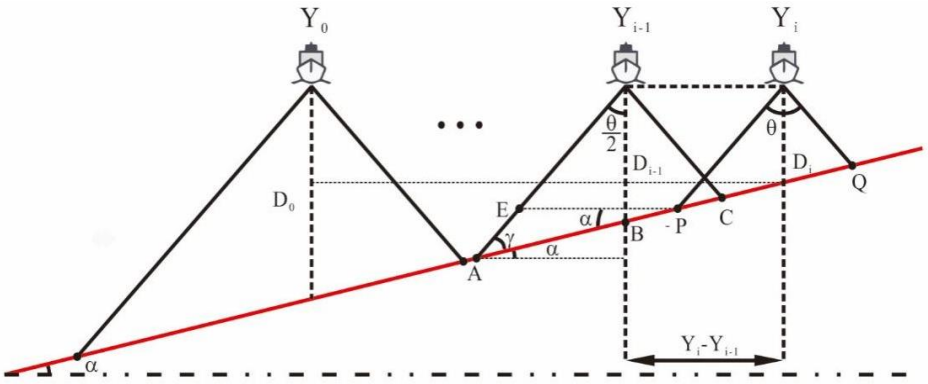


Figure 2. Measurement situation

Therefore, establishing a single angle model of ideal slope can be simplified as an equation problem.

From equation (1), the seawater depth measured by line i can be calculated:

$$D_i = \mathrm{D}_0 - (Y_i - Y_0) \times \tan\alpha \tag{1}$$

According to the sine theorem, we get:

$$\frac{AB}{\sin\frac{\theta}{2}} = \frac{D_0}{\sin\gamma} \tag{2}$$

$$\frac{BC}{\sin\frac{\theta}{2}} = \frac{D_0}{\sin(180-\theta-\gamma)} \tag{3}$$

$$\gamma = \frac{180-\theta}{2} - \alpha \tag{4}$$

Associating Eqs. (2) (3) (4) yields the coverage width of the slope as:

$$W_i = AC = AB + BC = \left[\frac{\sin\frac{\theta}{2}}{\sin\gamma} + \frac{\sin\frac{\theta}{2}}{\sin(180-\theta-\gamma)}\right] \times D_i \tag{5}$$

It can be seen that when the sea level terrain is flat, d is the spacing of two adjacent measurement lines, W is the cover width of the strip, then the overlap width is $(W-d)$. Therefore, the overlap rate [8] with the previous survey line:

$$\eta = \frac{Repetition\ Width}{Covering\ Width} = \frac{W-d}{W} = 1 - \frac{d}{W} \tag{6}$$

When the sea level terrain is slope, the overlap rate with the previous survey line:

$$\eta = \frac{Repetition\ Width}{Covering\ Width} = \frac{PC}{AC} = \frac{AC-AP}{AC} = 1 - \frac{AP}{AC} = 1 - \frac{AP}{W_{i-1}} \tag{7}$$

Formula (8) is obtained by sine theorem:

$$AP = \frac{Y_i - Y_{i-1}}{\sin\gamma}\sin\angle AEP = \frac{Y_i - Y_{i-1}}{\sin\gamma}\sin\angle(180^\circ - \alpha - \gamma) \tag{8}$$

There are

$$\eta_i = 1 - \frac{\frac{\sin(180^\circ-\alpha-\gamma)}{\sin\gamma}}{\left[\frac{\sin\frac{\theta}{2}}{\sin\gamma} + \frac{\sin\frac{\theta}{2}}{\sin(180^\circ-\theta-\gamma)}\right]} \times \frac{Y_i - Y_{i-1}}{D_i - 1} \tag{9}$$

The coverage width of multibeam bathymetry in a two-dimensional plane and the overlap rate between neighboring strips are obtained, and a single-angle model of an ideal slope is established.

3.3. Variable angle model of an ideal slope

On the basis of the single angle model of ideal slope, the angle β between the direction of survey line and the normal direction of submarine slope projected on the horizontal plane is added.

When the ship travels along the surveying line to any position, the plumb plane formed by multi-beam scanning at the position is ε, and the plumb plane where the surveying line is located is σ, as shown in Figure 3.

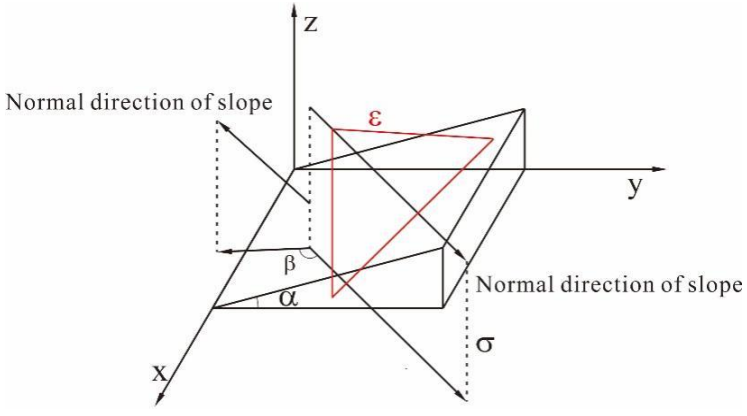


Figure 3. 3D model diagram

First of all, solve for the intersection line between the plane ε and the slope, which is converted to a problem of solving for the line of intersection of two planes in three-dimensional geometry. Project the known external normal vector of the slope onto the yoz plane, as shown in Figure 4.

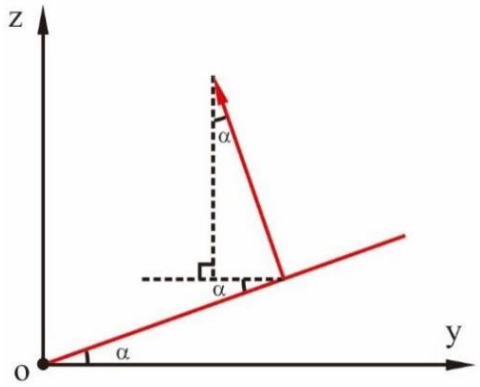


Figure 4. Projection of the external normal vector of the slope to the yoz plane

It can be seen that the unit normal vector of the slope is $\overrightarrow{n_1} = (0, -\sin\alpha, \cos\alpha)$.

The normal vector of the plane ε is identically $\overrightarrow{n_2} = \left(\cos\left(\beta - \frac{\pi}{2}\right), \sin\left(\beta - \frac{\pi}{2}\right), 0\right) = (\sin\beta, -\cos\beta, 0)$. The normal vector of the plane ε is cross-multiplied with the normal vector of the slope. From the geometric meaning of the cross product, the direction vector $\overrightarrow{l_1}$ of the intersection line of the two planes is obtained, as shown in equation (10).

$$\overrightarrow{l_1} = \overrightarrow{n_1} \times \overrightarrow{n_1} = (\cos\alpha\cos\beta, \sin\beta\cos\alpha, \sin\alpha\sin\beta) \tag{10}$$

At any position of the ship, the line and plane angle formed by the intersection line of the plane ε and the slope and the horizontal plane is α, as shown in Figure 5.

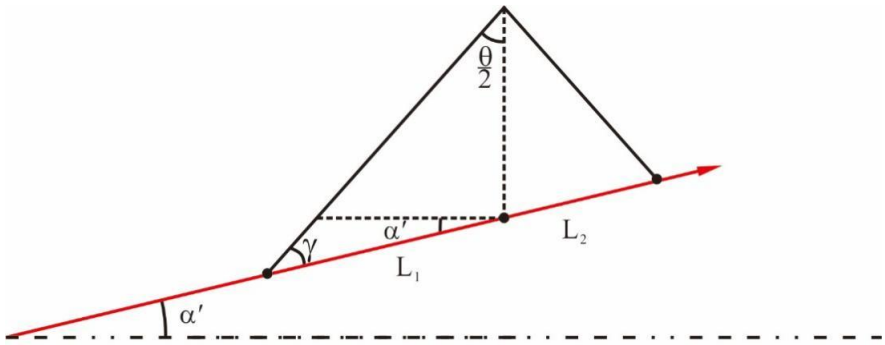


Figure 5. The line and plane angle α formed by the intersection line of the plane ε and the slope and the horizontal plane

Equation (11) can be obtained

$$\alpha = \tan^{-1}\frac{l_{1z}}{\sqrt[2]{(l_{1x})^2+(l_{1y})^2}} = \tan^{-1}\frac{\sin\alpha\sin\beta}{\sqrt[2]{(\cos\alpha\cos\beta)^2+(\sin\beta\cos\alpha)^2}} = \tan^{-1}\frac{\sin\alpha\sin\beta}{\cos\alpha} \tag{11}$$

Then solve the intersection line between the plane σ and the slope, the direction vector of the intersection line between the plane ε and the slope is $\overrightarrow{l_1}$. It is easy to know that the projection of $\overrightarrow{l_{1z}}$ on the horizontal plane is perpendicular to the plane σ, so the normal vector $\overrightarrow{n_3}$ of the plane σ is the projection of $\overrightarrow{l_1}$ on the horizontal plane, i.e., $\overrightarrow{n_3} = (\cos\alpha\cos\beta, \sin\beta\cos\alpha, 0)$.

The direction vector of the intersection line between the normal vector of the plane σ and the slope is $\overrightarrow{l_2} = \overrightarrow{n_1} \times \overrightarrow{n_3}$, as shown in equation (12).

$$\overrightarrow{l_2} = \overrightarrow{n_1} \times \overrightarrow{n_3} = (-\sin\alpha\cos^2\alpha, \cos\beta\cos^2\alpha, \sin\alpha\cos\alpha\cos\beta) \tag{12}$$

The line and plane angle formed by the intersection line between the plane σ and the slope and the horizontal plane is φ, as shown in figure 6.

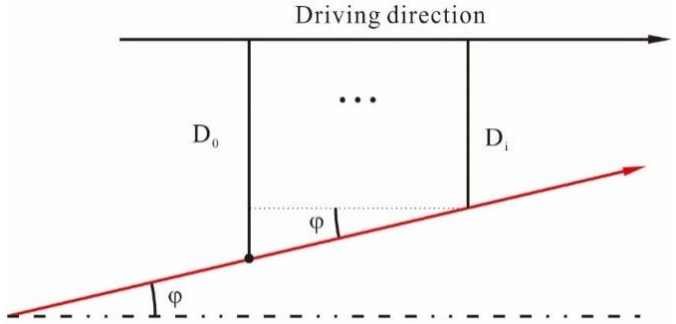


Figure 6. The line and plane angle φ formed by the intersection line between the plane σ and the slope and the horizontal plane

With equation (13)

$$\phi = \tan^{-1}\frac{l_{2z}}{\sqrt[2]{(l_{2x})^2+(l_{2y})^2}} = \tan^{-1}\frac{\sin\alpha\cos\alpha\cos\beta}{\sqrt[2]{(\cos^2\alpha\cos\beta)^2+(-\sin\beta\cos^2\alpha)^2}} = \tan^{-1}\frac{\sin\alpha\sin\beta}{\cos\alpha} \tag{13}$$

Discuss the effect of the β value on the calculation of seawater depth:

When $90° < \beta < 270°$, at this time, the ship is in an uphill state, the sea depth gradually becomes shallower;

When $0° \le \beta \le 90°$ or $270° \le \beta \le 360°$, at this time, the ship is in a downhill state, and the sea depth gradually becomes deeper.

As shown in equation (14)

$$D_i = \begin{cases} D_0 - \Delta d \times tan\alpha', 90^\circ < \beta < 270^\circ \\ D_0 + \Delta d \times tan\alpha',\ 0^\circ \le \beta \le 90^\circ \text{ or } 270^\circ \le \beta \le 360^\circ \end{cases} \tag{14}$$

$$\frac{AB}{\sin\frac{\theta}{2}} = \frac{D_i}{\sin\gamma'} \tag{15}$$

$$\frac{BC}{\sin\frac{\theta}{2}} = \frac{D_i}{\sin(180°-\theta-\gamma')} \tag{16}$$

$$\gamma' = \frac{180°-\theta}{2} - \alpha' \tag{17}$$

Associative formulations (15) (16) can be obtained:

$$W_i == \left[\frac{\sin\frac{\theta}{2}}{\sin\gamma'} + \frac{\sin\frac{\theta}{2}}{\sin(180-\theta-\gamma')}\right] D_i \tag{18}$$

Under the given ideal conditions, the coverage width of multi-beam sounding is shown in Table 2.

Table 2. Coverage width of multi-beam sounding

Coverage width /m		**Distance of the ship from the center point of the sea area/ nautical miles**							
		0	0.3	0.6	0.9	1.2	1.5	1.8	2.1
Angle of measure-ment line direction /°	0	381.0512	431.4500	481.8489	532.2478	582.6466	633.0455	683.4443	733.8432
	45	381.5089	417.1768	452.8448	488.5128	524.1807	559.8487	595.5166	631.1846
	90	381.9675	381.9675	381.9675	381.9675	381.9675	381.9675	381.9675	381.9675
	135	381.5089	345.8409	310.1730	274.5050	238.8371	203.1691	167.5012	131.8332
	180	381.0512	330.6523	280.2535	229.8546	179.4557	129.0569	78.6580	28.2591
	225	381.5089	345.8409	310.1730	274.5050	238.8371	203.1691	167.5012	131.8332
	270	381.9675	381.9675	381.9675	381.9675	381.9675	381.9675	381.9675	381.9675
	315	381.5089	417.1768	452.8448	488.5128	524.1807	559.8487	595.5166	631.1846

3.4. Optimal cabling model for ideal slope

First of all, based on the established variable angle model of an ideal slope, different β values are discussed and different survey line schemes are obtained. Then, the feasibility of the schemes is

evaluated, and the specific schemes of survey line layout and sea area survey are analyzed based on whether there is leakage detection and whether the overlap rate relationship between the two adjacent strips satisfies $10\%\sim20\%$.

3.4.1 Discussion of the general situation

Equation (18) shows the relationship between the covering width W_i of slope and the water depth D_i where the ship is located.

When $\beta \neq 0°, 90°, 180°, 270°$, for the general situation, the water depth of the ship traveling along the direction of the survey line changes constantly, that is, a linear variation of D_i leads to a linear variation of W_i. Therefore, the strips scanned along the survey line is trapezoid, as shown in Figure 7.

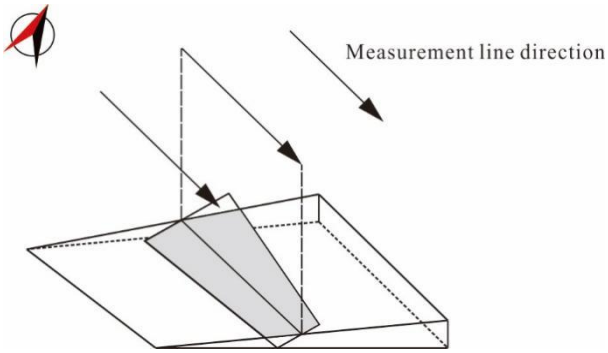
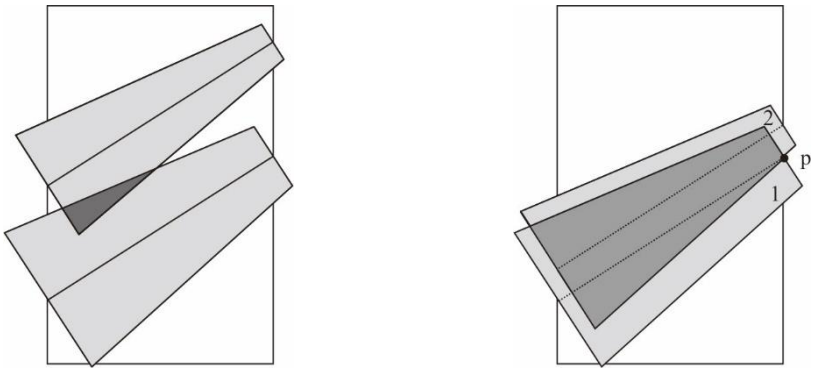


Figure 7. The case when β is a general value

When the measurement line spacing is too large, there will be a leakage test, as shown in Figure 8-(a), which does not meet the measurement specification [9]. In order to avoid missing measurement, the measurement line spacing should be reduced. Figure 8-(b) shows the maximum line spacing that can be achieved without missing test. In this case, according to geometric knowledge, the waist of Trapezoid 2 should pass through point p, and point p is the midpoint of the upper base of trapezoid 1. At this time, the overlap rate between the two adjacent strips is $\eta > 20\%$, which does not meet the requirements of the given overlap rate.



(a) Leakage test diagram (b) No leakage test diagram

Figure 8. Horizontal projection when β is a general value

3.4.2 Discussion of special cases

(1) When $\beta = 0° \; or \; \beta = 180°$, the plumb plane where the survey line is located is perpendicular to the submarine slope. Therefore, the strip formed by scanning along the survey line at any position is an isosceles trapezoid. The spacing d of measuring lines will affect the coincidence of scanning strips of adjacent measuring lines (that is, the area of isosceles trapezoid). The critical condition, that is, the maximum measuring line spacing d_{max} that can be achieved without missing measurement, is adopted for analysis, as shown in Figure 9.

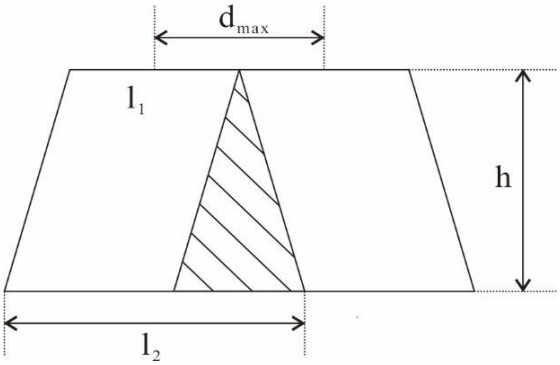


Figure 9. The critical condition when $\beta = 0° \; or \; \beta = 180°$

When measuring line spacing $d > d_{\max}$, at this time leakage between the two adjacent strips occurs and does not meet the requirements; When the line interval $d < d_{max}$, the overlap rate will be greater than the critical case, which does not meet the requirements; When the line interval $d = d_{\max}$, it is the maximum line spacing that can be achieved without missing measurement, as shown in equation (19), (20) and (21):

$$\eta = \frac{S_{\text{Overlap area}}}{S_{\text{Strip area}}} = \frac{S_{\text{triangle}}}{S_{\text{trapezium}}} \tag{19}$$

$$S_{\text{trapezium}} = \frac{(l_1 + l_2)}{2} \times h \tag{20}$$

$$S_{\text{triangle}} = \frac{(l_2 - l_1)}{2} \times h \tag{21}$$

Joining equations (19) (20) (21) we have:

$$\eta = \frac{S_{\text{Overlap area}}}{S_{\text{Strip area}}} = \frac{S_{\text{triangle}}}{S_{\text{trapezium}}} \tag{22}$$

$$\eta = \frac{S_{\text{Overlap area}}}{S_{\text{Strip area}}} = \frac{S_{\text{triangle}}}{S_{\text{trapezium}}} = \frac{l_2 - l_1}{l_1 + l_2} \quad (l_1, l_2 \text{is the upper and lower base of trapezoid}) \tag{23}$$

$$S_{\text{triangle}} = \frac{(l_2 - l_1)}{2} \times h \tag{24}$$

Knowing the depth of the sea water D_0 at the center point of sea area and the slope of the submarine slope α, l_1 =45.03m and l_2 =717.03m can be obtained. Substitution gives $\eta = 88.16\% > 20\%$, which does not meet the requirements. Therefore, the scheme is not feasible when $\beta = 0° \; or \; \beta = 180°$

(2) When $\beta = 90° \; or \; \beta = 270°$, the strips formed by scanning along different measurement lines are rectangles with different areas. As shown in Figure 10, this scheme will not miss detection, and the overlap rate η can be dynamically adjusted as needed.

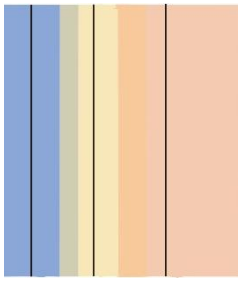


Figure 10. Scanning strip when $\beta = 90° \; or \; \beta = 270°$

3.4.3 Scheme implementation

The following design a group of survey line scheme with the shortest measurement length and complete coverage of the sea area. When $\eta = 10\%$, $\beta = 90°$ or $\beta = 270°$, that is, when the direction of measuring line layout is parallel to the strike of the isobath [10], the overlap area of each strip is the smallest, the total number of measuring lines is the smallest, and the measurement length is the shortest. The following analysis of the specific implementation of the program:

$l_{measure}$ is the single measurement line length, l_{total} is the total measurement line length, n is the number of measurement lines, S_{real} is the actual slope area, and S_{total} is the sum of the areas of the bands without repeated scans.

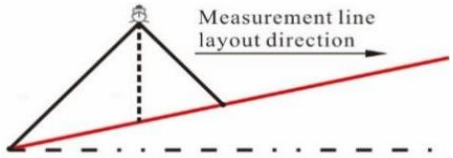
$$W_i = \left[\frac{\sin\frac{\theta}{2}}{\sin\gamma} + \frac{\sin\frac{\theta}{2}}{\sin(180-\theta-\gamma)}\right] \times D_i \tag{25}$$

$$\eta_i = 1 - \frac{\frac{\sin(180°-\alpha-\gamma)}{\sin\gamma}}{\left[\frac{\sin\frac{\theta}{2}}{\sin\gamma}+\frac{\sin\frac{\theta}{2}}{\sin(180°-\theta-\gamma)}\right]} \times \frac{T_i}{D_{i+1}} \in [10\%, 20\%] \tag{26}$$

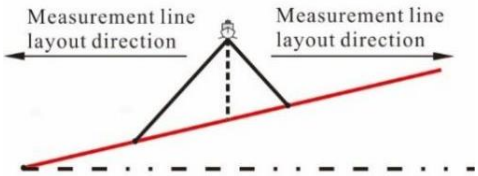
$$S_{total} = \sum_{i=0}^{n} W_i(1-\eta_i) \tag{27}$$

$$l_{total} = n \times l_{measure} \tag{28}$$

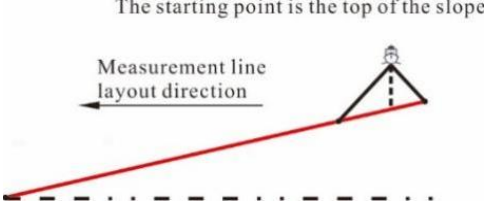
(1) Choice of starting point. There are three situations: the bottom of the slope is the starting point, the point between the slopes (taking the midpoint as an example) is the starting point, and the top of the slope is the starting point. As shown in Figure 11 below.



(a) The base of the slope is the starting point



(b) The starting point is between the slopes



(c) The top of the slope is the starting point

Figure 11. Schematic diagram of starting points for different measurements

The number of measured lines and total measured line length obtained from the above three different starting points are shown in the Table 3 below:

Table 3. The number of measured lines at different starting points

Starting point position	Number of measured lines	Total line length/meters
Top of slope	37	137048
Midpoint	35	129640
Base of slope	34	125936

It can be shown that when the starting point is the bottom of the slope, the shortest measurement scheme can be realized.

3.4.4 The optimal solution

When the starting point is the bottom of the slope, the shortest measurement scheme can be realized as shown in Figure 12:

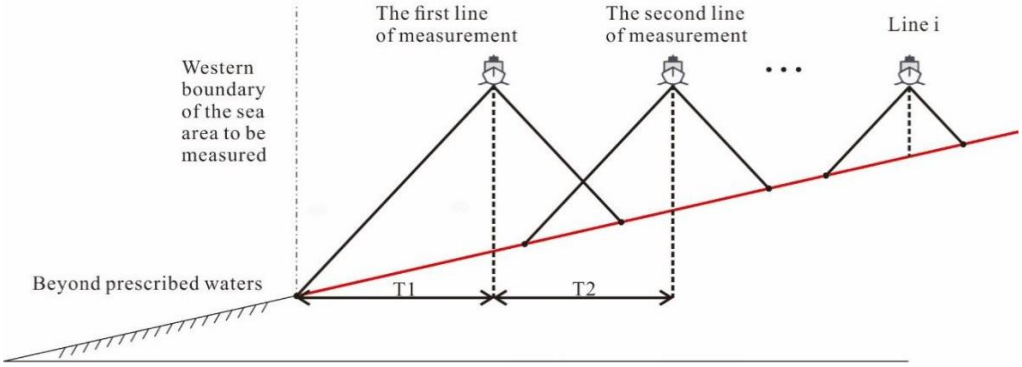


Figure 12. Schematic diagram of measuring line distribution

The specific parameters of the measurement scheme are shown in Table 4 below.

Table 4. Line numbers and horizontal intervals of the optimal solution

Measuring line number i	Horizontal direction interval T_i (m)	Measuring line number i	Horizontal direction interval T_i (m)	Measuring line number i	Horizontal direction interval T_i (m)
1	358.5217926	13	240.9355	25	90.80732
2	589.3399258	14	222.1191	26	83.7155
3	543.3139596	15	204.7721	27	77.17753
4	500.8825056	16	188.7799	28	71.15016
5	461.7648415	17	174.0367	29	65.59351
6	425.7021685	18	160.4449	30	60.47082
7	392.4558996	19	147.9145	31	55.7482
8	361.8060808	20	136.3628	32	51.39441
9	333.5499357	21	125.7132	33	47.38063
10	307.5005244	22	115.8953	34	43.68032
11	283.4855067	23	106.8442		
12	261.346	24	98.49991		

4. Conclusion

Aiming at the analysis of a given ideal sea area, this paper firstly establishes a single angle model of ideal slope, extends the flat seabed overlap ratio formula to the seabed of slope using plane geometry, and then computates the coverage width of beam sounding. Then, the angle between the direction of survey line and the projection of the normal direction of the seabed slope on the horizontal plane β is added, and the variable angle model of an ideal slope is constructed by applying solid geometry knowledge. Finally, the optimal routing model of the local ideal slope is established from the general and special angles, and the final optimal measurement scheme is obtained: when the

survey ship starts the measurement from the bottom of the slope, the direction of the survey line layout is parallel to the trend of the isobarth line, the total number of survey lines is minimum and the measurement length is shortest. In theory, the optimal routing scheme of multi-beam surveying and mapping in ideal rectangular sea area is provided. In engineering practice, the idea of plane approximation provides a new idea for the optimal route selection of underwater terrain surveying and mapping.

References

[1] Yan Wenbin, Application of multibeam bathymetry in hydrographic projects [J]. China Water Transport, 2019, (05): 48-50.

[2] Dong Yu. Application research of Multi-beam sounding system in ocean Hydrographic survey [J]. Research of Engineering Technology, 2023, 8 (15): 122-124.

[3] Yu Qiyi. Submarine topographic survey based on multi-beam sounding technology [J]. Surveying, Mapping and Spatial Geographic Information, 2022, 45 (09): 262-4. (In Chinese)

[4] Fu Yingying. Application of multi-beam sounding system in analysis of river bed change [J]. Urban Roads and Bridges and Flood Control, 2023 (07): 266-269+29.

[5] Yang Haidong. Influence analysis of Multi-beam sounder attitude on sounding and error correction technology [J]. Journal of Applied Acoustics, 2023, 42 (05): 1024-1032.

[6] Guan Xiaohan. Advantages of Multi-beam sounding technology and new ideas of ocean surveying and mapping [J]. China Water Transport, 2019, (05): 51-2.

[7] Li Fan, Jin Shaohua, Bian Gang, et al. Construction and verification of Multi-beam bathymetric error improvement Model [J]. Journal of Surveying and Mapping, 2022, 51 (05): 762-771.

[8] Procedure for Ocean multi-beam Bathymetry: DZ/T 0292-2016[S]. 2016.

[9] State Administration of Quality and Technical Supervision. Specification for Hydrographic Survey: GB 12327-1998 [S]. 1998.

[10] Zhu Huimin. Key technologies and accuracy evaluation of underwater topographic survey in multi-beam sounding system [J]. Jingwei Tiandi, 2022, (02): 4-6.