Hybrid Role Modeling of Lampreys Sex Ratio Environment Based on Lotka-Volterra Model

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Abstract. In this paper, the Lotka-Volterra model was used to study the sex ratio of lampreys under different food resources. The results showed that the male ratio of lampreys was relatively low in the environment with abundant food resources. In environments where food resources were scarce, the proportion of males increased significantly. In addition, the study also established a mixed-role model of lampreys' sex ratio environment. By comparing the reproductive efficiency of lampreys' populations under different sex ratio conditions, it was found that the reproductive efficiency of lampreys' populations with sex ratio adaptation was higher in the short term, but in the long term, the fluctuation of sex ratio would reduce the reproductive efficiency of lampreys' populations. Finally, the effects of sex ratio changes on ecosystem stability of lampreys were analyzed by constructing a food web model with multiple trophic levels. The results showed that the fluctuation of lampreys' sex ratio had a stabilizing effect on ecosystem stability.

Keywords: Lotka-Volterra model, Lampreys, Sex ratio, Environmental modeling, Ecosystem stability.

1. Introduction

Most species are essentially male or female, and many are born with a sex ratio of 1:1, but some species are characterized by adaptive sex ratio variation, and the sex ratio of the lampreys varies according to the external environment. The rate of growth in the larval stage of the eel determines whether they end up as males or females, with lower growth rates and higher proportions of males in environments where food is less available, and a relatively significant decrease in the proportion of males in the population observed in environments where food is more readily available.

In this paper, the Lotka-Volterra model of Lampreys was established to study the sex ratio of lampreys under different food resources. This study not only revealed the mechanism of sex ratio change in lampreys, but also established a mixed role model of sex ratio environment, compared the reproductive efficiency of lamprey's population under different sex ratio conditions, and analyzed the impact of sex ratio change on ecosystem stability. This study provides an important reference for understanding the effects of sex ratio changes on lamprey's ecosystem, and provides a theoretical basis for further research on the ecological mechanism of this species.

2. Predator-Prey Synergistic Change Model

The impacts of lamprey's populations on ecosystems can be modeled using population dynamics. The Lotka-Volterra model can be used to describe the interactions of sea lampreys with other species such as the prey and predators. Considering the impacts under different sex ratios, separate dynamics should be modeled for male and female lampreys.

In this model, the impact of the lamprey's eel as a predator on the whole ecosystem is mainly reflected in the change in the number of its own population and the number of its prey population, and the change in the number of its prey represents the change in bioenergetic changes and biotic abundance of the ecosystem, reflecting the situation of the whole ecosystem.

Here, the effect of human predation on the lamprey's population is modeled as the mortality rate of the lampreys (i.e., mortality rate = natural mortality rate + mortality rate due to human predation).

$$\frac{dM}{dt} = \mathrm{r_M M} - \mathrm{d_M M} + \mathrm{b_M aPM} \quad (1)$$

$$\frac{dF}{dt} = r_F F - d_F F + b_F aPF \quad (2)$$

$$\frac{dP}{dt} = r_P P - aP(M + F) \quad (3)$$

Table 1.

Symbol	Description
M	Population size of male sea lampreys
r_M	Reproduction rate of male sea lampreys
d_M	Mortality of male sea lampreys
b_M	Predation efficiency of male sea lampreys
F	Population size of female sea lampreys
r_F	Reproduction rate of female sea lampreys
d_F	Mortality of female sea lampreys
b_F	Predation efficiency of female sea lampreys
a	Probability of prey predation
P	Prey population size
r_P	Natural growth rate of prey

To solve this model, we can use the numerical method, which is the Eulerian method here, and from the large amount of statistical data, we assign the values:

$rM = 0.04, dM = 0.02, bM = 0.00, rF = 0.03, dF = 0.02, bF = 0.001, rP = 0.1, a = 0.001$, for the regression fitted through the nonlinear curve, we get the result of Figure 1.

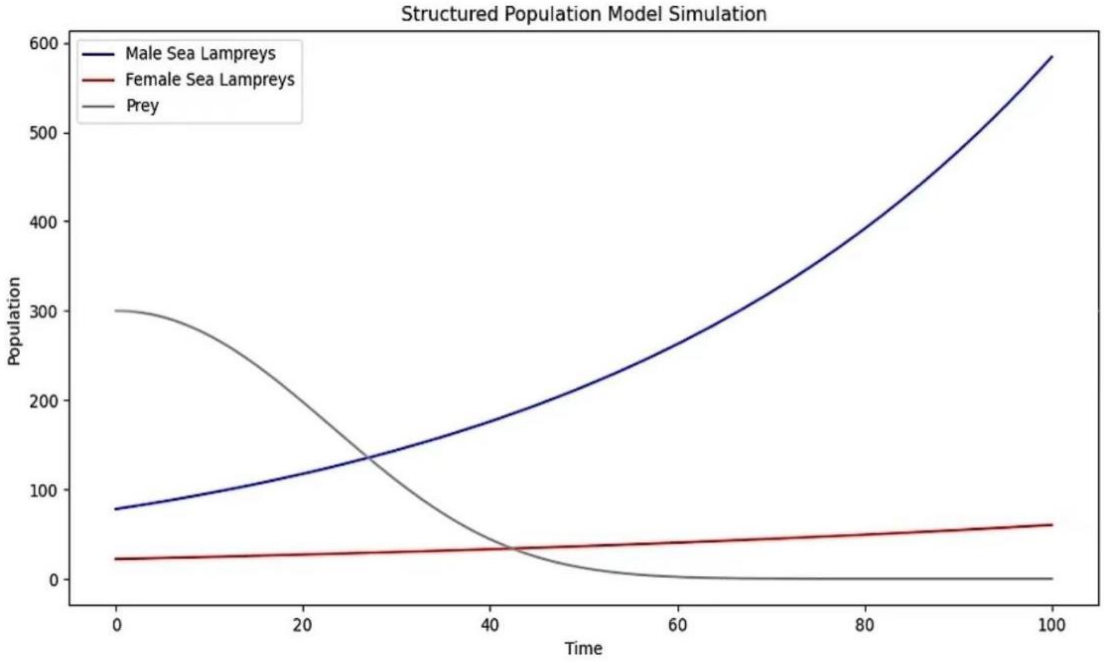


Figure 1. Sexes changes

Observation of Figure 1 shows that This model we can see that as the population of prey declines, the proportion of males becomes higher and the proportion of males and females rises.

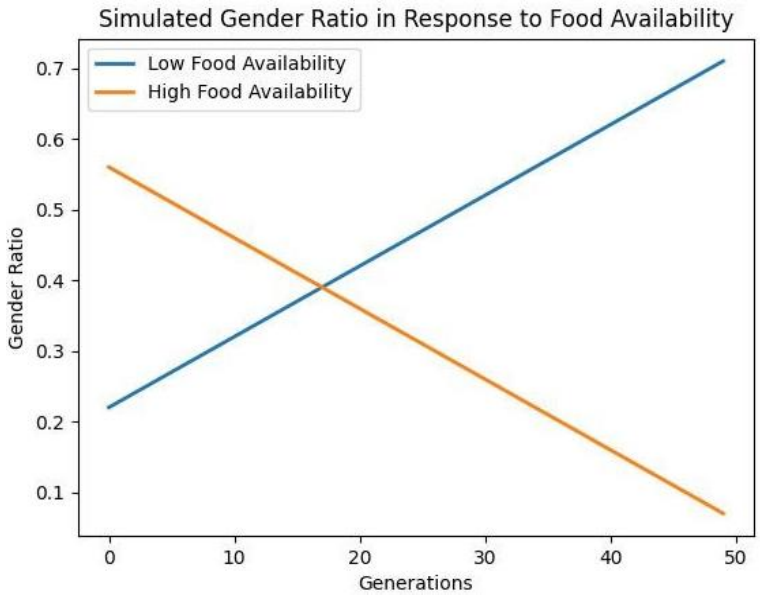


Figure 2. Gender ratio changes

Observation of Figure 2 shows that meanwhile, using the adaptive changes in the male-to-female sex ratio found in sea lampreys under different food supply environments, we plotted an increase in the proportion of males in sea lampreys under low food resources, and a relative decrease in the proportion of males in sea lampreys under high food resources.

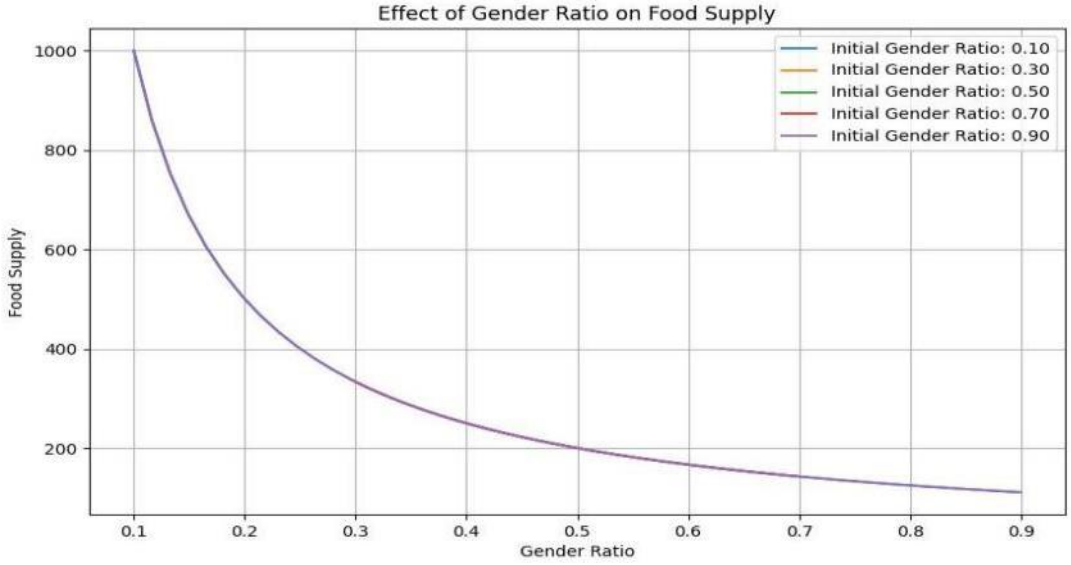


Figure 3. Effect gender on food

From the above figure 3, we plotted the ecological conditions that can be adapted to under different female to male sex ratios by fitting an on linear curve regression, and combined with 1,2 and 3, we can learn that along with the adaptive regulation of the sex ratio of the sea lamprey population, other species or resources in the ecosystem are also affected by its adaptive adaptation. The ecosystem is affected by the fluctuating sex ratio of the sea lamprey population, and the fluctuating sex ratio of the sea lamprey population has a stabilizing effect on the ecosystem.

3. Establishment of Sex Ratio Environmental Mixed Role Modeling

To examine the superiority or inferiority of lamprey's populations based on comparing the changes in reproduction in a population of lampreys with adaptive sex ratios versus a population with a fixed sex ratio of 1:1.

We know that when environmental resources are limited, the male ratio is greatly increased. It is also known that the feeding rate of lampreys during juvenile development greatly affects the final sex of individuals, and the slower the feeding rate, the higher the probability of individuals developing into males. Therefore, a sex ratio adaptive function g(P) can be defined, where P is the number of food populations. This function will output a different sex ratio adjustment value depending on the amount of food. This value can vary between 0 and 1, where 0 represents a complete preference for females and 1 represents a complete preference for males. However, g(P) only gives the tendency of the sex ratio of the seagrass eel, and cannot affect the dimension of the overall number of reproductions.

Therefore, we utilize the $\boldsymbol{r_M(M/M')(F/F')}$**:** the change in overall reproduction rate at the quantity level due to the change in the current number of males and females versus the previous number of males and females. Finally, changes in ecosystem food supply conditions affect not only sex ratios, but also the maximum environmental holding capacity of the population, so we use a description of the effect of the food resource itself on reproduction, a life activity, in conjunction with the Logistic model equations to create a solution to the model equations.

The logistic model is as follows:

$$\frac{dN}{dt} = r\mathrm{N}(1\text{-}\frac{\mathrm{N}}{\mathrm{K}}) \tag{4}$$

3.1. The Sex Ratio Environmental Mixed Role Modeling

And the reproduction rate of a population with a fixed sex ratio of 1:1 is divided equally between the two sexes, so the model can be expressed as:

$$\frac{dM}{dt} = g(P)r_M(M/M')(F/F')M(1 - M/\mathrm{K_M}) - d_M M + b_M aPM \tag{5}$$

$$\frac{dF}{dt} = g(P)r_F(M/M')(F/F')F(1 - F/\mathrm{K_F}) - d_F F + b_F aPF \tag{6}$$

$$\frac{dP}{dt} = r_P P - aP(M + F) \tag{7}$$

Table 2.

Symbol	Description
M'	Number of males after the last unit of time
F'	Number of females after the last unit of time
Km	Maximum environmental holding capacity of males
$\mathrm{K_F}$	Maximum environmental holding capacity of females
K	Maximum environmental holding capacity of the population

By comparing the changes in the two populations under different resource conditions (e.g., reproductive efficiency, population stability, population size, etc.), we can measure the advantages and disadvantages of the two in different aspects after fitting a nonlinear curve regression.

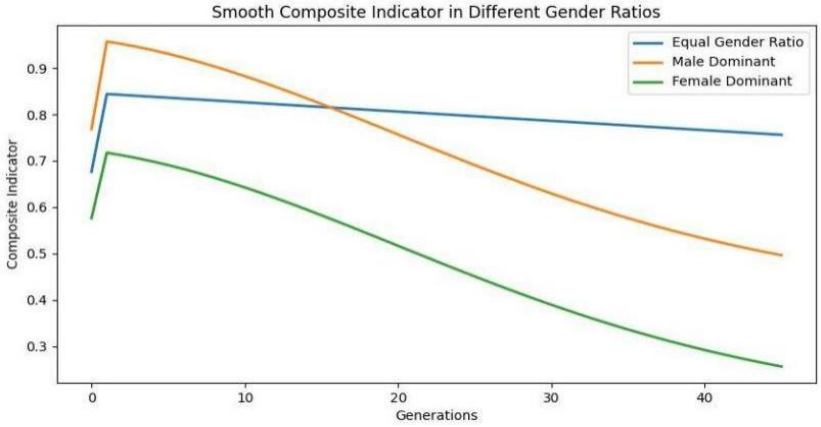


Figure 4. Smooth Composite Indicator in Different Gender Ratio

From the figure 4, it can be seen that under the limited resource conditions, in a relatively short period of time, sex ratio adaptive populations have a clear advantage in reproductive efficiency, especially those with a high proportion of males, but in the long term, the instability of the sex ratio affects the reproductive efficiency of population reproduction in the long run. Analyze the reason: It may be because the excessive number of males and the increase of species competition counteract the reproductive activity of male lamprey's eels and reduce their reproductive efficiency.

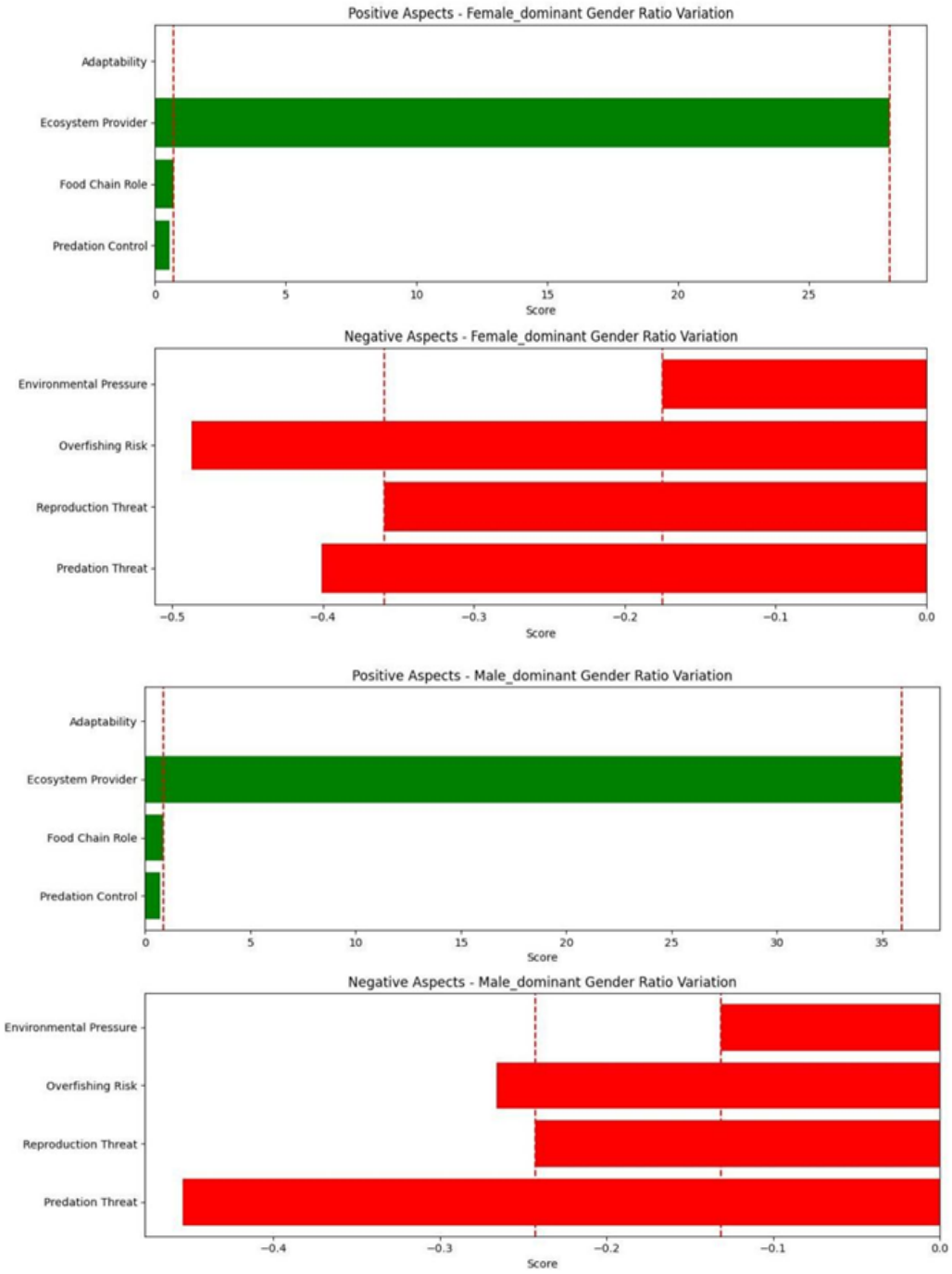


Figure 5. Positive aspects and Negative aspects of dominant gender ratio variation (Male and Female)

From the figure 5, Simulations in positive and negative environments allow us to compare the metrics of populations with higher proportions of males to those with higher proportions of females, concluding that in populations with higher proportions of males there is less pressure to reproduce despite elevated predation pressure.

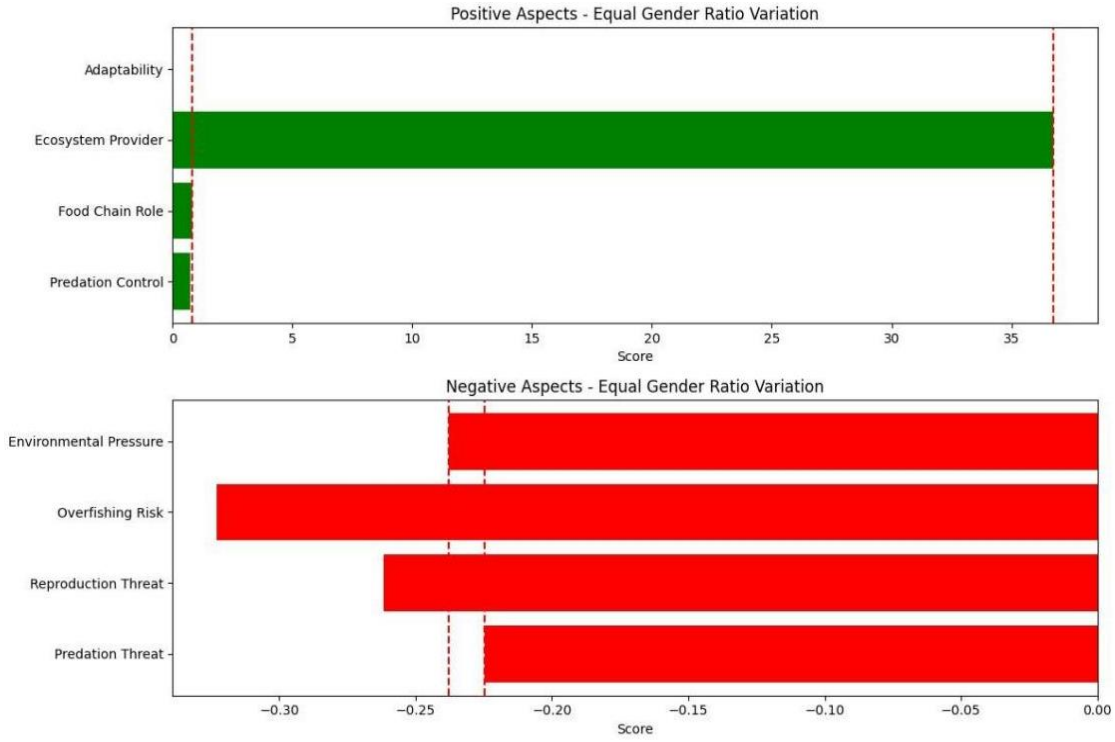


Figure 6. Positive aspects and negative aspects of equal gender ratio variation

Comparing Figures 6 and 7 with Figure 5 shows that Compared with the population with a sex ratio of 1:1, the lamprey is more relaxed in responding to environmental stresses, which is more favorable to the continuation of the population; however, the reproduction pressure of the lamprey population is greater, which makes it difficult for the population to grow.

Conclusion: The advantages of sex-ratio-adaptive populations are that they can adapt to the changing and complex environmental pressures in a timely manner, which is more favorable to the continuation of the population and less likely to die out; the disadvantages are that it is difficult to grow the population, and that fluctuating sex ratios are not conducive to reproduction.

3.2. Effects of Changes in Lampreys Sex Ratio on Ecosystem Stability

A more comprehensive assessment of the effects of changes in the sex ratio of lampreys on the stability of the entire ecosystem is needed, and we need a more complex ecological dynamics model that incorporates multiple species interactions and environmental factors. Therefore, a food web model containing multiple trophic levels is developed here. Such a model takes into account not only the interactions between predators and prey, but also the effects of other species in the ecosystem (including producers and competing species, etc.).

In this model, the ecosystem is divided into multiple trophic levels, including primary producers (e.g., hydrilla, etc.), primary consumers (e.g., fish parasitized or preyed upon by lampreys), and secondary consumers (the lampreys and its competing populations), among others. The model will describe species dynamics at each trophic level based on biomass transfer and energy flow. This multi-trophic food web modeling will allow us to more fully understand the potential impacts of changes in the sex ratio of the sea lampreys on the ecosystem as a whole.

3.3. Food Webs Under the Lampreys Ecosystem

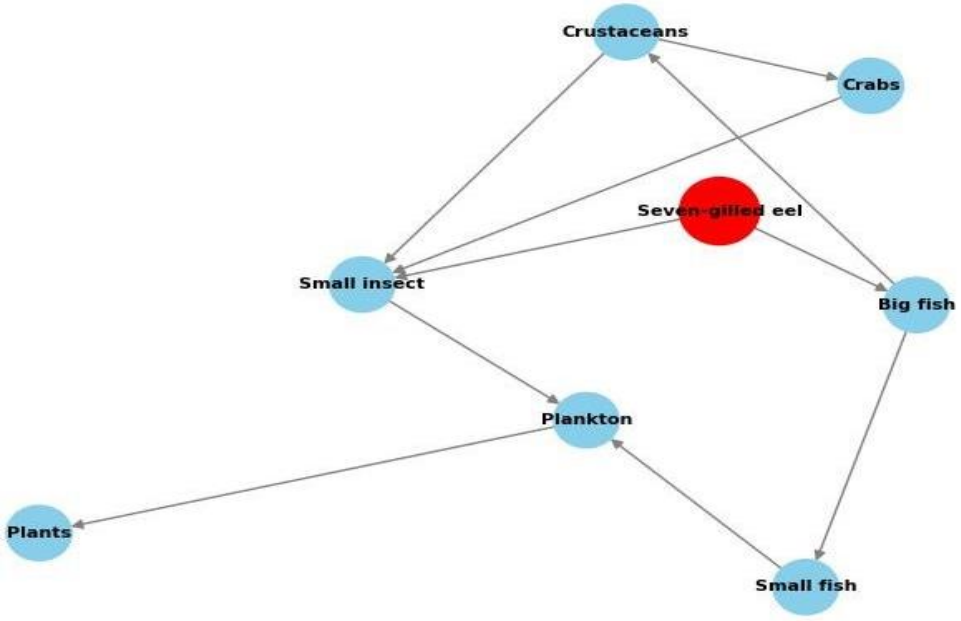


Figure 7. Food network

As can be seen from Figure 8:

For the food web model under the ecosystem in which the lampreys' lives, we set up plants (producers), zooplankton, small insects, small fishes, large fishes, crustaceans, and crabs (all consumers), and singled out the lampreys in order to simulate its impact on the ecosystem.

By setting different sex ratios by the above figure to get the impact on plants, other predators, and prey when the population of seven-gill eels has a higher proportion of males and a higher proportion of females.

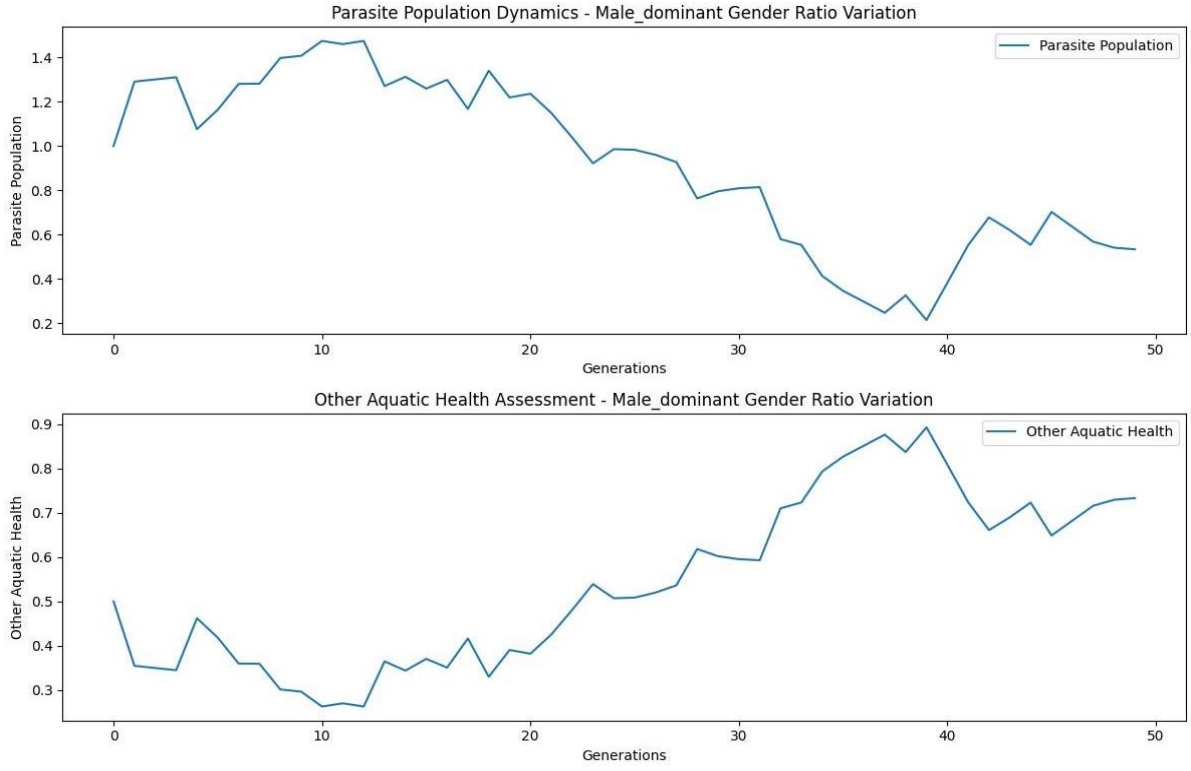


Figure 8. Aquatic health assessment

As can be seen from Figure 8 and 9:

By analyzing the changes in the populations of plants, other predators, and prey, we establish an ecosystem stability evaluation index that can represent the characteristics of the above changes, and the ecosystem stability evaluation takes into account the following indicators:

(1) Biodiversity indicators: including species richness (expressed as the number of different species in the model), species evenness (measured by Simpson's index)

(2) Local stability indicators: assessed by analyzing the Jacobian matrix of coefficients, which gives the response of the system to small perturbations at each equilibrium point of the ecosystem model. Under the condition that the real part of all significant eigenvalues of the Jacobian matrix is less than zero, the equilibrium point can be determined to be in a locally stable state.

(3) Competition intensity: to assess the competitive relationship between the sea lampreys and competing species and its impact on the stability of the ecosystem.

The above indicators were measured by the AHP-TOPSIS model to determine the relative importance, and then we obtained the effects of different sex ratios on different categories of evaluation indicators and the comprehensive evaluation index of the stability of the ecosystem, so that we could quantify how the change of the sex ratio affects the stability of the whole ecosystem.

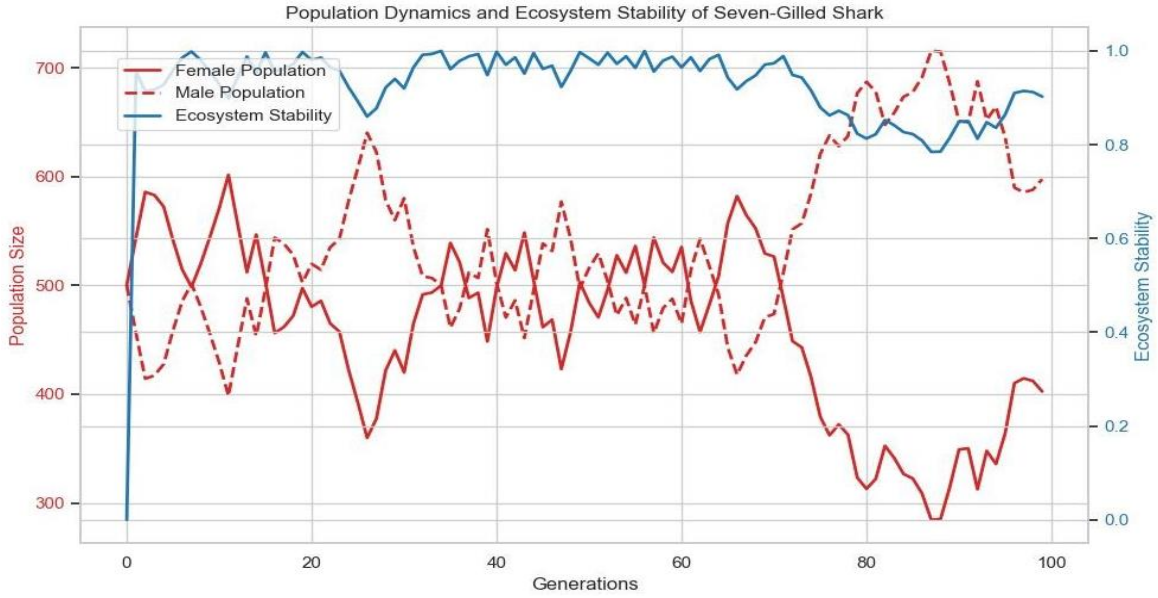


Figure 9. Ecosystem stability by gender ratio

Biodiversity indicators, local stability analysis, and competition intensity indicators were first analyzed initially through the establishment of an AHP model, and then combined with TOSIS to obtain evaluative indicators. Then we can get the result in Figure 10.

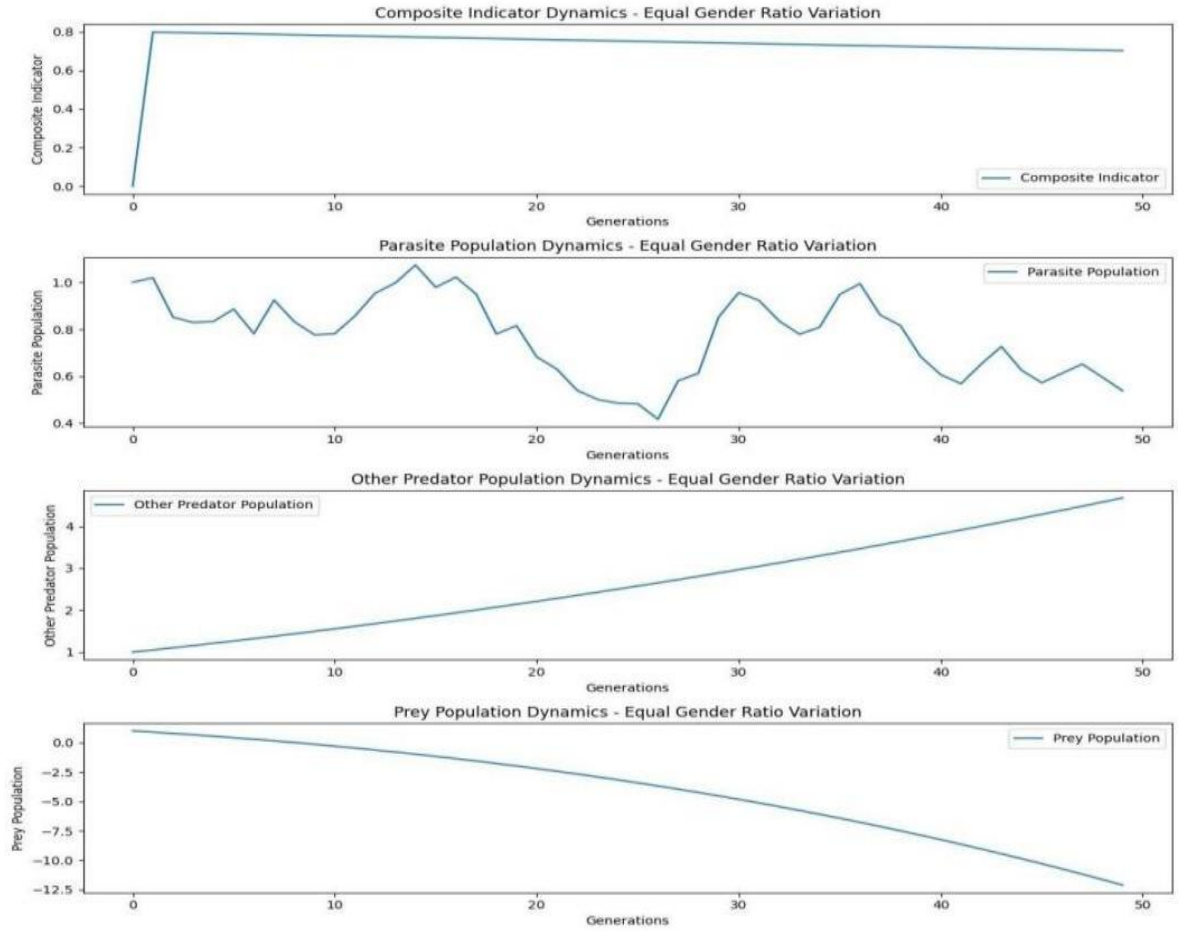


Figure 10. Impacts on organisms in other ecological niches

4. Conclusion

In this paper, the Lotka-Volterra model was used to study the sex ratio of Lampreys in different food resource environments, and a mixed role model of lampreys' sex ratio environment was established. The results show that sexual populations have higher reproductive efficiency than adaptive populations in the short term, but have adverse effects on long-term population growth. In addition, the fluctuation of lampreys' sex ratio has a stabilizing effect on ecosystem stability. This study provides an important reference for understanding the effects of sex ratio changes in lampreys on the ecosystem.

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