Development and Application of Flatness Analysis System based on Coordinate Measuring Data

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Abstract. To realize flatness error evaluation based on coordinate measuring data, the calculation process of four evaluation methods was analyzed, and a direct calculation method of minimum zone flatness was proposed. According to the calculation process of different evaluation methods, the flatness error analysis system based on coordinate measuring data was developed, and the accuracy of the system was verified by importing the case data of related research. The practicability of the system was verified by using the developed system in the measurement task.

Keywords: Flatness Analysis System; Coordinate Measuring Data; Four Evaluation Methods.

1. Introduction

Flatness is a common measurement characteristic in length measurement, such as flatness of optical flats. Flatness error is also one of the basic contents of shape error of mechanical parts. The national standard [1, 2] describes its definition, measurement and evaluation methods in detail. Flatness measurement can generally be divided into direct method and indirect method. Two transmission paths of line and plane in flatness measurement was studied [3]. The flatness measurement of level instrument, autocollimator, coordinate measuring machine, laser tracker and other instruments is indirect method. The essence of the indirect flatness measurement method is to use instruments to measure the spatial coordinate values of several points on the plane, and fit and calculate the flatness error results according to different evaluation methods (minimum area method, least square method, diagonal method, three distant points method). Due to the complexity of calculation and fitting process, special software needs to be developed for processing. A flatness calculation system was developed based on the least square method to realize the rapid calculation of flatness error [4,5]. The mathematical model of flatness measurement and its measurement uncertainty according to the diagonal evaluation method were discussed [6]. Reference [7] studied the concavity and convexity of the objective function of the minimum area evaluation method of flatness error, and proved that the global minimum of the objective function is unique. Reference [8] proposed a minimum area evaluation algorithm of flatness error based on geometric search approximation, which can accurately and stably calculate the minimum area flatness error. Reference [9] used Powell method to optimally solve the objective function of flatness error. Reference [10] studied the flatness error evaluation based on the clustering particle swarm optimization algorithm, give the algorithm flow and verifies it with an example. At present, many measuring instruments are equipped with special flatness calculation software, but the evaluation algorithm of the software is not comprehensive enough to meet the actual needs. Therefore, this paper studies the flatness calculation method, develops a flatness analysis system based on coordinate data, and verifies it in practical measurement.

2. Flatness Error and Evaluation Method

2.1 Definition of Flatness Error

Flatness error refers to the variation of the actual plane to the ideal plane. Its error area is the area between two parallel planes. The parallel plane must contain the actual plane, that is, all measuring

points are in the area between the two parallel planes, and the flatness error value is the distance between the two parallel planes. The schematic diagram of flatness error is shown in Figure 1.

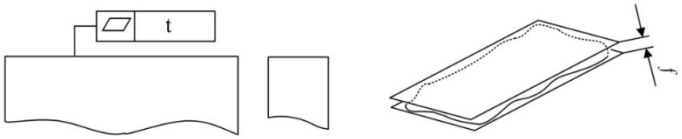


Figure 1. Flatness error region

It is assumed that the coordinates of each point on the measured plane are expressed as:

$$p_i(x_i, y_i, z_i) \quad i = 1, 2...n \tag{1}$$

The ideal plane S_{LX} represents:

$$z = Ax + Bx + C \tag{2}$$

Where, A, B and C are the parameters of the ideal plane S_{LX} equation.

Then the distance from point p_i to S_{LX} is calculated by the following formula:

$$d_i = \frac{z_i - (Ax_i + By_i + C)}{\sqrt{A^2 + B^2 + 1}} \tag{3}$$

According to the definition, the flatness error f calculation formula is:

$$f = d_{\max} - d_{\min} \tag{4}$$

Where d_{max} and d_{min} are the maximum and minimum deviation values of the measuring point relative to the ideal plane respectively (the measuring point is positive above the ideal plane, and vice versa).

Substituting equation (3) into equation (4) shows that the size of the parameter C of the ideal plane equation has no effect on the flatness error result. Therefore, it is unnecessary to calculate the parameter value when calculating the flatness error.

2.2 Flatness Evaluation Method

The flatness evaluation methods include minimum area method, least square method, diagonal method and three far points method [1].

2.2.1. Minimum Area Method

The minimum area method refers to the method of fitting the extracted elements of the measured elements with the Chebyshev method to obtain the ideal element position, that is, the maximum distance between the extracted elements of the measured elements and the ideal elements is the minimum. The flatness error is evaluated by the minimum inclusion area method, which takes the inclusion plane of the minimum inclusion area as the ideal plane. The actual contour contained by the two parallel planes shall be in contact with the two parallel planes at least at four points, and the three or four points shall meet the triangle criterion, intersection criterion and straight-line criterion.

2.2.2. Least Square Method

The least square method is used to evaluate the flatness error, which takes the least square plane as the ideal plane and takes the difference between the maximum and minimum deviation values of each measuring point on the measured actual surface and the least square plane as the flatness error.

2.2.3. Diagonal Method

The evaluation of flatness error by diagonal method takes the plane passing through one diagonal of the measured plane and parallel to the other diagonal as the ideal plane, and takes the difference

between the maximum and minimum deviation values of each measuring point on the measured actual surface and the ideal plane as the flatness error.

2.2.4. Three Far Points Method

The evaluation of flatness error by the three far points method is based on the flatness error of forming an ideal plane through three points far apart on the measured plane, and taking the difference between the maximum and minimum deviation values from each measuring point on the measured actual surface to the ideal plane as the flatness error.

3. System Development and Verification

3.1 Design of Evaluation Algorithm for Flatness Error

The coordinate data of each measuring point of the actual plane are expressed as p_i (x_i, y_i, z_i) (i = 1,2... n). The calculation process of the above flatness evaluation methods is described.

3.1.1. Minimum Area Method

In the algorithm design of using the minimum area method to evaluate flatness, the base conversion method in accordance with the above criteria can usually be used, or the minimum area flatness error function can be optimized. These algorithms have high computational efficiency, but the algorithm design is cumbersome. Considering the powerful computing power of the current computer, this paper directly evaluates the definition of flatness according to the minimum area method, and carries out fitting calculation for the coordinates of all measuring points. The specific process is as follows.

Among all the measuring points of the measured plane, first take any group of three measuring points that can form the surface and calculate the distance $d_{(n-3)}$ from other measuring points to the surface. If all (n-3) points are on the same side of the surface (i.e., all $d_{(n-3)} \geq 0$ or $d_{(n-3)} \leq 0$), the flatness error $f_1 = \min(|d_{(n-3)}|$; Select three measuring points in the next group and repeat the above calculation steps until all measuring points are traversed and all qualified flatness errors fi are calculated. Then the flatness error F of the minimum area method is calculated by the following formula:

$$f = \min(f_1, f_2 \ldots f_i) \tag{5}$$

In this paper, the above method is called the minimum area flatness direct calculation method, and the calculation flow of this method is shown in Figure 2.

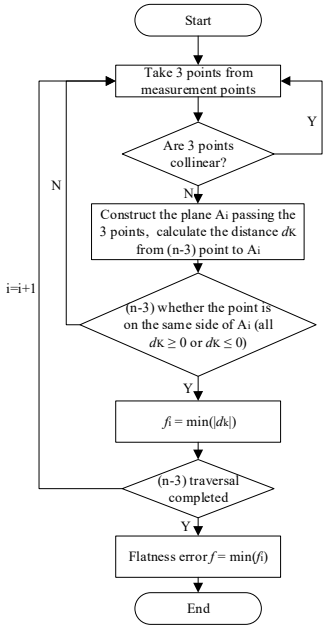


Figure 2. Flow chart of minimum zone flatness error direct calculation method

3.1.2. Least Square Method

According to the definition of the least square plane, when the sum of squares of the distance between each measuring point and the fitting plane is the smallest, the plane is the least square plane. When the ideal plane is the least square plane, the plane equation parameters a and B can be calculated by the following formula:

$$\left.\begin{aligned} A &= \frac{S_{12}S_{23} - S_{13}S_{22}}{S_{12}^2 - S_{11}S_{22}} \\ B &= \frac{S_{12}S_{13} - S_{11}S_{23}}{S_{12}^2 - S_{11}S_{22}} \end{aligned}\right\} \tag{6}$$

$$\left.\begin{aligned} S_{11} &= \sum_{i=1}^{n} x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2 \\ S_{12} &= \sum_{i=1}^{n} x_i y_i - \frac{1}{n}\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i \\ S_{13} &= \sum_{i=1}^{n} x_i z_i - \frac{1}{n}\sum_{i=1}^{n} x_i \sum_{i=1}^{n} z_i \\ S_{22} &= \sum_{i=1}^{n} y_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} y_i\right)^2 \\ S_{23} &= \sum_{i=1}^{n} y_i z_i - \frac{1}{n}\sum_{i=1}^{n} y_i \sum_{i=1}^{n} z_i \end{aligned}\right\} \tag{7}$$

After calculating the parameters of the least squares plane equation, the flatness calculation results of the least squares method can be obtained according to equations (3) and (4).

3.1.3. Diagonal Method

The ideal plane of the diagonal method is the diagonal plane, which passes through two diagonal points on one diagonal of the actual plane and is parallel to the other diagonal. According to the definition, the equation coefficients of the diagonal plane can be obtained. Suppose that the endpoints of the two diagonals in the diagonal plane are P_1, P_3 and P_2, P_4 respectively, and the diagonal direction vector is $\overrightarrow{p_1p_3}$ and $\overrightarrow{p_2p_4}$. Then the normal vector of the diagonal plane can be obtained from the following formula [9]:

$$\vec{n} = \overrightarrow{p_1p_3} \times \overrightarrow{p_2p_4} = (n_x, n_y, n_z) \tag{8}$$

Then the diagonal plane equation parameters A and B are:

$$\left.\begin{aligned} A &= n_x / n_z \\ B &= n_y / n_z \end{aligned}\right\} \tag{9}$$

After calculating the parameters of the diagonal plane equation, the calculation results of the flatness of the diagonal method can be obtained according to equations (3) and (4).

3.1.4. Three Far Points Method

The three-apogee plane is an ideal plane composed of three points far apart on the measured plane. According to the definition, the equation coefficients of the three-apogee plane can be obtained. Suppose that the three points are P_1, P_2 and P_3 respectively, and the direction vector of the two connecting lines is $\overrightarrow{p_1p_3}$ and $\overrightarrow{p_2p_3}$. Then the normal vector of the plane with three far points can be calculated by the following formula [9]:

$$\vec{n} = \overrightarrow{p_1p_3} \times \overrightarrow{p_2p_3} = (n_x, n_y, n_z) \tag{10}$$

Then the parameters A and B of the three far point plane equation can be calculated by equation (9). After obtaining the plane equation parameters of the three far points, the flatness calculation results of the three far points method can be obtained according to equations (3) and (4).

3.2 System Development and Verification

Based on the algorithm analysis of the above flatness evaluation methods, a flatness analysis system based on coordinate data is developed by using MATLAB software. When running the system, the three-dimensional coordinate data of the measuring points are imported through Excel file, and the flatness error results can be calculated by selecting different evaluation method buttons. Among them, the coordinates of the initial point of the evaluation base plane of the diagonal method and the three far point method need to be input manually. Take the operation example data in reference [9] as the object, input it into the system for verification, and the verification results are shown in Table 1.

Table 1. Verification results of flatness analysis system. unit: μm

evaluation method	reference [9] result	measured result
minimum area method	25.79	25.79
least square method	22.50	22.50
diagonal method	27.06	31.62
three far points method	46.11	32.00

It can be seen from the verification results in Table 1 that the flatness error evaluation results of reference [9] are in good agreement with those of the minimum region method and the least square method in this paper. At the same time, since the reference does not specify the coordinates of the ideal plane datum point selected in the diagonal method and the three far point method, there is a certain difference between the flatness error evaluation results of this paper and those of the diagonal method and the three far point method.

4. Conclusion

A direct calculation method of flatness of minimum area method is proposed and a flatness analysis system based on coordinate data is developed. The accuracy of the system is verified by taking the example data of relevant research as the object. The minimum area method in this paper adopts the algorithm of directly traversing all the measuring points. The algorithm is simple and easy to implement. Although the actual computation time is long, it has little influence on the actual measurement task. Next, the minimum area optimization algorithm will be studied to improve the calculation efficiency of flatness analysis system.

References

[1] GB/T 11337-2004 Measurement of departures from flatness. Standards Press of China, 2004.

[2] GB/T 1958-2017 Geometrical Product Specifications(GPS)- Geometrical tolerance-Verification. Standards Press of China, 2017.

[3] Wang Qing, Gu Yang. Flatness metrology - value transfer and control of point, section and surface[J]. Acta Metrologica Sinica, 2019, 40(2), 189-195.

[4] Wang Wei. Fast calculation of flatness based on Matlab[J]. Information Technique & Control, 2011, 21 (6), 11-12.

[5] Xing Minjian, Chen Hao, Xu Jingyang. An automatic verification system for flatness error of large-scale flat plate based on least squares method[J]. SHANGHAI MEASUREMENT AND TESTING, 2017, 2 (258), 18-21.

[6] Xu Jingyang, Chu Xuening. Study on mathematical model in flatness measurement[J]. SHANGHAI MEASUREMENT AND TESTING, 2006, 3(193), 29-31.

[7] Liu Ping. Study on the convex and concave character of flatness error objective function[J]. Journal of Astronautic Metrology and Measurement, 2002, 22(6), 14-17.

[8] Lei Xian-qing, Li Fei, Tu Xian-ping, Wang Shi-feng. Geometry searching approximation algorithm for flatness error evaluation[J]. Optics and Precision Engineering, 2013, 21(5), 1312-1316.

[9] Li Shujuan. Software Develop of Form Error Evaluation based on Coordinate Measuring Datas[D]. Xi'an University of Technology, 2006.

[10] Bi Li-heng, Zhu Yan-qi. Flatness Error Evaluation based on Grouped Partical Swarm Optimization Algorithm [J]. Acta Metrologica Sinica, 2019, 40(6), 980-985.