

Damage Inference of Truss Structure Based on Bayesian Updating

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Abstract. In the long-term use of civil engineering structures, affected by natural disasters and man-made disasters, its performance gradually deteriorated, so it is very important to find structural damage in time. This paper discusses the problem of damage inference for truss structures, and uses Bayesian updating theory as a solution. By combining Bayesian updating with structural damage inference method, Monte Carlo sampling method is used for repeated variable experiment and analysis. The experimental results show that in a specific truss structure, when the load point is applied at a specific position, the experimental effect is the best. Moreover, with the increase of load and measuring points, the accuracy of experimental results is gradually improved. This method obviously improves the existing problems of the traditional method, and proves its effectiveness and accuracy. The combination of structural damage inference and Bayesian updating will bring more intelligent, reliable and efficient structural health monitoring and maintenance methods to the field of engineering construction. Realizing real-time reliability update and damage assessment of structures has a positive impact on improving the safety, sustainability and longevity of structures.

Keywords: Structural damage inference, Bayesian update, Monte Carlo method, rejection sampling method.

1. Introduction

With the passage of time, the performance of civil engineering structures gradually deteriorate in all aspects. Cumulative damage to the structure can also occur during its service life due to exposure to earthquakes, hurricanes, fires, corrosion and explosions. From the perspective of structural applicability and safety, it is an important problem to detect structural damage before failure. Failure to detect structural damage in time and take appropriate measures may lead to catastrophic structural damage and a large number of casualties [1].

In structural engineering, many problems involve the estimation of structural parameters, such as material properties, load characteristics. The prior probability distribution can reflect the initial knowledge of these parameters. Bayesian model updating is a method to obtain more accurate and reliable parameter estimation based on prior knowledge and the latest observational data. Through the combination of Bayesian updated statistics framework and reliability method, real-time monitoring and fault diagnosis of complex structural systems can be realized, and the probability distribution of various parameters of structures can be understood more clearly.

The Bayesian inference approach treats uncertain parameters as random variables, utilizing both prior knowledge and subsequently acquired conditional information to determine the posterior probability density distribution and the most precise parameter estimates. Therefore, it is widely used to solve various uncertainty problems [1]. The structural reliability method is to analyze the probability of the engineering structure to complete the predetermined function under certain conditions. However, conventional reliability methods have some common shortcomings: they are not good at dealing with problems involving discrete variables, and they cannot be updated when new information comes out. Bayesian networks can make up for these shortcomings [2]. Bayesian method has important application in reliability analysis. Reliability analysis is characterized by less data. Most of the objects of reliability analysis have small sample size, and some even have only one or two test results. In this case, in order to analyze the reliability index, all kinds of prior experience must be collected and synthesized as much as possible, and the prior distribution of parameters should be

sorted out and deduced. The prior distribution is not arbitrarily determined, but rather is derived through a logical and rational process. The employment of prior distribution serves as a reasonable complement to the limited size of the post-test sample [3]. Straub and Der Kiureghian first introduced Bayesian networks in structural reliability updating problems. After discretizing the observation information of the equation, a conditional probability table was established to construct the Bayesian network, and a node elimination algorithm was proposed to realize the reliability update of the structure based on Bayesian network [4, 5]. Sun et al. proposed earlier in China to use Bayesian networks to process reliability update under the condition of known detection information [6]. Through Bayesian updating, combining the knowledge of statistics and probability with structural engineering, the probability distribution of structural parameters can be estimated more accurately and the reliability of the model can be improved. In health monitoring, Bayes network is combined with traditional reliability methods to make up for the defects of traditional reliability methods.

The main purpose of Bayesian updating is to obtain the posterior distribution of model parameters through probability axioms by combining the prior distribution and the likelihood function derived from the observed data. Most of the main solving methods are based on stochastic simulation. One of the best known is the Markov chain Monte Carlo method. It constructs a Markov chain whose stationary distribution is a posteriori distribution. When the Markov chain reaches the limit distribution, the sample of the posteriori distribution is generated. These samples are correlated, and the traditional Markov chain Monte Carlo method is sometimes inefficient [7]. Later, some improved algorithms have been developed, such as adaptive Metropolis-Hastings (MH) method, transitional Markov chain Monte Carlo method and hybrid Monte Carlo method [8-10]. The research of structural engineering combined with Bayesian updating has great potential engineering value.

2. Methodology

2.1. Research Method

For civil engineering structures, structural damage identification is essentially a strong uncertainty problem under the influence of non-uniformity of structural materials and other factors. Uncertainty is essentially a manifestation of the lack of all kinds of information or knowledge, such as random variables representing material properties, geometric parameters, resistance calculation models and action statistical models all contain uncertainty [11]. Therefore, in order to better deal with the problem of damage identification in civil engineering, the fundamental approach is to reasonably consider and deal with the uncertainty in damage identification. Because of its solid theoretical foundation of probability and its rigor in dealing with uncertainty problems, Bayesian reasoning theory provides a possibility to solve this problem [1]. This project uses Bayesian updating theory to discuss the loading scheme for damage determination of specific truss structures, and repeatedly changes the variables to carry out experiments. The analysis is carried out with Bayesian updated Monte Carlo sampling method, and the obtained data is calculated by Tcl code and Opensees software. The calculated results are analyzed intuitively by excel, and the experimental scheme with the best accuracy is compared and selected. After calculating the desired actual displacement value of each measurement point, a set of actual data is obtained into the selected scheme, so as to obtain the final result after the end of the experiment.

2.2. Structural Model Likelihood Function

The basic formula of the Bayesian method is as follows:

$$p(A/B) = \frac{p(A,B)}{p(B)} = \frac{p(B/A)p(A)}{p(B)} = cp(B/A)p(A) \quad (1)$$

The posterior probability distribution of the parameter to be corrected is denoted as $p(A/B)$, while the likelihood function is represented by $p(B/A)$. Additionally, $P(B)$ represents the edge density function of the event B , which is a constant.

For a given structural model M , it is assumed that the vector of structural parameters to be identified is the vector of structural observation data, then the given posterior joint probability distribution can be obtained from the Bayesian formula.

$$p(\mathbf{x}|\tilde{\mathbf{y}}, \mathbf{M}) = \frac{p(\tilde{\mathbf{y}}|\mathbf{x}, \mathbf{M})p(\mathbf{x}|\mathbf{M})}{p(\tilde{\mathbf{y}}|\mathbf{M})} \quad (2)$$

- The prior probability distribution of $p(\mathbf{x}|\mathbf{M})$ is the parameter vectors, which generally needs to be derived from engineering experience and historical data.

- The $p(\mathbf{y} | \mathbf{x}, \mathbf{M})$ is the probability of obtaining the observed data when the structural physical parameter is $\mathbf{x}$, which reflects the updated structure model explains the observed data, which is called the likelihood function.

- The $p(\mathbf{y}|\mathbf{M})$ is equivalent to a normalized constant so that the integration value is equal to 1.

Generally, multiple observations are adopted. These data are divided into equal types and inequality types according to the definition of observation error. For m observations, assuming they are all independent, the likelihood function can be defined as follows:

$$L(\mathbf{x}) = \prod_{i=1}^m L_i(\mathbf{x}) \quad (3)$$

$$L(\mathbf{x}) = \prod_{i=1}^m L_i(\mathbf{x}) \propto \prod_{i=1}^m \exp\left\{-\frac{1}{2} \frac{[y_i - h_i(\mathbf{x}, \mathbf{M})]^2}{\sigma_i^2}\right\} \quad (4)$$

$$L(\boldsymbol{\mu}) = \prod_{i=1}^n f_x(X_i|\boldsymbol{\mu}) = \prod_{i=1}^n \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{x_i - \mu}{\sigma}\right)^2\right] = \prod_{i=1}^n \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{\mu - x_i}{\sigma}\right)^2\right] = \prod_{i=1}^n N_u(x_i, \sigma) \quad (5)$$

$$f_x(X|Y) = \frac{1}{c} L(X) \cdot f_x(x) \quad (6)$$

During the Bayesian update process, the direct integration of the posterior joint distribution of structural parameters poses significant challenges. Consequently, to derive the edge probability density distribution of the parameters, addressing this issue becomes imperative.

2.3. Monte Carlo Method and Rejection Sampling Algorithm

The Monte Carlo method, alternatively referred to as the statistical simulation method or the random sampling technique, is a highly significant numerical calculation approach rooted in the utilization of "random numbers" and grounded in the principles of probability statistics.

The Monte Carlo method leverages the capabilities of information-intensive and high-performance computing platforms to transform intricate research objects or computational challenges into simulations and calculations involving random numbers and their numerical characteristics, achieved through scientifically sound and rationally designed statistical modeling. Thus, it essentially simplifies the research problem, reduces the computational complexity, and obtains the approximate solution with excellent properties [11]. In the common programming software R, Python or Excel, only sample some common distributions (Gamma, Beta, or normal). However, the expectation of these distributions itself has an analytical form, so it is not necessary to use the Monte Carlo method. For those calculated and formally rare posterior distributions, there are no readily available functions in the programming software to help with sampling. In most cases, the difficulty can determine the full form of the objective function, let alone find the cumulative distribution function or its inverse function. Therefore, it's important to find a sampling method that works for any distribution. One effective method is to use the rejection sampling method (Rejection Sampling) [12]. The rejection sampling method can produce a random sample following the probability density function of incomplete form (the integral is not 1 on the real field) [13]. The corresponding sample receptive field was constructed according to the target distribution, and the random samples meeting the target distribution were selected [14]. Finally, random sampling of molecular kernel density is realized in the posterior probability density function of random variables [15].

3. Experiment and Discussion

3.1. Experimental Subject

With the truss as shown in Fig. 1, the bar stiffness EA was inferred through the experiment as $10^5 \text{ kN} \cdot \text{m}^2$. The bars 4-11 and 10-11 are two damaged rods, and the stiffness of the remaining rods is known. Bayesian update according to the measurement results to give the posterior distribution of the stiffness of the two damaged rods. The rod 4-11 prior follows uniform distribution $U(0.68, 1.26)$, and the rod 10-11 prior follows uniform distribution $U(0.76, 1.54)$. The maximum load of each node is 100 kN vertically downward, and the structure is always in the linear phase.

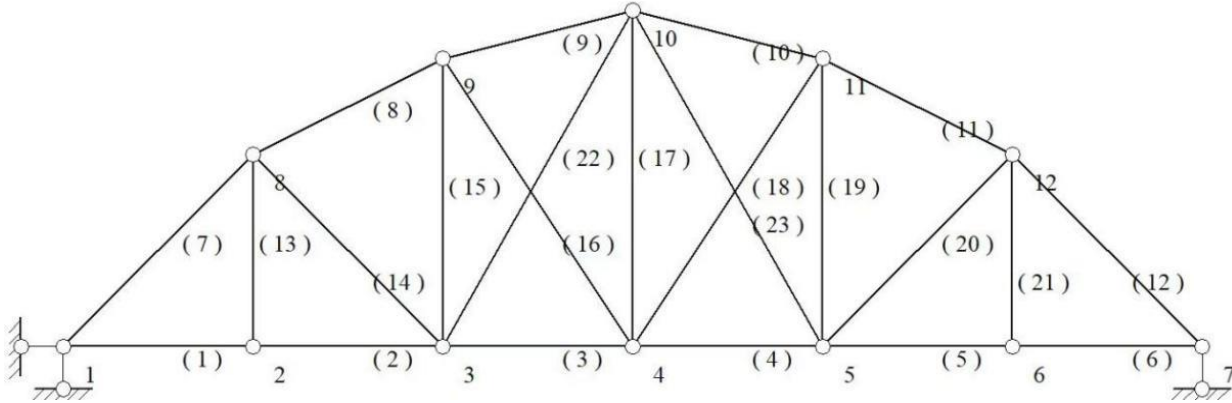


Fig. 1 Experimental truss

3.2. Experimental Procedures

During the experiment, the corresponding load point, load size and measurement point in the code were changed to output the results. 5000 samples were taken for each experiment, and the error was set at 0.05.

3.2.1 Changing the point of load

The load applied at the load point is 100 kN vertically, the displacement points are 4, 5 and 6, and the experimental error is set to 0.05. Load is applied at point 12. The findings of the experimentation are depicted in Fig. 2.

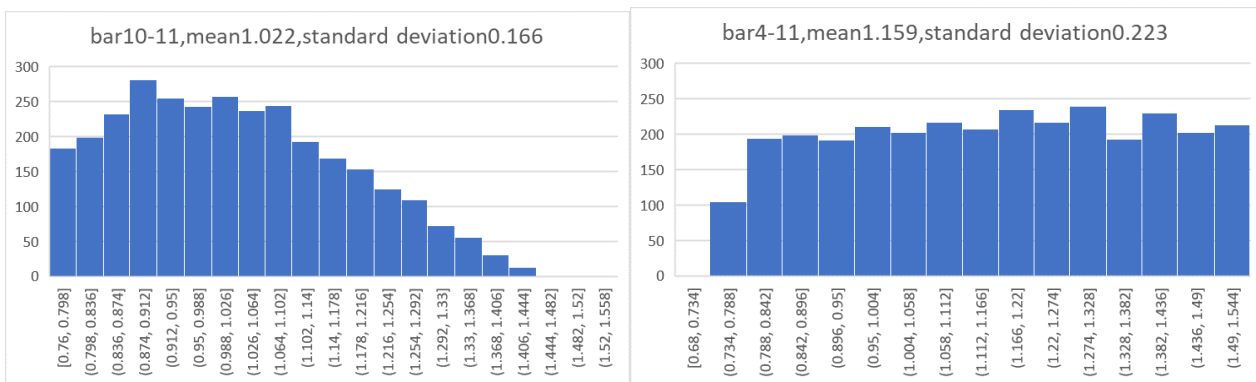


Fig. 2 Experimental results of the load applied at point 12

The load is applied at point 11, and the experimental results are exhibited in Fig. 3.

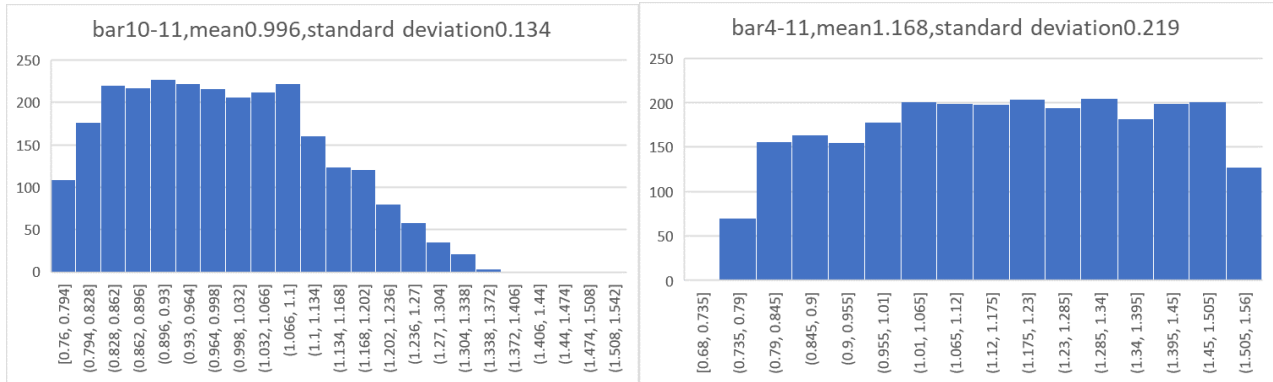


Fig. 3 Experimental results of the load applied at point 11

The load is applied at point 10, and the experimental outcomes are presented in Fig. 4.

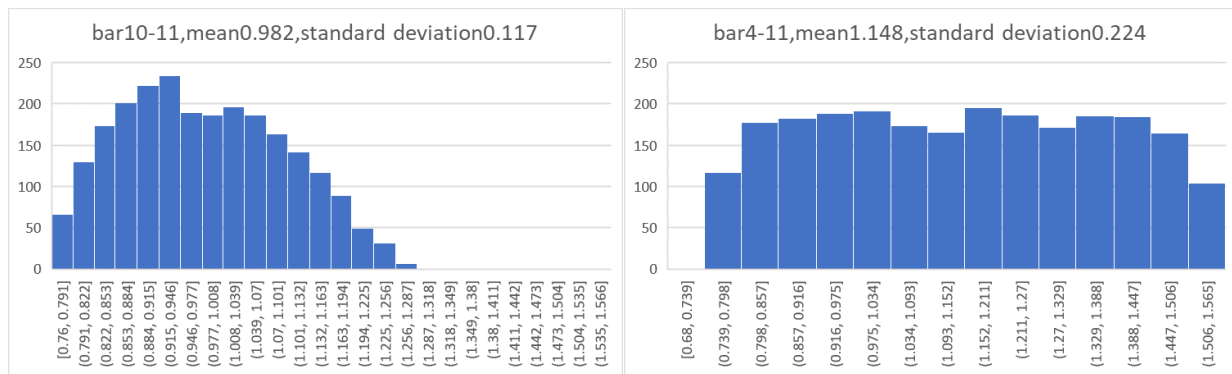


Fig. 4 Experimental results of the load applied at point 10

Loads were applied simultaneously at points 10 and 11, and the experimental results are presented in Fig. 5.

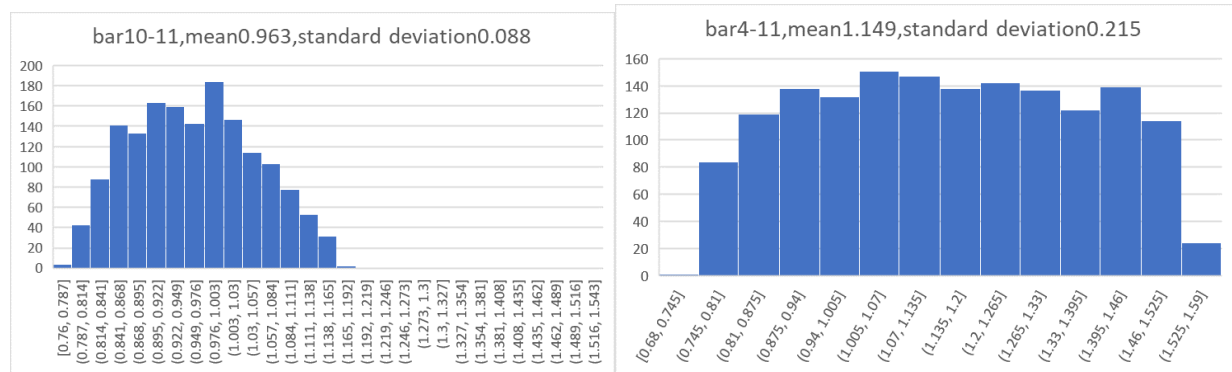


Fig. 5 Experimental results of the load applied at points 10 and 11

According to the comparison of Fig. 2, Fig. 3, Fig. 4 and Fig. 5, when the load is added at 10 and 11 points, the standard deviation is small, so the experimental effect is better.

3.2.2 Changing the size of load

The load is applied at points 10, 11, the load direction is vertically downward, the measured displacement points are point 4, 5, 6, and the experimental error is set to 0.05. A 50 kN load was applied, and the experimental results are presented in Fig. 6.

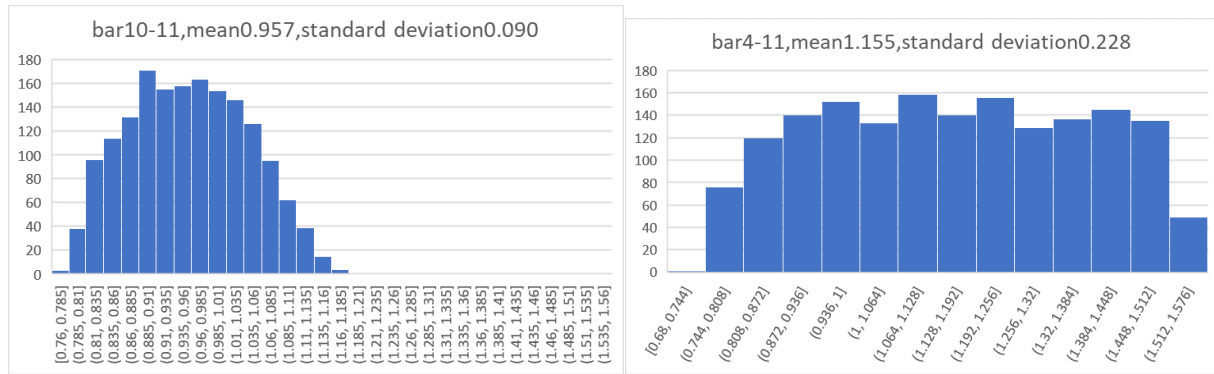


Fig. 6 Experimental results under 50 kN loading

The load size is changed again and 75 kN load is applied. The experimental results are presented in Fig. 7.

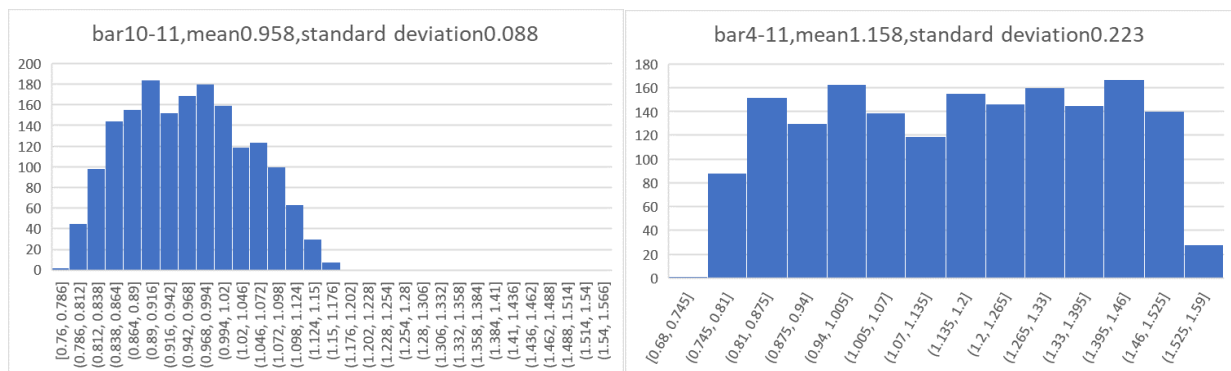


Fig. 7 Experimental results under 75 kN loading

A load of 100N has been applied, and the corresponding experimental outcomes are depicted in Fig. 8.

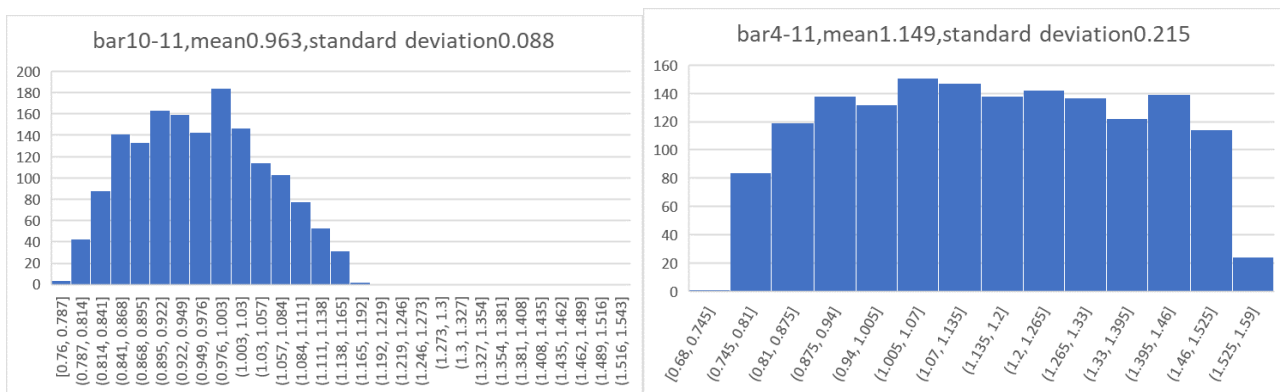


Fig. 8 Results of the experiment under 100 kN loading

According to Fig. 6, Fig. 7, and Fig. 8, the larger the load, the smaller the standard deviation, the less the data dispersion, the more accurate the results. However, the difference between the experimental results is small, and there is no significant difference.

3.2.3 Changing the number of measuring points

The load is applied at points 10 and 11 at 100 kN in a vertical direction. The displacement is measured at four measuring points at 4, 5, 6 and 10, and Fig. 9 shows the experimental results.

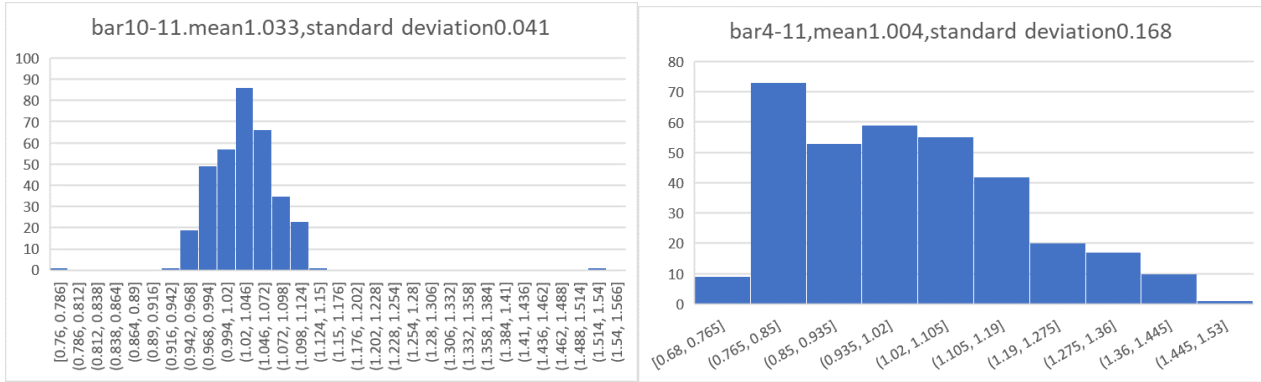


Fig. 9 Results of the displacement measured at points at 4, 5, 6 and 10

The displacement is measured at five measuring points include 4, 5, 6, 10, 11, and the experimental results are presented in Fig. 10.

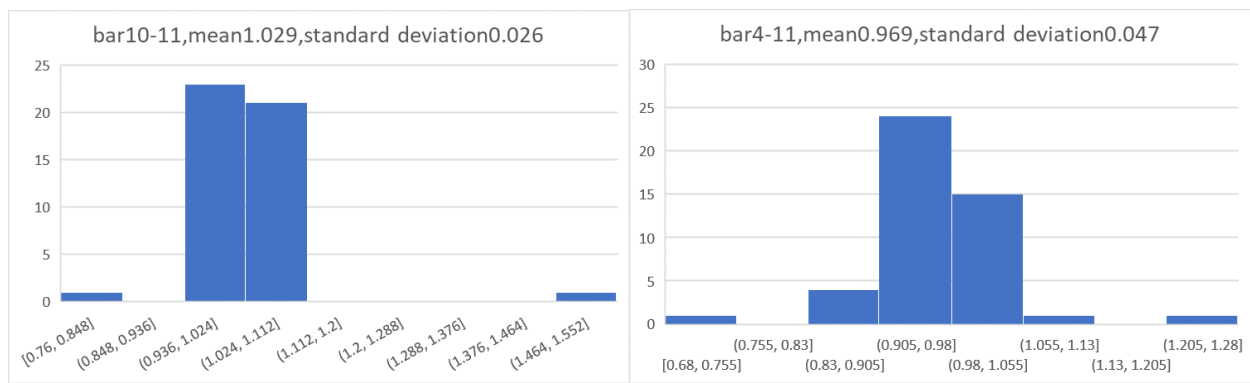


Fig. 10 Results of the displacement measured at 4, 5, 6, 10, 11

The displacement is measured at six measuring points in 4, 5, 6, 10, 11, 12. The experimental outcomes are depicted in Fig. 11.

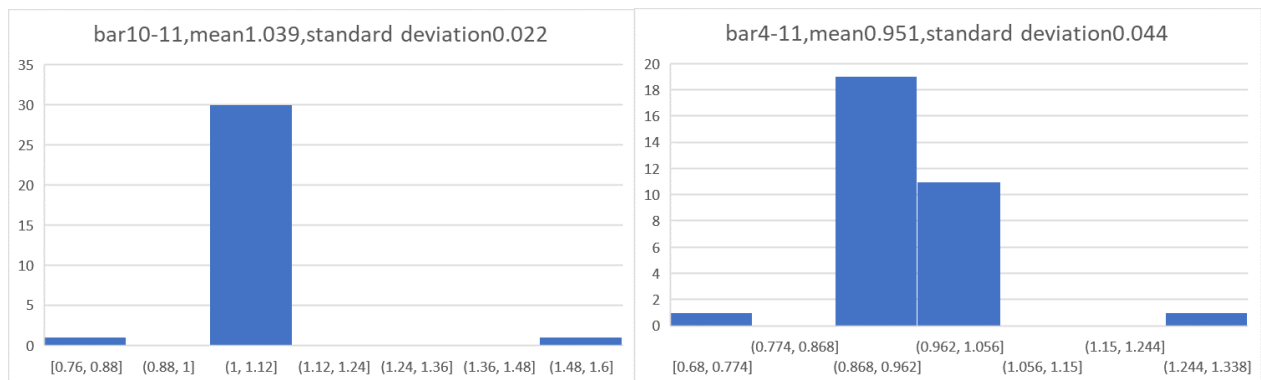


Fig. 11 Results of displacement measured at 4, 5, 6, 10, 11, 12

From the above experiments, it can be clearly seen that the best results at the applied load points are at 10 and 11. With the increase of applied load and the number of measuring points, the experimental results are also becoming more and more accurate. It can be seen from the standard deviation and the degree of divergence of images in the figures. And the results also show that changing the size of the load has little influence on the experimental results, while increasing or decreasing the number of measuring points significantly affects the experimental results.

4. Conclusion

Based on the Bayesian update method, this paper studies the problem of damage inference of truss structure and obtains the following main conclusions:

(1) It is clear from the experiment that when loading points are applied, the best results are obtained at points 10 and 11. With the increase of applied loads and measuring points, the accuracy of the results is also improved, and the experimental results are more in line with the requirements.

(2) In this paper, Bayesian updating and structural damage inference are combined to improve the problems of traditional methods. At the same time, the results are analyzed with Monte Carlo sampling method, which proves the effectiveness and accuracy of the proposed method. The combination of structural damage inference and Bayesian updating will bring more intelligent, reliable and efficient methods of structural health monitoring and maintenance to the engineering and construction fields. When the evidence information appears, the real-time update of structural reliability and damage assessment are realized. A series of problems related to the structure's own state and parameters can be solved more reasonably and efficiently by combining with Bayesian updating. This has a positive impact on the safety, sustainability and longevity of the structure.

(3) However, this paper only makes a preliminary exploration and research on improving the accuracy of truss damage inference. The variables taken in the experiment are limited, and the influence of other factors such as the setting of the relative error size is not considered. Subsequently, the application effect of the theory on structural damage identification in practical engineering projects can be explored. In addition, the problem of the best scheme considering the actual cost and other factors has not been solved, and further research is still needed.

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