Sex Ratio Analysis of Lampreys Based on Logistic and Lotka-Volterra Models

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Abstract. Because of the influence of larval food availability on the sex ratio of lampreys s, the study of changes in the sex ratio of lampreys is of great practical importance for understanding various aspects of the lamprey's ecosystem. First, this paper analyzes competition and predation relationships within the ecosystem and models Logistic models and Lotka-Volterra models for each relationship. Second, to determine whether changes in the sex ratio of lampreys are favorable or unfavorable, this paper develops a comparative model that analyzes populations of lampreys with different sex ratios and populations with a fixed 1:1 sex ratio. Finally, competitive, inhibitory and predatory relationships were inferred in conjunction with ecological dynamics and fitted using Hurwitz determinants and Jacobi matrices. The results found that changes in the sex ratio of lampreys favored ecosystem stability.

Keywords: Lotka-Volterra model; Logistic model; Hurwitz determinant; Jacobi matrix.

1. Introduction

Despite the fact that many species have a 1:1 sex ratio at birth, adaptive sex ratio variations cause the actual species sex ratio to vary, and lampreys are a very good example of this [1]. The rate at which food becomes available, or the growth rate of the larval stage, affects the sex ratio of lampreys. In situations when food is abundant, the percentage of males can reach 78%, whereas in situations where food is scarce, the percentage of males is 56%. Because the lampreys are considered an important food source and parasite that has a significant impact on the ecosystem in some areas, this paper explores the interactions throughout the lamprey's ecosystem through mathematical modeling [2]. First, lampreys exhibit a variety of relationships with other species in the ecosystem, and we analyzed competition and predation relationships within the ecosystem. We modeled Logistic models and Lotka-Volterra models for each relationship. Then, using Matlab, we fitted these three relationships, created ecosystem simulation equations, and provided visualizations. Next, to determine whether changes in the sex ratio of lampreys were favorable or unfavorable, we developed a comparative model. Based on these models, we analyzed populations of lampreys with different sex ratios and populations with a fixed 1:1 sex ratio. Finally, we based our analysis on a single-population logistics model to assess ecosystem stability by considering the amplitude of population oscillations. Competition, inhibition, and predation relationships were inferred and fitted in conjunction with ecological dynamics. For the linear part of the simulation equations, we constructed Hurwitz deterministic series. For the nonlinear part, we constructed a Jacobi matrix and performed calculations using Matlab. We found that the quantifiable area was minimized when the number of female individuals was equal to 0.85, suggesting that variation in the sex ratio of lampreys in this range is beneficial to ecosystem stability.

2. Ecosystem Impact Analysis

2.1. Ecosystem-Competitive Relationship Analysis Model

We first examined the competitive connection between populations that have comparable living conditions to lampreys and lamprey populations, as lamprey populations interact with other populations in the ecosystem and their survival is influenced by upstream and downstream biotic chains.

First, based on the changes in population X_1 of lampreys and population X_2, which has similar living conditions to lampreys, over time t, we developed a logistic model to study the relationship between the sex of lampreys and the change in population characteristics [3]. From this, we developed the following logistic equation:

$$\frac{dX_1}{dt} = r_1 X_1 \left(1 - \frac{X_1}{K} - \alpha \frac{X_2}{K}\right) \tag{1}$$

$$\frac{dX_2}{dt} = r_2 X_2 \left(1 - \frac{X_2}{K} - \beta \frac{X_2}{K}\right) \tag{2}$$

In the above equation, r is the population growth rate, K is about the limit carrying capacity of the ecological environment where the lampreys population is located about the environment of a certain population, then each additional individual in the population has an impact of $\frac{1}{K}$ on the environment, where α and β are the competition coefficients of the populations X_1 and X_2, respectively, and from this, the integral equation can be introduced:

$$\frac{dX}{dt} = rX \left(1 - \frac{X}{K}\right) \tag{3}$$

In the above analysis, the effect of each individual in population X_2 on population X_1 is $\frac{\alpha}{K_1}$, and the effect of each individual in population X_1 on population N_2 is $\frac{\beta}{K_2}$. Therefore, when species 2 can suppress species 1, then it can be assumed that the effect of species 2 on species $1 >$ the effect of species 2 on itself, i.e., $\frac{\alpha}{K_1} > \frac{1}{K_2}$.

The collation gives: $K_2 > \frac{K_1}{\alpha}$, and the same can be derived. Species X_2 cannot suppress species 1: $K_2 > \frac{K_1}{\alpha}$. Species X_1 can suppress species 2: $K_1 > \frac{K_2}{\beta}$.

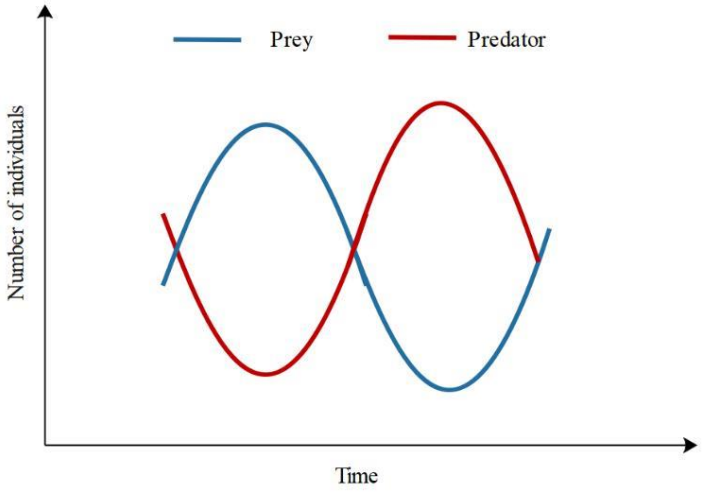


Fig. 1 Analysis of the competitive relationship between predators and prey

As shown in Fig. 1. Predators rise somewhat later than prey, i.e., the two are asymmetric but alternate, as the predator-prey-time dynamics relationship above illustrates. However, as prey numbers rise, so do predators.

The calculated derived images show that $K_1 < \frac{K_2}{\beta}$ and $K_2 < \frac{K_1}{\alpha}$ indicating that lamprey's population X_1 competes with population X_2 and can form a stable longterm coexistence relationship.

2.2. Ecosystem-Predator Relationship Analysis Model

On the basis of the competitive relationship model, we introduce the effect of sex change on the population in the lampreys population X_1, assuming that the number of males within the population

of lampreys is A and the number of females within the population of lampreys is B, and it is known that "the sea-lampreys become males or females depending on the growth rate of these larvae in the larval stage, which is affected by food availability". The growth rate of these larvae is affected by the availability of food". Observing the specific changes at a certain time t, the LotkaVolterra model was developed as follows [4].

$$\frac{dX}{dt} = rX - eXN \tag{4}$$

$$\frac{dN}{dt} = -pN + e\theta XN \tag{5}$$

In the above equation, r is the within-population reproductive growth rate of lampreys, e is the within-population mortality rate of lampreys, respectively, N is the number of populations feeding on lampreys, p is the natural growth rate of populations feeding on lampreys, and θ is the conversion factor between the number of neonatal lamprey's individuals and the number of neonatal individuals of the predatory lamprey's population.

Based on the Lotka-Volterra model, we perform dynamic planning for lampreys with their predators, modeled as follows:

$$X_t = X_0 e^{rt} = A(t) + B(t) \tag{6}$$

$$\frac{dA}{dt} = R_A A + r_A A - e_A A - E_A NA \tag{7}$$

$$\frac{dB}{dt} = R_B B + r_B B - e_B B - E_B NB \tag{8}$$

$$\frac{dN}{dt} = p_N N - E_A AN - E_B BN \tag{9}$$

In the above equations, R_A and R_B are the intra-population supply food acquisition efficiencies for males and females, respectively, r_A and r_B are the intrapopulation reproduction growth rates for males and females, respectively, e_A and e_B are the intra-population mortality rates for males and females, respectively, and E_A and E_B are the probabilities of predation within the intra-population for males and females, respectively. The results are shown in Fig. 2.

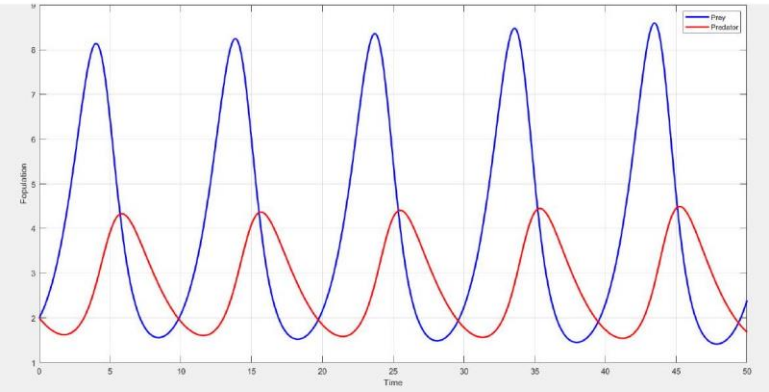


Fig. 2 Dynamics of predator and prey Lotka-Volterra model relationships

Plotted by Matlab solution, the mean, maximum and minimum values of predators of lampreys for different male proportions were obtained as shown in Fig. 3.

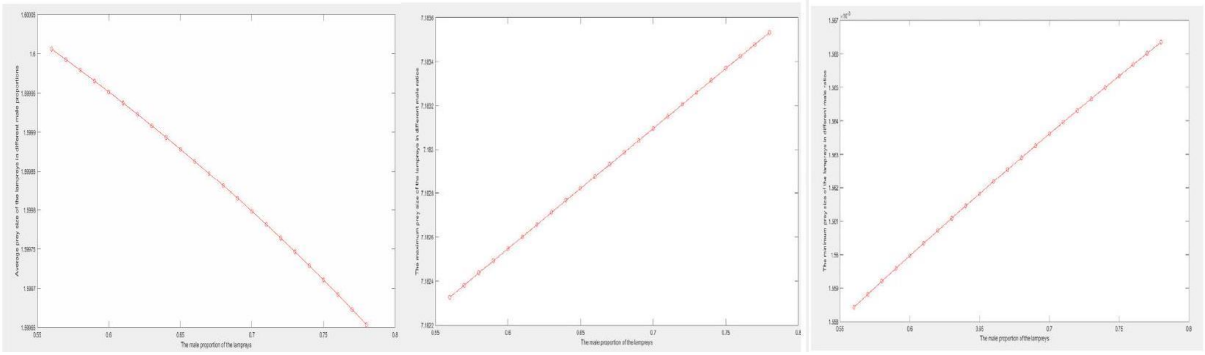


Fig. 3 Predators of lampreys at different male ratios

The result in Fig.3 reveals that the mean value of the prey for lampreys decreases as the number of males rises, that the ecosystem develops specific interactions as a result of the predation relationship, and that the maximum and minimum values of the prey exhibit an increasing tendency.

3. Is Lampreys Good or Bad

3.1. Modeling in Comparison

The main characteristic of lamprey's populations is that their larval period is subject.

to changes in the sex ratio of lamprey's populations because of the influence of the efficiency of food acquisition from internal supply.

In order to more closely match the adaptive change of sex ratio of lamprey's population in ecological environment, $G(X)$ is defined as the function of adaptive change of sex ratio of lamprey's population, where the number of lampreys population is X.

When the number of lampreys population changes, it causes the number of male and female individuals A and B in the lamprey's population to undergo intra-population adaptive changes, and the Lotka-Volterra model is optimized as follows:

$$\frac{dA}{dt} = R_A A + \rho r_A A G(X) - e_A A - E_A NA \tag{10}$$

$$\frac{dB}{dt} = R_B B + (1-\rho) r_B B(1 - G(X)) - e_B B - E_B NB \tag{11}$$

$$\frac{dN}{dt} = p_N N - E_A AN - E_B BN \tag{12}$$

$$\frac{dN}{dt} = p_N N - E_A AN - E_B BN \tag{13}$$

Where ρ is the coefficient of proportional allocation of the reproductive growth rate of the lamprey's population to the number of male and female individuals. In the above equation, as $G(X)$ approaches 1, the population tends to be completely male within the population, when ρ tends to 1; as $G(X)$ approaches 0, the population tends to be completely female within the population, when ρ tends to 0.

Relevant information shows that many species have a sex ratio of $1:1$ at birth, so to understand the characteristics of the above population with a sex ratio of $1:1$ at birth, it is necessary to exclude the adaptive change function of the sex ratio $G(X)$ in the above model, and at the same time, it is possible to take the proportional distribution coefficient ρ of the number of male and female individuals in the population of lampreys to be 0.5 , and the above model can be fixed to ensure that the variables are unique, and the optimization of the Lotka-Volterra model for the population at birth with a sex ratio of 1:1 is organized as follows:

$$\frac{dA}{dt} = R_A A + 0.5(r_A + r_B)A - e_A A - E_A NA \tag{14}$$

$$\frac{dB}{dt} = R_B B + 0.5(r_A + r_B)B - e_B B - E_B NB \tag{15}$$

$$\frac{dN}{dt} = p_N N - 0.5(E_A + E_B)AN - 0.5(E_A + E_B)BN \tag{16}$$

The Lotka-Volterra model of this population described above was investigated by varying internal factors (e.g., internal reproductive growth rate r, efficiency of access to supply food R, etc.) and comparing it to a population with a fixed sex ratio of $1:1$ at birth, and examining the differences in stability and fitness between the lampreys population and a population with a fixed sex ratio of $1:1$ at birth resulting from the variations in each factor.

On the basis of the above, sensitivity analysis was utilized to discuss the advantages and disadvantages of adaptive sex ratio changes in the lamprey's population compared to a population with a fixed sex ratio of 1:1 at birth in the ecosystem.

3.2. Comparison Results Solving and Analyzing

The statistical analysis software gives the comparative trend presented in Fig. 4 below:

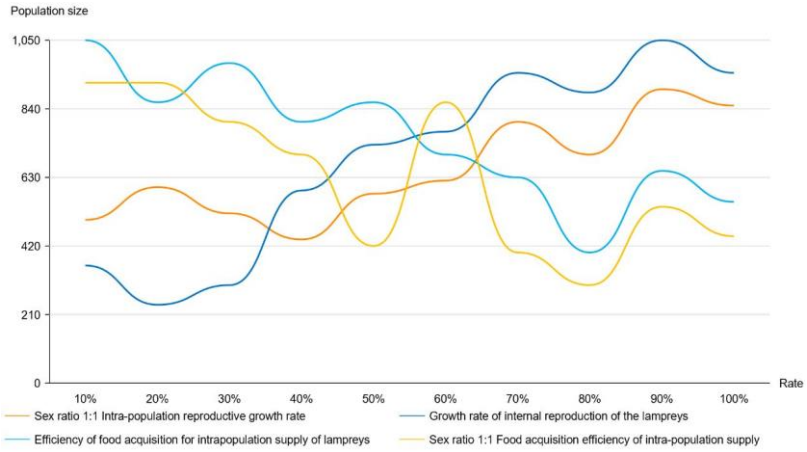


Fig. 4 Sensitivity analysis of comparative relationships

As can be seen from Fig. 4 above, the population of lampreys has undergone an adaptive change in sex ratio over time, making it more significant in terms of internal value-added rate and food acquisition efficiency than the population with a fixed sex ratio of 1:1 at birth, which causes the population to rise more significantly.

From this it can be deduced that the advantages of lampreys are as follows:

Resource Utilization Optimization: When there is a shortage of food, having a larger percentage of males may help to lower the total demand for scarce resources.

Flexible survival tactics: Lamprey populations can adapt their survival strategies to varying environmental conditions by means of adaptive changes in sex ratios.

Population Ecological Stability: Adaptive Sex Ratio Changes in Lampreys allow populations to respond more flexibly to changes in the environment because different sexes may have different adaptations in reproduction, resource acquisition, and survival strategies.

Nevertheless, the shifting sex ratio of lampreys has the following negative effects on the ecology in which it lives:

Reduced reproductive efficiency: The reproduction rate will drop if there are too many males, and the males will put the population in intense rivalry with one another, which will lower the pairing rate and ultimately reduce the population's capacity for reproduction.

Ecological Role Imbalance: Too much of one sex can lead to increased competition and predation pressures that affect the survival and reproduction of other species, which in turn can lead to an ecological role imbalance that can affect ecosystem function and stability.

Blocked gene flow: Too much of one sex may limit gene flow and exchange, leading to reduced genetic diversity. This may make populations less resilient to environmental change and increase their risk of extinction.

4. Ecosystem Stability Analysis

4.1. Modeling of Ecosystem Networks of Relationships

After reviewing the relevant literature, we know that in ecosystems, the number of groups is used as the state variable, while the structure of interactions and coefficients at the level of effectiveness (e.g., internal growth rate r and food acquisition efficiency R, etc.) are used as model coefficients [5]. The stability of ecosystems is then categorized into state stability (where the covariates are constant and perturbed by the state variables) and structural stability (where they are perturbed by the covariates).

Because in general ecosystem dynamics, a linear Lotka-Volterra model with unsaturated type I functional responses is commonly used, but the model suffers from the disadvantage of oversimplification of nature. In a single-population Logistics model, if the population dynamics are monotonically varying and converge to the limiting.

carrying capacity K of the population environment, then stability increases as ξ is the rate of growth within the resource; conversely, if the population dynamics are oscillatory (e.g., in the case of a discrete or time-lagged scenario), the amplitude of the oscillations increases as ξ increases, and thus is more unstable.

The basis of an ecosystem is constructed by a food web, and the basic components of the food web are producers, consumers, and other abiotic resources (e.g., temperature, light, etc.). We fitted the following magnitudes based on the Logistic model of a single population by portraying the ecosystem stability through the amplitude of oscillations of the population, combined with the dynamics of the food web building blocks, as follows:

Consumers to the amount of resource coupling:

$$\frac{-fSC}{\xi+\xi_0} \tag{17}$$

The amount of resource-to-consumer coupling:

$$\frac{\eta fSC}{\xi+\xi_0} \tag{18}$$

The amount of linear loss in consumers:

$$-DC \tag{19}$$

Resources are subject to extreme carrying capacity:

$$\frac{-\xi S^2}{K} \tag{20}$$

Resource uptake:

$$\xi S \tag{21}$$

In the above equation, S denotes the population density of the resource, C denotes the population density of the consumer, ξ is the internal growth rate of the resource (equivalent to the amount converted by the producer into the amount available to the consumer or the amount converted by a consumer from a higher level consumer), K is the limiting carrying capacity of the environment of the population, f is the shock rate (i.e., the probability of acquiring the access to a particular resource, which creates a shock to the amount of the resource), ξ_0 is the half-saturation density of the resource, η is the proportion of the biomass of the resource that is converted to the consumer, and D is the individual mortality rate of the consumer.

In the case of the lamprey's population, under the above constraints, the impact of adaptive changes in its own lamprey's sex ratio on the stability of the ecosystem in which it is located should also be considered, and the established ecosystem mimicry equations for the ecosystems containing the lamprey's population should be optimized for the introduction of the C-S interaction:

$$V(t) = v(A(t), B(t), C(t), S(t)) \tag{22}$$

$$\frac{dV}{dt} = \xi S\left(1 - R_A \frac{dA}{dV} - R_B \frac{dB}{dV}\right) + (1-\eta)\frac{fSC}{\xi+\xi_0} - DC \tag{23}$$

In order to better examine the strength of the coupling quantity relative to the depletion quantity, we weaken the negative impact of the subeffective mimetic equation in the expression, and the coupling strength is centralized in Eq. $(1-\eta)\frac{fSC}{\xi+\xi_0} - DC$, where a higher coupling strength implies that the consumer at that level has a higher demand for that resource.

Although both linear and nonlinear models can be centrally reflected in the composition of their matrix characteristic polynomial coefficients, the matrices of linear planning models are nonstationary matrices and those of nonlinear planning models are stationary matrices. Therefore, in order to determine how strong the coupling between the trophic levels of the food chain in an ecosystem is (i.e., whether it has reached a locally stable state), we analyze the stability of the linear and nonlinear parts of the model separately.

Under the linear part of the model, this criterion can be related to the feedback in the system by examining the nth order feedback of the system according to the construction of the Hurwitz determinant series [6].

The local steady state implies that all eigenvalues of the community matrix are negative in real part, which is centrally manifested by the fact that all orders of feedback in the system are generally negative; secondly, higher order feedback are either absent or weak to weaken the amplitude of the community oscillations.

For the nonlinear part of the model, the stable point is the point in the nonlinear model where the analyzed rate of change tends to zero. For the analysis of unstable points, it is necessary to calculate the Jacobi matrix listed in the vicinity of the approximation of the stable point and analyze its eigenvalues.

Using the Jacobi matrix to solve the eigenvalues, according to the eigenvalue positive and negative to determine the size of the system oscillation amplitude in the anthropomorphic equation, and then determine the stability of the constructed ecosystem.

4.2. Stability Modeling Results and Analysis

After running the ecosystem fitting equations, Matlab was utilized to plot the following phase trajectory graph. The result is shown in Fig. 5.

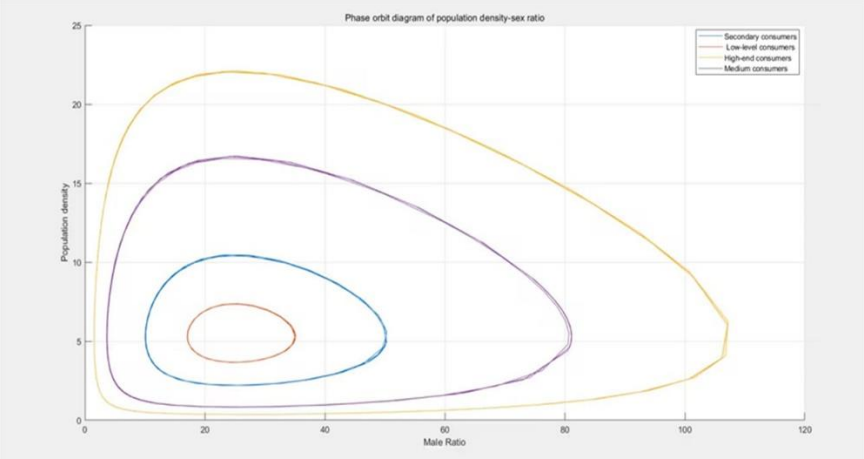


Fig. 5 Phase orbit diagram of population density-sex ration

From the general trajectory in Fig. 5, it can be determined that under multiple factor variations, the gradual decrease in the resources available for transfer by consumption $(1-\eta)\frac{fSC}{\xi+\xi_0}$ between trophic levels due to the elevation of the trophic levels causes the phase-track curves to appear as outwardly expanding, but not intersecting, potentials.

At the same time, fitting each phase-track curve can be done by scaling up $(1-\eta)\frac{fSC}{\xi+\xi_0}$ in the derivation process to degrade and gradually approach the equilibrium point of the phase-track curve.

We can also find that the equilibrium point of each curve is basically close to a quantifiable region, so the stabilization points of the ecosystem fitting model is also quantified with this convergence and then fixed, then it can be shown that the ecosystem fitting equations are more in line with the actual ecosystem.

Combined with the Jacobi determinant code, it can be concluded that when the number of males: number of females $= 0.85$, the equilibrium points of the curves of the track diagram are closest to each other, and the quantifiable area is the smallest, i.e., the ecosystem is the most stable in this area.

5. Summary

The purpose of this paper is to explore the interactions throughout the lamprey's ecosystem through mathematical modeling. First, competitive and predatory within the ecosystem were analyzed, and Logistic and Lotka-Volterra were developed for each relationship. The results showed that lampreys populations and the populations they compete with can coexist for long periods of time with alternating extremes. As the proportion of lamprey's males increased, there was a decreasing trend in both the number of prey and food availability for lampreys s. Second, based on the above model, populations of lampreys with different sex ratios and populations with a fixed 1:1 sex ratio were analyzed. It was determined that changes in the sex ratio of the lampreys were conducive to the optimization of resource use, adjustment of survival strategies, maintenance of population ecological stability and conservation of genetic diversity. Finally, distinct competitive, inhibitory and predatory relationships were inferred and fitted using CS interactions in conjunction with ecological dynamics. We found that the quantifiable area was minimized when the number of females was equal to 0.85, suggesting that variation in the sex ratio of lampreys in this range is beneficial to ecosystem stability. The study in this paper describes population changes in the ecosystem and provides a reference value for real-world environments.

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