Optimization of UAV formation positioning accuracy based on double circle crossing method

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Abstract. This paper delves into the optimization of positioning accuracy for Unmanned Aerial Vehicles (UAVs) in a circular formation. The study commences by establishing a polar coordinate system leveraging the positions of two reference UAVs, FY00 and FY01, whose locations are presumed to be stable and error-free. Utilizing the triangulation method within this coordinate system, we determine the positions of the remaining UAVs based on the direction information received from signal transmissions. Under the premise that positional deviations are confined to a range of ±5°, we employ an exhaustive method to discern the correlation between interval numbers and direction information. This approach facilitates the precise positioning of UAVs, despite minor location discrepancies. Subsequently, an iterative adjustment scheme is introduced to refine the distribution of UAVs along a circular path. Through a series of iterative computations, the UAVs are adjusted to achieve an even distribution, with the positioning error maintained within an acceptable margin of approximately 3 meters. The methodology integrates mathematical modeling with computational simulations, demonstrating the efficacy of the double circle crossing method in enhancing the accuracy of UAV formation positioning. The paper concludes with the validation of the proposed models through Matlab simulations, confirming the potential for practical application in optimizing UAV formation positioning tasks.

Keywords: UAV formation; Positioning Accuracy; Double Circle Crossing Method; Triangulation; Iterative Adjustment.

1. Introduction

Unmanned aerial vehicles (UAVs) are unmanned aircraft operated by radio remote control devices and program controls. On the military side, drones can be used as reconnaissance aircraft. The UAV formation flight can achieve a single and efficient completion of military reconnaissance, target attack, communication relay and other tasks. In order to adjust the position of UAVs without external interference and keep the relative position of a UAVs and other UAVs unchanged during the execution of tasks, azimuth-only positioning method should be adopted, that is, several UAVs in the formation send signals, and other UAVs receive signals, locate and adjust their positions through the direction information in the signals. The directional information obtained by one of the receiving drones is: the Angle between the drone and any two transmitting drones. Since it is an important way to win the military advantage to master the formation flight of UAVs, it has long been the focus of all countries to study its formation formation and flight path planning [1, 2, and 3].

This paper studies how to locate the circle formation of UAVs effectively. Firstly, the position of UAVs is determined by establishing polar coordinate system and applying triangulation method. Under the assumption that the position deviation is not more than ±5°, the relationship between the interval number and the direction information is found by using the exhaustive method to realize the positioning. Finally, an iterative adjustment scheme is proposed, which makes the UAV evenly distributed on the circumference through multiple adjustments, and the error is controlled to about 3 meters.

2. Model based on double circle crossing method

When the UAV transmitting the signal and the UAV receiving the signal are in the same plane, the UAV positioning problem is transformed into determining the target position in the two-

dimensional plane. For passive positioning in the case of two-dimensional plane, each UAV can be regarded as a point on the plane with a certain position relationship, and the position of a certain target can be determined by using the knowledge of plane triangle and two-dimensional plane direction finding cross positioning. In addition, since UAVs form a circle formation at the same height, the establishment of a double-circle cross positioning model can simplify the calculation by using the circumferent Angle theorem and the characteristics of the inner triangle of the circle, and finally realize the uniform distribution of UAVs on a circle, as shown in Fig. 1.

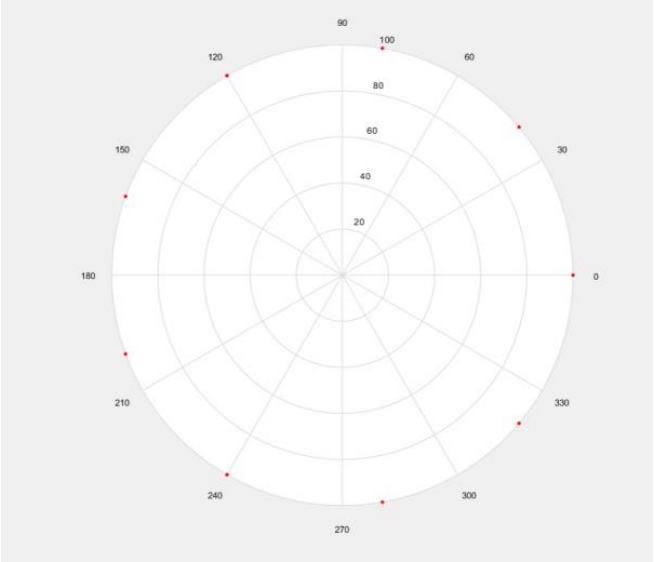


Fig 1. Site equipartition map.

2.1. Construction of model

Assuming that all UAVs fly in the same direction and remain unchanged, the flying direction of UAVs is assumed to be the positive direction, with FY00 as the pole, and the polar coordinates established by the ray at the position of the pole and FY01 as the polar axis. When ten UAVs are moving as a whole, the position of FY00 and FY01 UAVs transmitting signals is unbiased, so if you want to maintain the formation formation, you only need to keep the other UAVs evenly distributed in this polar coordinate and the coordinate is unchanged.

Assume that the two UAVs with known numbers are FY0a and FY0b, and their positions are unbiased. FY0X is the UAV receiving the signal, and the extracted direction information includes three included anglesα_1,α_2 and α_3, where $\alpha_3 = \alpha_1 + \alpha_2$(default b>a). If the drones are evenly distributed on a circle with radius R=100m, then the polar Angle of each drone is a multiple of 40°, so the polar coordinates of FY0a (100m, 40a°) and FY0b (100m, 40b°) are set.

The three vertices of ΔAXO make the peripheral circle, the center of the circle is O_1, the radius is r_2, and in the peripheral circle of ΔAXO, α_2 is the circumference Angle. The constructed outer circle is shown in Fig. 2.

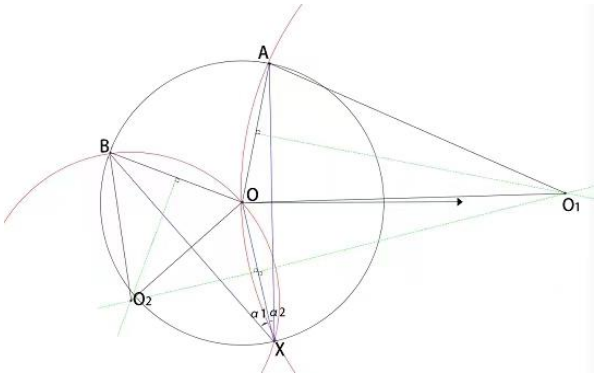


Fig 2. Schematic diagram of the external circle.

The formula can be obtained as follows.

$$r_1 = \frac{R}{\sin\alpha_1}. \tag{1}$$

The same formula can be found in the outer circle of ΔBXO.

$$r_2 = \frac{R}{\sin\alpha_2}. \tag{2}$$

OX is the same string in O_1 and O_2, where O_1 is $[r_1, 40b + (90° - \alpha_1)]$ and O_2 is $[r_2, 40b + (90° - \alpha_2)]$. Fig. 3 is a diagram of the common string.

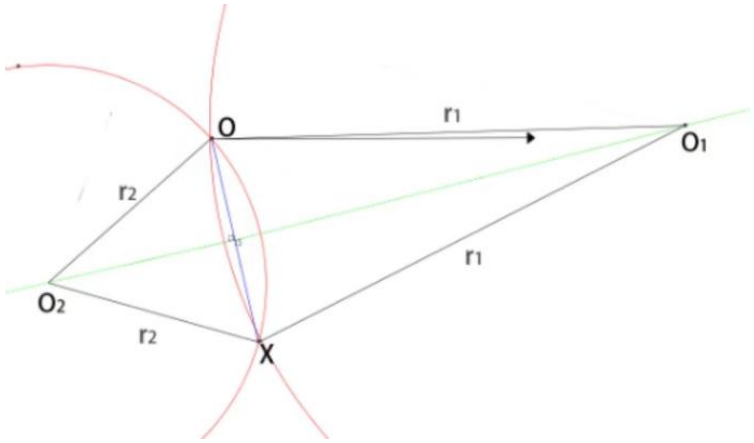


Fig 3. Common chord diagram.

2.2. Model solving

The following formula can be derived by considering the polar distance formula.

$$\begin{aligned} O_1O_2 &= \sqrt{r_1^2 + r_2^2 - 2r_1r_2[(40°b + 90° - \alpha_2) - (90° - \alpha_1 - 40°a)]} \\ &= \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos[(40°a + 40°b - \alpha_2 + \alpha_1)]} \end{aligned}. \tag{3}$$

In ΔOO_1O_2, by Helen's formula:

$$S = \sqrt{p(p - r_1)(p - r_2)(p - O_1O_2)}. \tag{4}$$

Where,

$$p = \frac{r_1 + r_2 + O_1O_2}{2}. \tag{5}$$

The chord length obtained by the equal area method is shown below.

$$OX = \frac{4S}{O_1O_2} = \frac{4\sqrt{p(p-r_1)(p-r_2)(p-O_1O_2)}}{O_1O_2}. \tag{6}$$

Then, the polar Angle of the target position is obtained as follows.

$$\theta_x = \arccos\left(\frac{OX}{2r_2}\right) + 40°b + 90° - \alpha_2. \tag{7}$$

Therefore, if a UAV can obtain azimuth information α_1 and α_2, it's position in the polar coordinates established by the two drones FY00 and FY01 can be calculated as follows.

$$\left(OX, \arccos\left(\frac{2\sqrt{p(p-r_1)(p-r_2)(p-O_1O_2)}}{r_2O_1O_2}\right) + 40°b + 90° - \alpha_2\right). \tag{8}$$

Then use Matlab to simulate a situation: the simulation parameters are α_1=10.55° and α_2=45.63°. FY02, FY05 and FY00 transmit signals, and FY07 receive signals. Through the mathematical model established by us, the polar coordinates can be calculated as (242.2186, 110.4633).

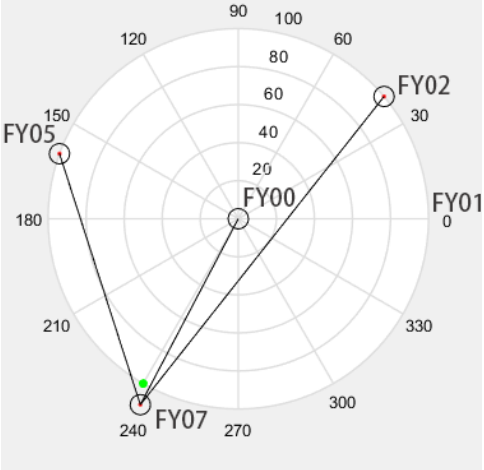


Fig 4. Verification result.

3. Model application and error analysis

For UAVs receiving signals with a slight deviation in position, we assume that the angular deviation is within 5°, and the minimal deviation is negligible. We first discuss the situation that in addition to the two UAVs of FY00 and FY01, there is also a need for a UAVs to transmit signals, and judge whether the effective positioning of UAVs can be achieved. We first put the specific number into the established model for verification, assuming that the drone number to be located is FY06, when the direction information extracted is α_1=10°, α_2=20°, as shown in Fig. 5, by calculating the isosceles triangle formed by two 7-point lines in the circle and the law that the degree of the circumference Angle is half of the degree of the center Angle, we can see that The third UAV transmitting the signal can be FY03 or FY09, and the result is not unique, so it is not possible to use the three UAV transmitting signals to achieve effective positioning of the target UAV [5].

Now we discuss the situation of positioning by four UAVs, that is, in addition to the two UAVs of FY00 and FY01, two UAVs are also needed to transmit signals, and the numbers of these two UAVs are recorded as FY0c and FY0d, assuming that the number of UAVs to be located is FY0y. The Angle formed by the connection between FY0y and FY00, FY0c and FY0d is the direction information obtained by the UAV receiving the signal. In the case of no deviation, the Angle of the center Angle formed by the line between any two points on the circumference and the center of the circle is known, and the Angle relationship shown in Fig. 6 can be obtained by using the law that "the degree of the circumference Angle is half of the degree of the center Angle".

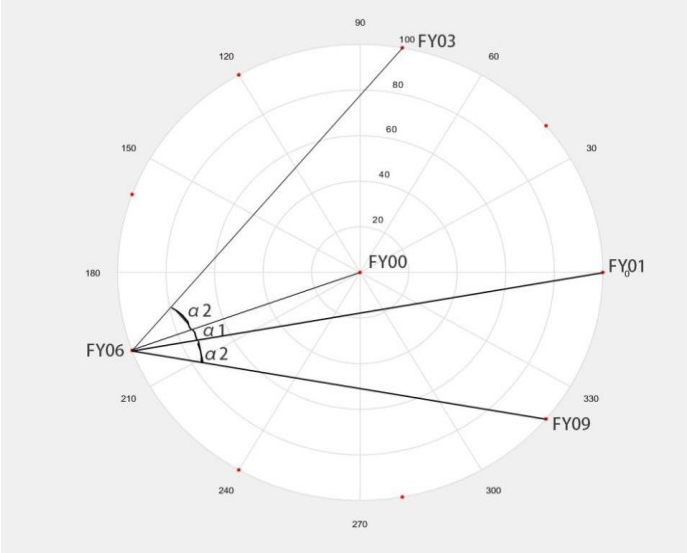


Fig 5. A hypothetical location with three drones.

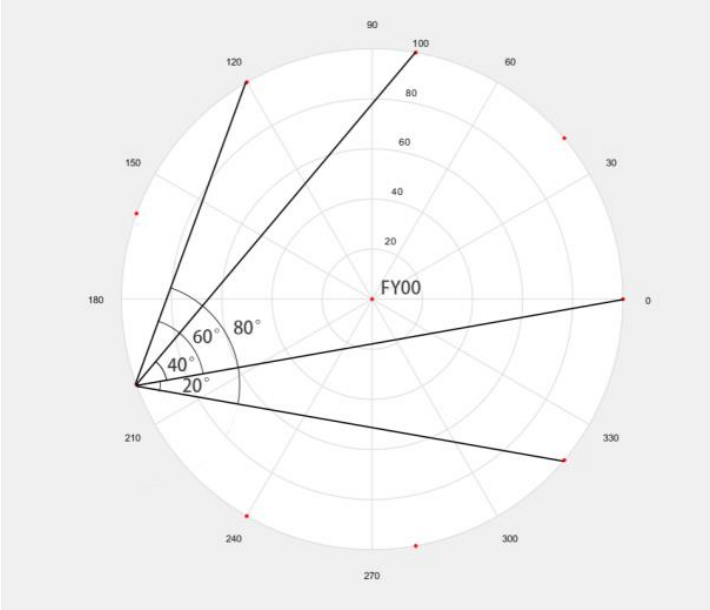


Fig 6. The relationship between different angles.

For any numbered target UAVs, the Angle formed by the connection between them and the number of UAVs with fixed intervals has the same Angle relationship. Table 1 can be summarized, where "number of UAVs with interval" is the number of UAVs with interval of less than or equal to 3 between FY0c and FY0d on the circular trajectory. "The Angle formed by the connection" is the Angle formed by FY0y and FY0c and FY0d respectively.

Table 1. Comparison table of the number of intervals and the included angles

The number of drone intervals	The Angle formed by the connection
0	20
1	40
2	60
3	80

The Angle of Fig. 7 can be inferred by using the isosceles triangle formed by two points on the circumference and the center of the circle.

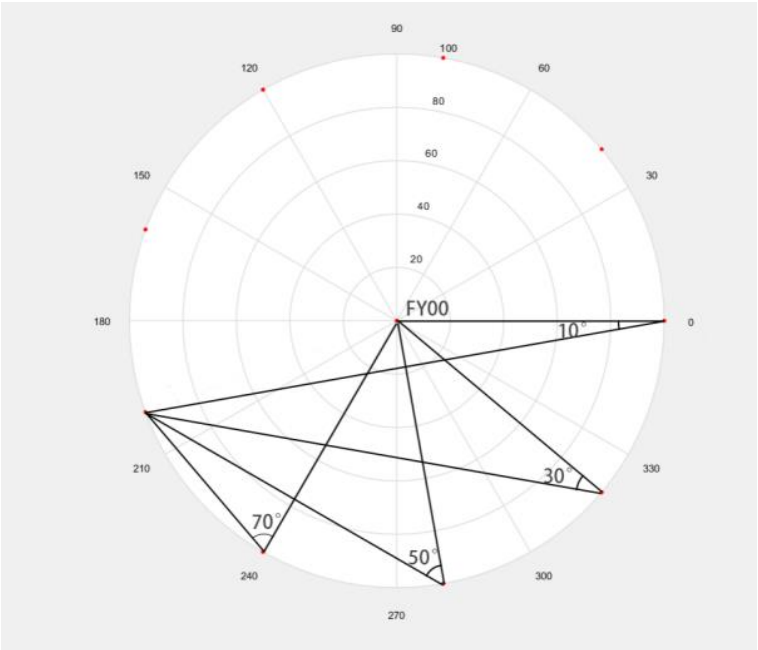


Fig 7. The relationship between different angles (10.30.50.70).

Table 2 shows the results of the summary. "Number of drones at intervals" in the table refers to the number of drones less than or equal to 3 separated between FY0y and FY01 on the circular trajectory. One side of the included Angle formed by the connection is the line between FY0y and FY00. The other side is the connection between FY0y and a certain UAV separated by several UAVs.

Table 2. Comparison table of the number of intervals and the included angles.

The number of drone intervals	The Angle formed by the connection
0	70
1	50
2	30
3	10

From the above research, we can find that the four lines formed by the target positioning UAV and the four transmitting UAV can be formed into a total of 6 angles in any pair. The position relationship between the transmitting UAV and the receiving UAV on the circumference can be judged by the odd Angle, and the position relationship between any two transmitting UAV on the circumference can be judged by the even Angle. These angles are included in the above eight degrees if they are approximately equal (or rounded) to a whole number. For example, the drone number receiving the signal is FY08, and the Angle formed by the connection with FY00 and FY01 is 50°. If the degree of the Angle information received by the drone contains 10, 20, 40, it can be determined that one of the drones transmitting the signal is numbered FY04; If the Angle information it receives contains 10, 20, 60 degrees, then it can be determined that one of the UAVs transmitting signals to it is numbered FY03. Same way to determine the number of the other drone.

As long as we know the Angle within the error range (±5°) through the direction information, the number and position of the UAV FY0c and FY0d can be determined by comparing the relationship table between the interval number and the included Angle. According to the conclusion of the first question, FY0c, FY0d and FY00 can be used to effectively locate the target UAV.

4. Iterative optimization

Further introducing the error parameter Δ, we extract one UAV from the UAV numbered FY02 ~ FY09 and FY01 to form a combination containing three UAVs, that is, three UAVs in each group transmit signals, and the remaining UAVs numbered receive signals and adjust them. Based on the conclusion of the first question, the position of all other UAVs receiving signals is adjusted after each group is adjusted, and the obtained result is different from the standard position (evenly distributed on a circle with a radius of 100m), and the position deviation error is obtained. The errors of each group are then added together.

Iteration method, also known as the tossing method, is a method that constantly uses the old value of a variable to introduce a new value to solve the problem. The iterative algorithm is generally used for numerical calculation. Accumulation and multiplication are the basic applications of iterative algorithms. The iterative process is to gradually solve the solution of the large-scale problem through the solution of the small-scale problem. On the surface, it is exactly the opposite of recursion, but it also finds the relationship between the large-scale problem and the small-scale problem [6].

After three iterations of Matlab adjustment, we found that the sum of errors of most UAV combinations was about 3m, which roughly met our requirements. Finally, the position adjustment shown in Fig. 8 is achieved. (Red site is before adjustment, green site is after adjustment)

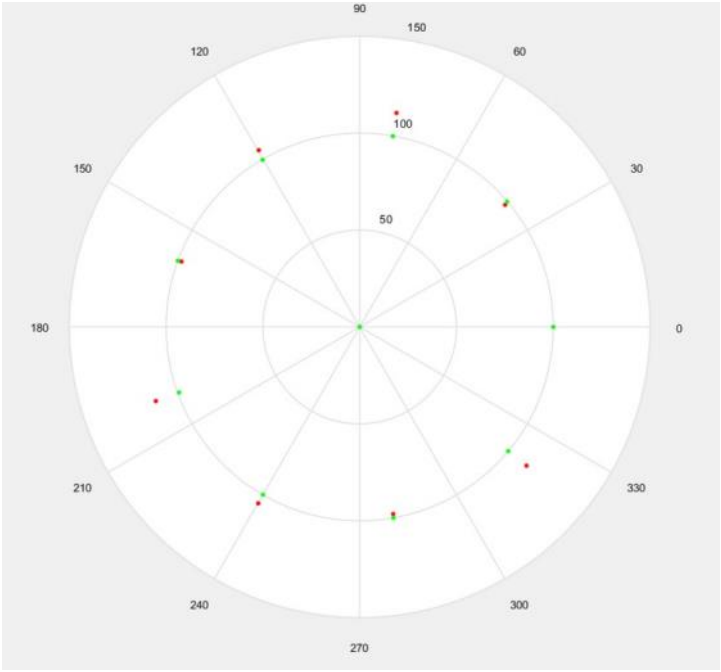


Fig 8. Result diagram based on iterative strategy.

5. Summary

This paper presents a comprehensive study on how to optimize the positioning accuracy of UAVs by using the double-circle crossing method. The research successfully demonstrates the effectiveness of establishing polar coordinate system and applying triangulation method to determine the position of UAVs in circular formation. Under the assumption of minimal position deviation, the exhaustive method is effectively used to establish the relationship between the interval number and the directional information, so that precise positioning can be achieved despite small inaccuracies. The innovative iterative tuning scheme proposed in this paper allows the drone to be evenly distributed along a circular path with an error control within an acceptable range of about 3 meters. Matlab simulation verifies the theoretical model and provides a practical method to improve the positioning accuracy of UAV formation.

However, the study also acknowledges inherent limitations in assumptions, such as the minimum deviation hypothesis, that may not always hold true in practical applications. Subsequent research can focus on refining the model to take into account larger positional biases and environmental factors (such as wind and turbulence) that may affect positioning accuracy. In addition, exploring 3D formations and integrating machine learning algorithms for dynamic adjustment can further improve the robustness and adaptability of drone formations in various operating environments.

In conclusion, this paper has made important progress in the field of UAV formation positioning, which provides a solid foundation for further research and development. These findings have important implications for military and civilian applications that require accurate formation control.

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