Integrating Seabed Topography and Ocean Current Dynamics for Submarine Trajectory Analysis

Zhihang Fu [*, #], Zhengkai Jia [#]

School of Information Science and Engineering Harbin Institute of Technology, Weihai, China, 264209

* Corresponding author: 18954671363@163.com

[#]These authors contributed equally.

Abstract. In the face of growing global ocean exploration, the research focuses on overcoming the challenges of deep-sea navigation and the unpredictability of ocean currents to improve submarine pathway planning and trajectory prediction. This study proposes a method for improving submarine navigation and trajectory prediction through digital seabed mapping of the Ionian Sea and ocean current simulation. Using NCEI data, it transforms latitude and longitude into a geographic model, bypassing seawater variables like salinity and temperature. The approach combines sine functions and Gaussian noise to mimic ocean currents' natural variability, enhancing position predictions. Simplifying the submarine as a sphere, it integrates ocean currents into motion analysis and employs MATLAB for simulation, including scenarios of contact loss and breakup. Recognizing model limitations, it recommends using sensors on submarines for real-time data collection, aiming to refine navigational accuracy amidst environmental uncertainties.

Keywords: Submarine pathway planning, ocean currents simulation, Submarine topography reconstruction, Ocean Exploration.

1. Introduction

As global ocean exploration activities increase, navigation and pathway planning in deep-sea environments have become crucial areas of marine technology research [1]. The complex and variable seabed topography is foundational for safe and efficient navigation of submarines and other deep-sea exploration devices, as well as key to deep-sea resource exploration and scientific studies [2, 3]. However, the extreme conditions of the deep-sea environment, such as high pressure, low temperatures, and lack of light, present significant challenges in acquiring and analyzing seabed topography [4]. Furthermore, the unpredictability and complexity of ocean currents add difficulty to submarine pathway planning, making the prediction of submarine trajectories fraught with uncertainty.

In embarking on this comprehensive study, we first digitally reconstruct the seabed topography of the Ionian Sea, utilizing data from the National Center for Environmental Information (NCEI) to create a geographical model. Following this, we simulate the dynamics of ocean currents by integrating sine functions with Gaussian noise, aiming to capture the unpredictable nature of these currents accurately. This preparatory work forms the basis for our subsequent analysis, which includes a mechanical and kinematic examination of submarine movement influenced by these environmental factors. Through MATLAB simulations, we anticipate the trajectory of a lost submarine, considering various scenarios including potential breakdowns. This workflow is meticulously designed to predict submarine positions with greater accuracy, without venturing into the outcomes, thereby setting a solid foundation for addressing the complexities involved in underwater navigation and trajectory prediction.

2. Submarine Topography Reconstruction

To facilitate the planning of submarine pathways beneath the sea, our study first necessitates a digital reconstruction of the seabed topography. We sourced relevant data from the National Center

for Environmental Information (NCEI), a division of the National Oceanic and Atmospheric Administration (NOAA), known for its extensive collection of climatic, meteorological, oceanographic, geoscientific, and geospatial data. For enhanced reliability and robustness of our model, we strategically selected a segment of the Ionian Sea, capturing elevation data within the latitude range of 35.7354 to 38.1187 and longitude range of 16.2938 to 20.5354. This approach, supplemented with a TIFF file for comprehensive sea mapping, aims to generalize our findings across the entire Ionian Sea.

Because the index of an array in MATLAB requires integers, and the information we get for latitude and longitude is in decimals, we need to process the latitude and longitude data to get a new data set and create the X, Y axes for geographic modeling.

$$X = \left[1 + \frac{CurrentLongitude \ - \ MinLongitude}{step} \right] \tag{1}$$

[] indicates downward rounding, and *step* indicates the step size, which is the value of the longitude spanned by a frame. And the formula for *step* is calculated as follows:

$$step = \frac{MaxLongitude \ - \ MinLongitude}{NumData} \tag{2}$$

In the formula, *numdata* is the number of points of the same latitude and longitude in the downloaded map data.

This is how we obtained the geographic model. Our team believes that for a lost submarine, its trajectory will be more affected by ocean currents. For the density of seawater, we used ρ=1025 kg/m^3 while the salinity and temperature of seawater were not considered. This may make our model not accurate enough. Note: The converted X and Y have no units. However, they can be converted to latitude and longitude.

The downloaded TIFF file is imported into ArcGIS to generate a shape file, which can be imported into Matlab to obtain a set of coordinates and elevation information. Then, the latitude and longitude are processed into integers and then transformed into matrices, and each matrix element represents the elevation information of the corresponding coordinates. Finally, the original and reconstructed images are shown in Figure 1.

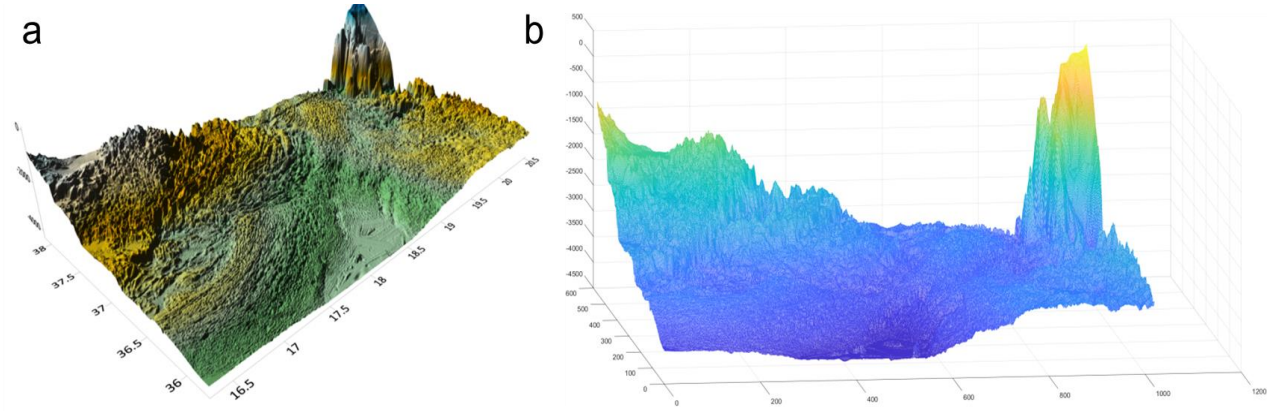


Figure 1. (a) Seafloor diagram (b) Submarine topography reconstruction images

3. Constructing a Model for Ocean Current Analysis

Ocean currents, the vast movements of water within the oceans, are influenced by a myriad of factors such as winds, the rotation of the Earth, and the underwater landscape. Given the inherent unpredictability of ocean currents, any model attempting to predict their behavior inevitably contains a degree of error [5]. To enhance the universality of our model, allowing it to predict positions in

various oceanic currents scenarios, we introduced a simulated random ocean current into our analysis. This simulation employs a combination of sine functions and Gaussian noise to mimic the natural variability of ocean currents. It's important to note that Gaussian noise refers to a sequence of random numbers following a Gaussian, or normal, distribution, adding a realistic element of unpredictability to our simulation.

$$a*\sin(wt)+b\times G(t) \tag{3}$$

In the formula, $G(t)$ is a Gaussian function that generates a random value that conforms to a Gaussian distribution.

We have generated two sets of ocean current data: east-west and north-south. The numerical values represent the magnitude of the speed of the currents, while positive and negative represent the direction of the currents. North and east are positive and south and west are negative.

Since the values of the image obtained by the above method change abruptly and the amplitude changes are large and not realistic, to ensure that the model is more realistic during simulation, we have smoothed the image:

$$\text{NewData} = k\times \text{LastData} + (1-k)\times \text{NewData} \quad (k<1) \tag{4}$$

Where k is a relatively large number. In this way, the newly generated current data will be more dependent on the previous current data, making the data smoother.

This is just an example, to make the change in the curve more obvious and intuitive, we increase the amplitude of the data. Herein, as shown in figure 2, the images we processed to simulate the speed of the currents:

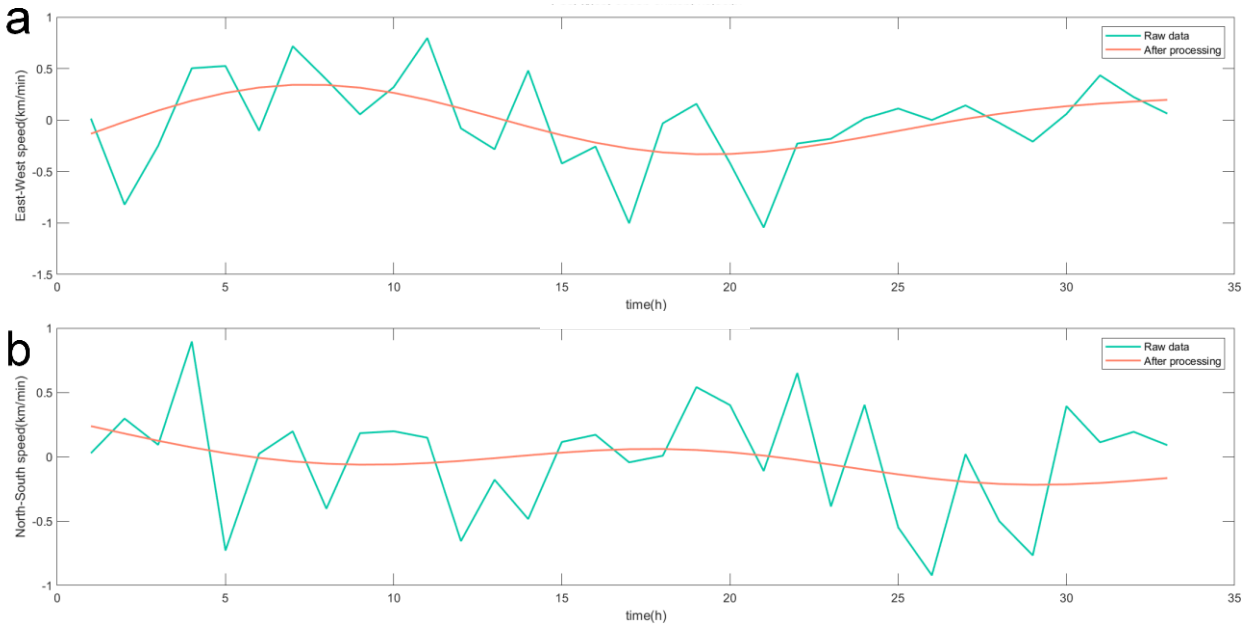


Figure 2. Chart of real-time changes in ocean current speed. a b are two examples.

Next, we vectorially superimpose the velocities of the ocean currents to form data with direction and magnitude and display it in a 2D plane by arrow direction.

To understand the Figure 3, we have analyzed it here: For each time, the corresponding column of vectors is the same, and this whole column of vectors represents the direction of the ocean currents corresponding to that part of the ocean at that moment. The direction of the current is a combination of the X-axis (east-west) speed and the Y-axis (north-south) speed. For example, the red field in the image represents the current direction in this area at the 7th hour, which is about 45° north by east.

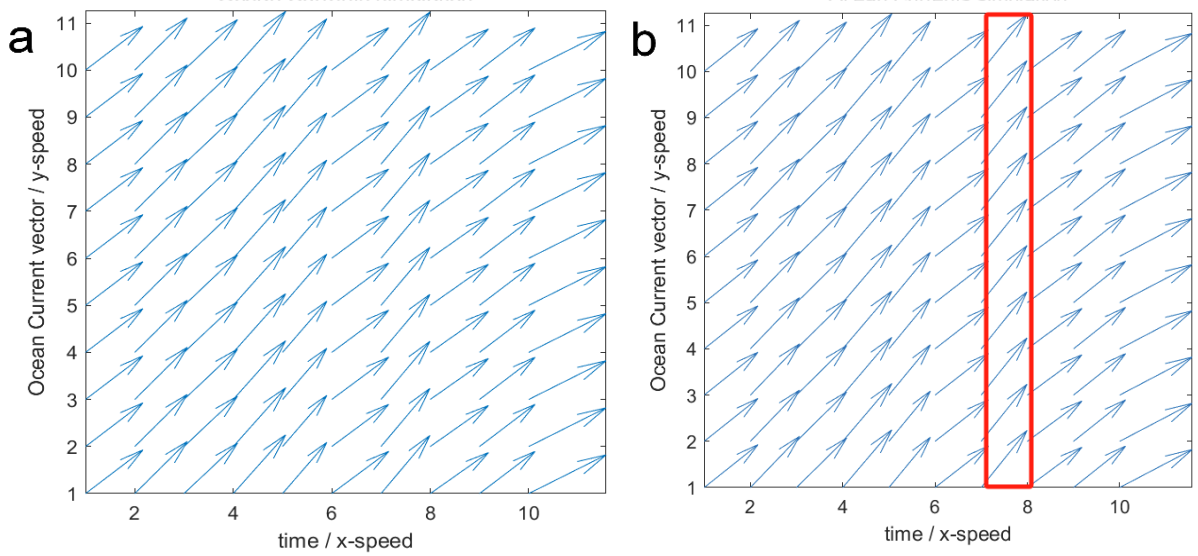


Figure 3. Vector of ocean currents. a b are two examples.

In this way, we have simulated a model in which the direction of the ocean currents changes over time, but the overall direction does not change much. It also verifies that the ocean currents we simulated are more in line with the actual situation.

4. Constructing a Model for Predicting Submarine Positions

To thoroughly analyze this process, our study will first undertake a mechanical and kinematic analysis of the submarine.

4.1. Force Analysis of a Submarine

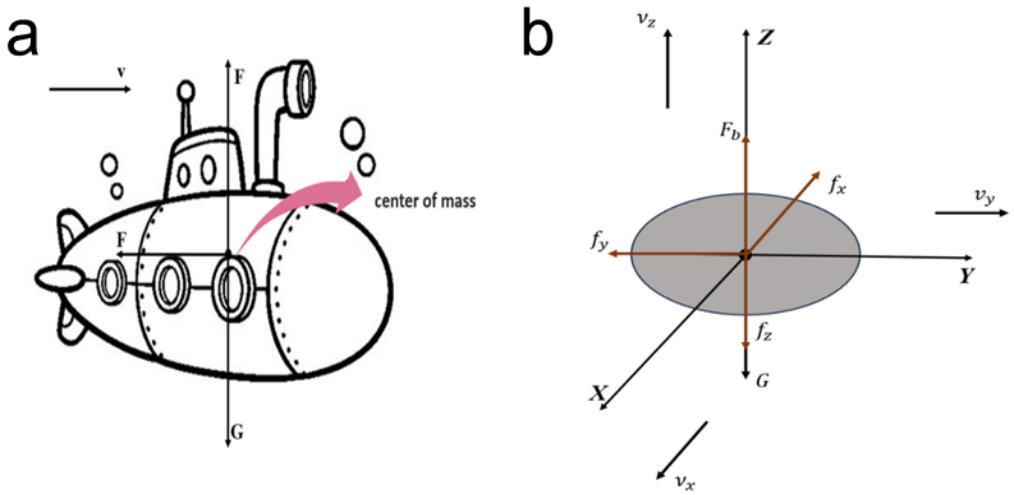


Figure 4. Force Analysis Diagrams. (a) Illustrates the scenario where the submarine possesses a combined velocity *v* relative to the current. (b) Demonstrates the analysis of the submarine's three-dimensional motion and forces.

Firstly, as shown in figure 4, for a relatively small commercial submarine, it's practical to consider it as a point mass within the vast ocean. Refining the model to include angular velocity not only enhances precision but primarily influences the submarine's own state (orientation, etc.) without significantly altering its trajectory over extended periods. Therefore, for the sake of simplification, we model the submarine as a sphere. By synthesizing the relative velocity of ocean currents from all directions and denoting it as velocity v, we can construct a two-dimensional force analysis diagram. However, to refine our analysis and accurately determine the submarine's three-dimensional coordinates, we establish a coordinate system. This system utilizes orthogonal decomposition across

three dimensions: the X-axis represents the east-west direction, the Y-axis the north-south direction, and the Z-axis the vertical direction, facilitating a comprehensive force analysis on the submarine.

With the knowledge of hydrodynamics, Fluids are conventionally classified as either liquids or gases. it is known that an unpowered submarine in ocean currents will have a fluid resistance, The fluid resistance is as follows:

$$f = \frac{1}{2} C_d \rho A v^2 \tag{5}$$

In the equation, ρ is the density of seawater, v is the velocity of the submarine relative to the seawater, C_d is the coefficient of drag, and S is the force area of the submarine.

Also, since the submarine experiences not only fluid resistance but also gravity and buoyancy, we can obtain the forces on the submarine in the three directions of the XYZ coordinate axis. Next, we analyze the forces in each of the three directions.

It is easy to know that, in the ideal case, a submarine is propelled in the vertical direction only by gravity, buoyancy, and the force of the fluid pushing against it.

$$F_z = F_b + G + \frac{1}{2} \rho v_z^2 C_d S_z \tag{6}$$

And for the horizontal direction, it is only pushed against it by the fluid:

$$F_x = \frac{1}{2} \rho v_x^2 C_d S_x \tag{7}$$

$$F_y = \frac{1}{2} \rho v_y^2 C_d S_y \tag{8}$$

4.2. Kinematic Analysis of a Submarine

From Newton's second law we can get the acceleration of the submersible.

$$a = \frac{F}{m} \tag{9}$$

Using the acceleration, we can then analyze the motion of the submarine. Again, using the equations of motion, we can determine the distance traveled per unit of time.

$$\Delta s = v \times dt + \frac{1}{2} a \times dt^2 \tag{10}$$

Knowing its movement distance, we should update its latitude and longitude and translate them into coordinates on the map, making it possible to display the movement trajectory on the map. Suppose the ship's movement distance on the X-axis is Δx, and the movement distance on the Y-axis is Δy.

Based on the change in displacement on the earth, we can deduce the change in latitude and longitude of the submarine. The derivation is as shown in Figure 5.

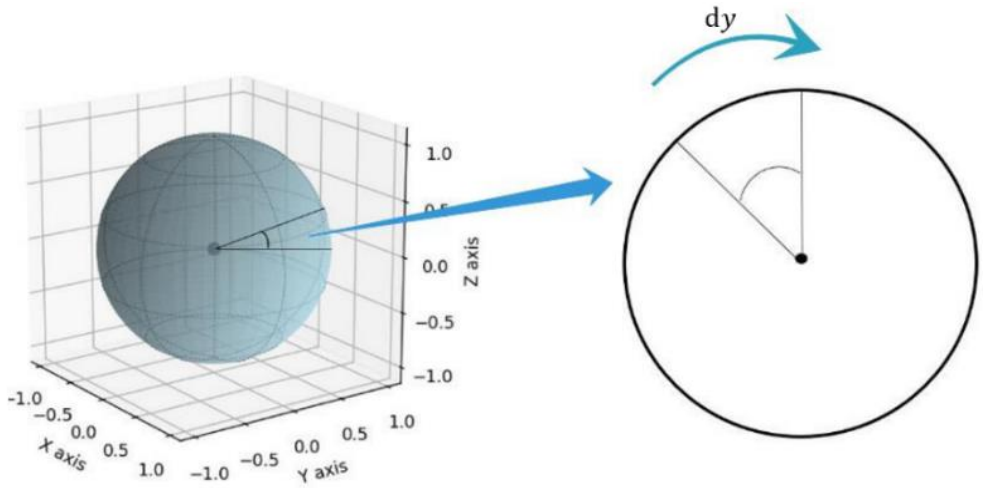


Figure 5. Schematic diagram of latitudinal changes

The Earth, as an approximate sphere whose radius is denoted as *R*, first decomposes the displacements into sub-displacements along both the x and y axes. We can look only at the displacement in the direction of the y-axis when deriving the formula for the change in latitude, taking any circle that crosses the sphere and is perpendicular to the equator, as shown in Figure 6, we can see that the shift in the direction of the y-axis corresponds to an angle that is equal to the change in latitude.

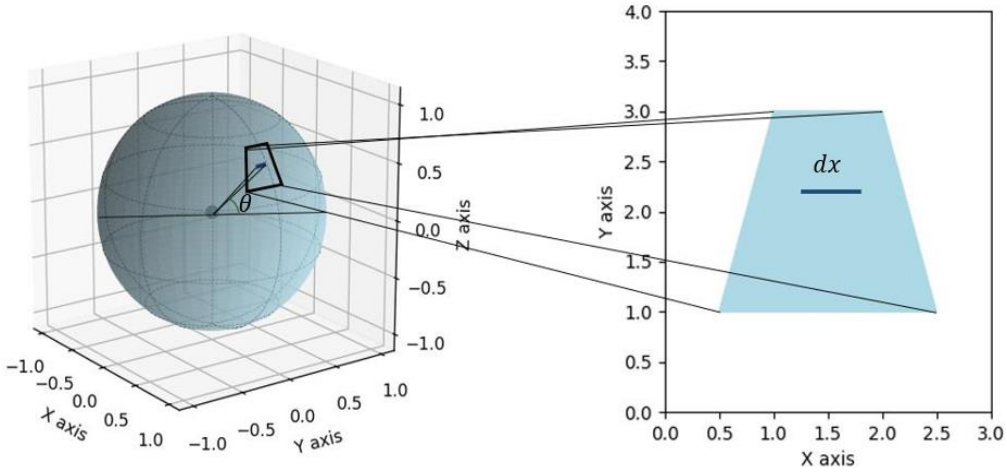


Figure 6. Schematic diagram of longatorial changes

For the analysis of the change in longitude, we can only look at the displacement in the x-axis direction, take a circle parallel to the equator, the radius of the circle is $R\cos\theta$, we can find that the x-axis displacement is in this small circle, so according to the x-axis direction of the displacement we can deduce to get the change in longitude.

The final two equations derived are as follows:

$$\Delta Latitude = \frac{\Delta y \times 180}{\pi R} \tag{11}$$

$$\Delta Longitude = \frac{\Delta x \times 180}{R\cos\theta \times \pi} \tag{12}$$

Where θ represents the current latitude and R is the radius of the Earth.

The above equation then gives the new latitude and longitude coordinates.

4.3. Trajectory prediction

With the above conclusion, the simulation of the motion trajectory can be carried out.

If the submarine has lost contact at a certain location and its power system is damaged and it cannot float up, down, forward, or backward, its motion will be determined by the ocean currents. Run the program and plot the trajectory as shown in Figure 7 a and b to see the submarine's motion in three dimensions and where it will eventually stop (hit a rock formation and stop). In the worst case scenario where the submarine breaks up, it will first accelerate and then sink, reaching a certain speed and then sinking at a constant speed. We have successfully simu-lated this scenario using MATLAB, and the simulated trajectory graph is as shown in Figure 7c:

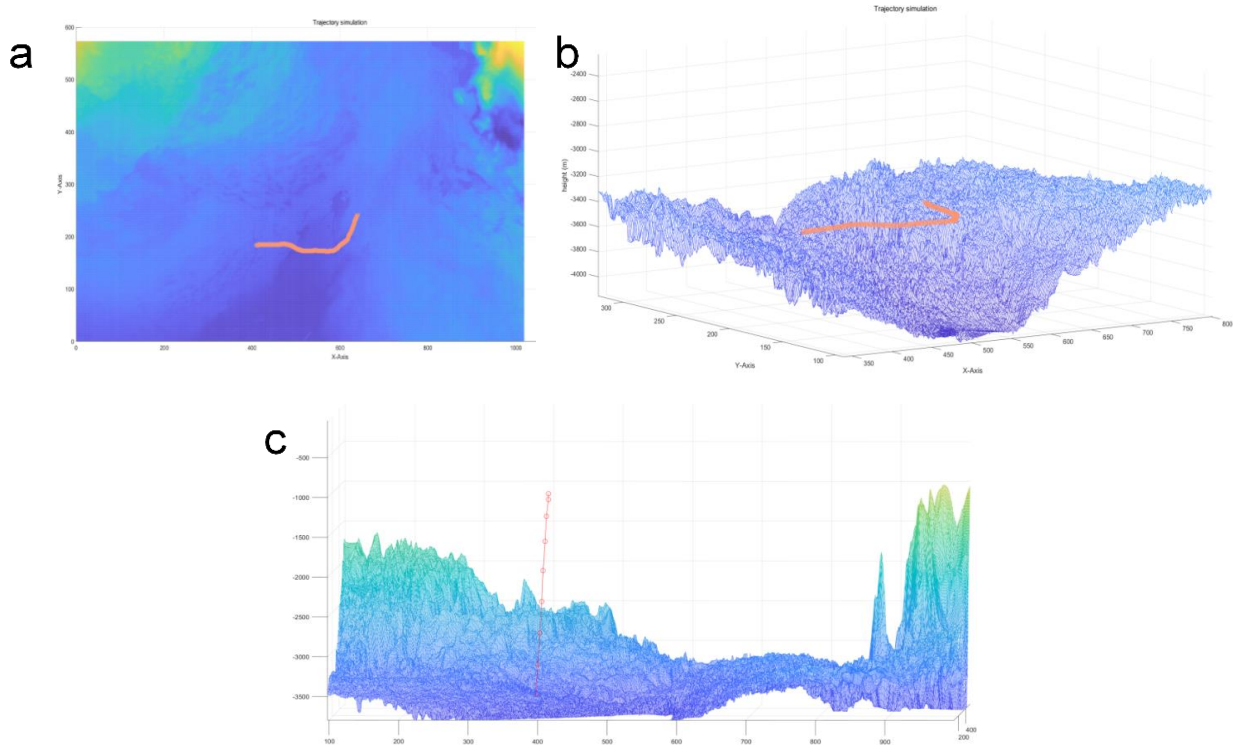


Figure 7. (a) and (b): Trajectory Simulation Chart. (c) Submarine post-breakup path simulation

Furthermore, the uncertainties involved in predicting submarine trajectories in this study are as follows:

(1) The temperature, salinity, and density of the seawater are constant in the model's assumptions. There must be variations, and these small variations will have some effect on the movement of the submarine [6].

(2) In the model, it is assumed that the submarine will stop moving when it touches the bottom of the sea. The frictional force on the submarine may be less than the force of the ocean currents on it, and it may still move some distance. Similarly, if it hits a mountain range, it should roll some distance [7].

(3) A submarine at the bottom of the ocean cannot communicate with the submarine every moment, but it should communicate occasionally, then if it is lost at some point in the communication interval, our prediction of the initial position of the submarine wreck will be off [8].

(4) The currents at the bottom of the ocean are not necessarily in the same direction as the currents at the surface [9]. The currents at the bottom of the ocean called deep currents, are usually caused by differences in the density of seawater, which are not necessarily in the same direction as the currents at the surface of the ocean. At the same time, the master ship has access to the currents at the surface of the ocean, which may bias the trajectory prediction.

To mitigate these uncertainties, it's advisable for submarines to be outfitted with an array of sensors designed to monitor the surrounding ocean environment. These sensors should not only collect data on the immediate aquatic conditions but also transmit this information, along with precise positional details, back to the command ship. This approach ensures a continuous flow of real-time data,

enhancing the accuracy of submarine trajectory predictions and facilitating more informed navigational decisions [10].

5. Conclusion

In summary, this comprehensive study successfully leverages a combination of digital seabed reconstruction, ocean current simulation, and mechanical and kinematic analysis to predict submarine trajectories with a notable degree of precision. By integrating elevation data, simulating variable ocean currents with Gaussian noise for realism, and applying advanced force and motion equations, the research provides a robust framework for understanding submarine movements influenced by ocean dynamics. However, the study acknowledges potential inaccuracies due to assumptions of constant seawater properties and the complexities of deep ocean currents. It suggests enhancing predictive accuracy through the deployment of submarine sensors that monitor and transmit real-time environmental data. This multifaceted approach not only advances our understanding of submarine navigation and safety but also highlights the importance of continuous data collection and analysis in maritime operations.

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