The study on the force analysis model of wave energy devices in heaving and pitching motion

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Abstract. This study establishes a mathematical model for the heave and pitch motions of a wave energy converter. Force equations for the buoy and oscillator are derived based on a force analysis model. Utilizing Newton's second law, equations for the linear and angular accelerations of both components are obtained. Torque equations are then derived from the torques acting on the system during heave and pitch motions. These are used to calculate the related velocity equations for the buoy and oscillator. The displacement equations are formulated to determine the heave displacement and velocity, as well as the pitch angular displacement and velocity of the buoy and oscillator at specific time points, given the damping coefficients of linear and rotary dampers. The study finally develops a mathematical model to optimize the damping coefficients for linear and rotary dampers based on the aforementioned model. Open-source data are integrated into this model to establish the range of damping coefficients for both dampers. Exhaustive search and multithreading techniques are employed to enhance computational efficiency and fully utilize computer performance in deriving the relevant data.

Keyword: Wave Energy Device, Simulation, Planning Concept.

1. Introduction

Wave energy, as a sustainable marine resource, has gained increasing importance in the global energy transition due to its substantial potential and eco-friendliness. The surge in global renewable energy demand has brought Wave Energy Converters (WECs) to the forefront of research and development. These devices, by converting the kinetic energy of waves into electrical power, provide a dependable solution for coastal regions.

The research by Wang Zhenchun showcases the latest advancements in WEC control technologies, underscoring the significance of power maximization as a control goal and discussing the application of optimal and sub-optimal control techniques[1]. Moreover, with technological progression, innovative control strategies become pivotal in enhancing the conversion efficiency and economic feasibility of wave energy. Numerous studies have concentrated on augmenting the energy capture efficiency of WECs and refining their Power Take-Off (PTO) systems. For example, Huang Shuo conducted an optimization design of the PTO system for an Eagle-type wave energy device using ADAMS software, significantly enlarging its energy capture width and thereby boosting its power generation efficiency[2]. In the broader context of marine energy, Ma Changlei analyzed the current development and technological trends of China's marine energy industry, pointing out the vital role of industry coordination and policy incentives in advancing technological and industrial progress[3]. Li Xinyu revealed trends and patterns in China's marine wind and wave energy through extensive data analysis, providing valuable insights for energy assessment and climate prediction[4]. Yao Tao demonstrated the impact of structural parameter design on the efficiency of wave energy conversion by optimizing a resilient support chain parallel mechanism in WECs[5]. Additionally, Huang Yi employed direct thrust control and optimal control theory in their research to optimize the power output of direct-drive wave power systems, illustrating the potential for performance enhancement through advanced control strategies[6].

This paper develops a mathematical model for the heave and pitch motions of wave energy devices. It begins with a force analysis to derive the force equations for the buoy and oscillator, using Newton's second law to formulate their angular momentum and acceleration equations. These equations facilitate the calculation of corresponding torque and velocity equations. Subsequently, displacement equations are formulated to compute the buoy and oscillator's displacement and velocity under specific damping coefficients. Ultimately, the damping coefficients for linear and rotary dampers are optimized based on these models.

Reference data from:

http://www.mcm.edu.cn/html_cn/node/388239ded4b057d37b7b8e51e33fe903.html

2. Force Analysis and Model Establishment for Heaving and Pitching Motions of Wave Energy Devices

Analyze the forces acting on the oscillator and buoy under heaving and pitching motions and establish the system force analysis equations:

$$F_{ho} = F_{hdc} + F_{hfs} \tag{1}$$

$$F_{hf} = F_{\omega} + F_{w} + F_{ww} - F_{ho} * \cos(\theta_o) \tag{2}$$

In this context, F_{ho} represents the force on the oscillator under heaving and pitching motions, while F_{hf} denotes the force on the buoy in the same motions. Furthermore, F_{hdc} is the damper force under heaving and pitching motions, F_{hfs} is the spring force in these motions, F_{ω} is the wave excitation force, F_{w} is the wave-making damping force, F_{ww} is the hydrostatic restoring force, and θ_o is the angle of the oscillator.

Damper force in heaving and pitching motions:

$$F_{hdc} = -v_o * getdc \tag{3}$$

Spring force in heaving and pitching motions:

$$F_{hfs} = -m_o * k + (1 - \sin(\omega_o)) * g * M_{\text{pendulumc}} \tag{4}$$

Buoy force in heaving and pitching motions:

$$F_{hf} = F_{\omega} + F_{w} + F_{ww} - F_{ho} * \cos(\theta_o) \tag{5}$$

Wave excitation force:

$$F_{\omega} = f * \cos(\omega t) \tag{6}$$

Wave-making damping force:

$$F_{w} = -d_{hwmdc} * v_f \tag{7}$$

Hydrostatic restoring force:

$$F_{ww} = -m_f * \pi * R^2 * \rho * g \tag{8}$$

In the formula, ω represents the incident wave frequency, v_o is the velocity of the oscillator along the central axis in heaving motion, m_o denotes the total displacement of the oscillator along the central axis in heaving motion, k is the spring stiffness of 80000 N/m, ω_o is the angular velocity of the oscillator, g stands for the gravitational acceleration of 9.8 m/s^2, $M_{pendulum}$ represents the mass of the oscillator, which is 2433 kg, f indicates the amplitude of heaving excitation force, d_{hwmdc} denotes the heaving wave-making damping coefficient, v_f is the velocity of the buoy in heaving motion, m_f represents the total displacement of the buoy in heaving motion, m_f also refers to the base radius of the buoy, which is 1m, and ρ is the density of seawater, 1025 kg/m^3.

The acceleration equations for the buoy and oscillator in heaving and pitching motions are established using Newton's second law[7]:

$$a_{hf} = \frac{F_{hf}}{M_{\text{buoy}}+aq} \tag{9}$$

$$a_{ho} = \frac{F_{ho}}{M_{\text{pendulum}}} \tag{10}$$

$$a_{wf} = \frac{P_{fhf}}{J_f+ari} \tag{11}$$

$$a_{wo} = \frac{F_{ho}}{J_o+(0.75+m_o)^2*M_{\text{pendulum}}} \tag{12}$$

Here, a_{hf} is the acceleration of the buoy in heaving and pitching motions, a_{ho} is the acceleration of the oscillator in heaving and pitching motions, a_{wf} is the angular momentum acceleration of the buoy, and a_{wo} is the angular momentum acceleration of the oscillator.

Also, P_{fhf} represents the torque on the buoy in heaving and pitching motions.

The torque on the buoy in heaving and pitching motions:

$$P_{fhf} = P_{F\omega} + P_{Fw} + P_{Fww} - P_{fho} \tag{13}$$

$$P_{fho} = P_{ddc} + P_{fs} \tag{14}$$

Here, $P_{F\omega}$ is the wave excitation torque, P_{Fw} is the wave-making damping torque, P_{Fww} is the hydrostatic restoring torque, P_{fho} is the torque on the oscillator in heaving and pitching motions, P_{ddc} is the rotary damper torque, and P_{ddc} also represents the torsional spring torque[8].

Wave excitation torque:

$$P_{F\omega} = L * \cos(\omega t) \tag{15}$$

Wave-making damping torque:

$$P_{Fw} = -d_{pwmdc} * w_f \tag{16}$$

Hydrostatic restoring torque:

$$P_{Fww} = -d_w * \theta_f \tag{17}$$

Rotary damper torque:

$$P_{ddc} = (w_f - w_o) * P_{dc} \tag{18}$$

Torsional spring torque:

$$P_{fs} = (\theta_f + \theta_o) * K_t \tag{19}$$

In the formula, M_{buoy} represents the mass of the buoy, which is 4866 kg, aq denotes the heave added mass, J_f is the moment of inertia of the buoy, ari indicates the pitch added moment of inertia, J_o represents the moment of inertia of the oscillator, and K_t is the torsional spring stiffness, 250000 N/m.

Using the principles of kinematics, the velocity equations for the buoy and oscillator in heaving and pitching motions are established:

$$V_{hf} = \int a_{hf}\, dt = a_{ht} * t \tag{20}$$

$$V_{ho} = \int a_{ho}\, dt = a_{ho} * t \tag{21}$$

Here, V_{hf} is the velocity of the buoy in heaving and pitching motions, and V_{ho} is the velocity of the oscillator in heaving and pitching motions.

Using the principles of kinematics, the displacement equations for the buoy and oscillator in heaving and pitching motions are established:

$$X_{hf} = V_{hf} * \Delta t + \frac{1}{2} * a_{hf} * (\Delta)^2 \tag{22}$$

$$X_{\omega f} = \omega_f * \Delta t + \frac{1}{2} * a_{\omega f} * (\Delta)^2 \tag{23}$$

$$X_{ho} = V_{hf} * \Delta t + \frac{1}{2} * a_{ho} * (\Delta)^2 \tag{24}$$

$$X_{\omega o} = \omega_o * \Delta t + \frac{1}{2} * a_{\omega f} * (\Delta)^2 \tag{25}$$

Here, X_{hf} is the displacement of the buoy at an instant in heaving and pitching motions, $X_{\omega f}$ is the pitch angle displacement of the buoy, X_{ho} is the displacement of the oscillator at an instant in heaving and pitching motions, and $X_{\omega o}$ is the pitch angle displacement of the oscillator[9].

Based on the time interval provided in the problem, the update equations for heave displacement and velocity, and pitch angle displacement and angular velocity, of the buoy and oscillator during heaving and pitching motions are derived:

$$m_{ho} = m_{ho} + x_{ho} \tag{26}$$

$$m_{hf} = m_{hf} + x_{hf} \tag{27}$$

$$m_{\omega f} = m_{\omega f} + a_{\omega f} * \Delta t \tag{28}$$

$$m_{\omega o} = m_{\omega o} + a_{\omega o} * \Delta t \tag{29}$$

$$m_{x\theta f} = m_{x\theta f} + X_{\omega f} \tag{30}$$

$$m_{x\theta o} = m_{x\theta o} + X_{\omega o} \tag{31}$$

Here, m_{ho} represents the total displacement of the oscillator in heaving and pitching motions, m_{hf} represents the total displacement of the buoy in heaving and pitching motions, $m_{\omega f}$ is the updated equation for the buoy's angular velocity (total angular velocity), $m_{\omega o}$ is the updated equation for the oscillator's angular velocity (total angular velocity), $m_{x\theta f}$ is the total pitch angle displacement of the buoy, and $m_{x\theta o}$ is the total pitch angle displacement of the oscillator[10].

Due to the torque experienced by the oscillator during heaving and pitching motions, and considering the total spring length to the center of mass of the oscillator is 0.75 (0.5 + 0.25), the actual displacement analysis of the oscillator under these motion conditions is illustrated in Figure 1.

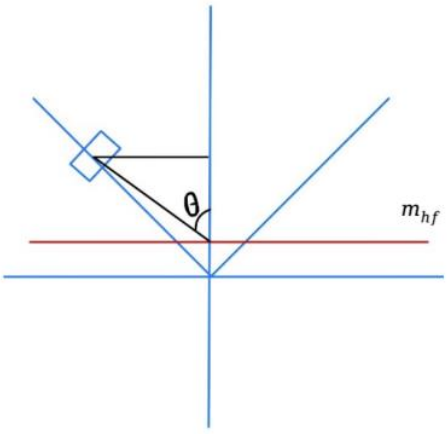


Figure 1. Displacement Analysis Diagram of the Oscillator During Heaving and Pitching Motions

$$m_{ho} = (0.75 + m_{ho}) * \cos(\theta_o) - X_{hf} \tag{32}$$

$$m_{x\theta o} = arctan \frac{(0.75+m_{ho})*\sin(\theta_o)}{(0.75+m_{ho})*\cos(\theta_o)-m_{hf}} \tag{33}$$

$$m_{ho} = \frac{m_{ho}}{\cos(\theta_o) - 0.75} \tag{34}$$

Here, m_{ho} is the total displacement of the oscillator in heaving and pitching motions, m_{hf} is the total displacement of the buoy in heaving and pitching motions, m_{wf} is the updated equation for the buoy's angular velocity (total angular velocity), m_{wo} is the updated equation for the oscillator's angular velocity (total angular velocity), $m_{x\theta f}$ is the total pitch angle displacement of the buoy, and $m_{x\theta o}$ is the total pitch angle displacement of the oscillator.

3. Result

Simulate the model equation using Java:

By setting the damping coefficients of the linear and rotational dampers as constants, denoted as c_L and c_R respectively, the equation is solved. The data results are visualized using Python, and the solution is shown in the figure below:

From Figures 1 and 2, the heave displacement and velocity variation curves of the buoy can be determined within the time interval [0, 100] s.

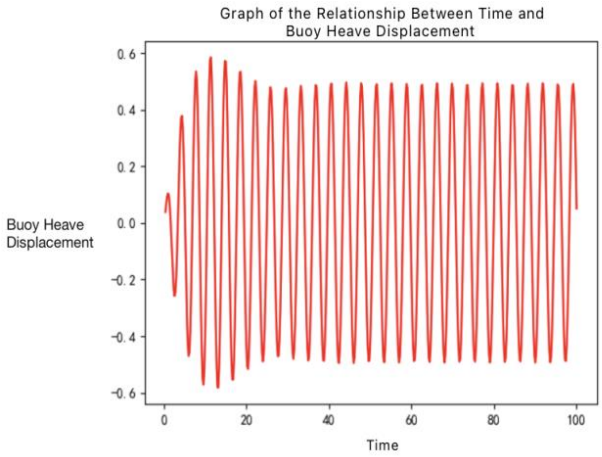


Figure 2. Graph of the Relationship Between Time and Buoy Heave Displacement

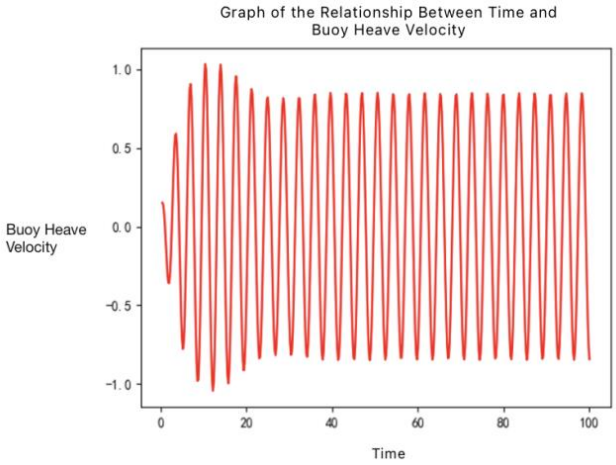


Figure 3. Graph of the Relationship Between Time and Buoy Heave Velocity

From Figures 3 and 4, the pitch angle displacement and pitch angle velocity variation curves of the buoy can be determined within the time interval [0, 100].

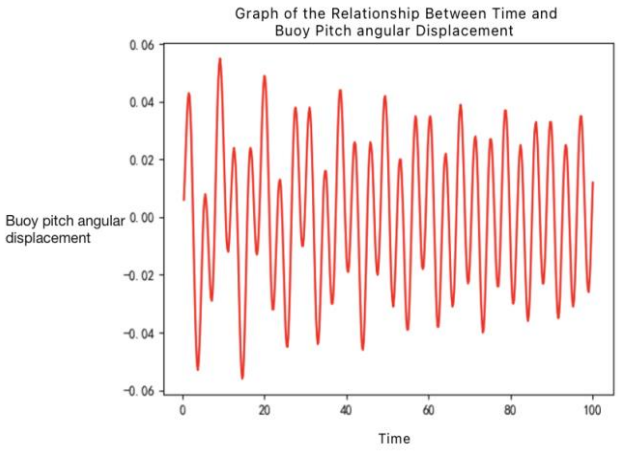


Figure 4. Graph of the Relationship Between Time and Buoy Heave Velocity

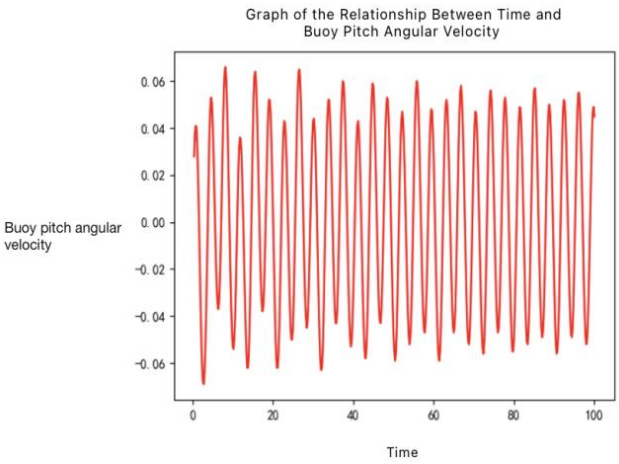


Figure 5. Graph of the Relationship Between Time and Buoy Pitch Angular Velocity

From Figures 5 and 6, the heave displacement and heave velocity variation curves of the oscillator can be determined within the time interval [0, 100] s.

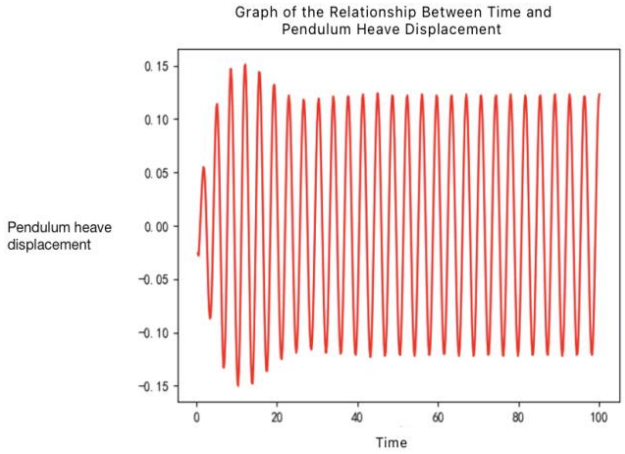


Figure 6. Graph of the Relationship Between Time and Pendulum Heave Displacement

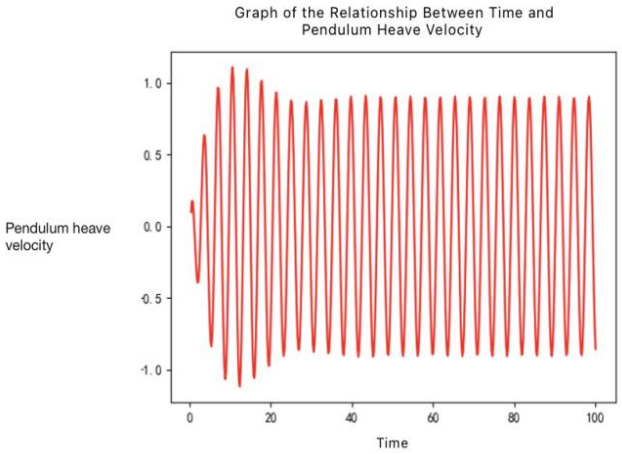


Figure 7. Graph of the Relationship Between Time and Pendulum Heave Velociy

From Figures 7 and 8, the pitch angle displacement and pitch angle velocity variation curves of the oscillator can be determined within the time interval [0, 100] s.

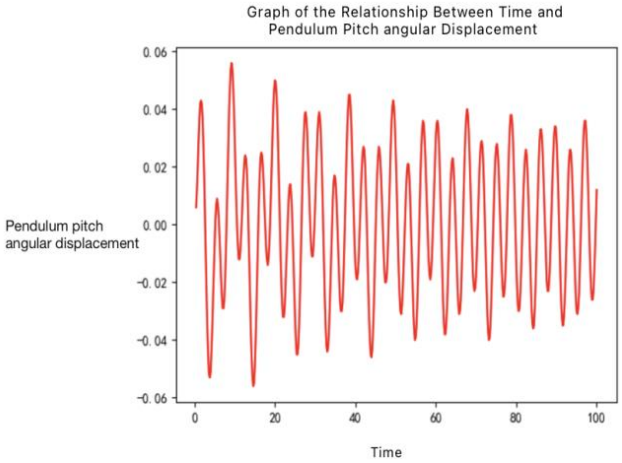


Figure 8. Graph of the Relationship Between Time and Pendulum Pitch angular Displacement

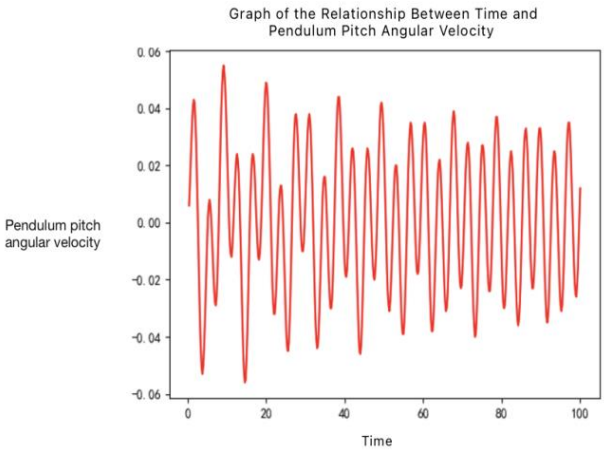


Figure 9. Graph of the Relationship Between Time and Pendulum Pitch angular Velocity

Establish the optimal damping coefficient model for linear dampers and rotational dampers.

From the above, it can be known that the mathematical equations for work done by the float and the oscillator in vertical pendulum motion and longitudinal sway operation are as follows:

$$P_{hw} = |xh_o * F_{hdc}| \tag{35}$$

$$P_{pw} = \left|\left(x_{wf} - x_{wo} * F_{hfs}\right)\right| \tag{36}$$

Consequently, the work update equation can be derived as:

$$P_{twp} = P_{twp} + P_{hw} + Ppw \tag{37}$$

Model Solution

To simulate the model data using Java:

As the problem states, the range of values for the damping coefficient of the linear damper, d_{dc}, and the rotational damper, d_{cc}, is [0, 100000].

An exhaustive search method is applied to determine the values for d_{dc} and d_{cc}, with a step size of 400 for both. From a practical standpoint, the values of d_{dc} and d_{cc} are meaningless when set to 0, hence the initial values are both set to 400.

The data visualization result using Python is shown in Figure 10:

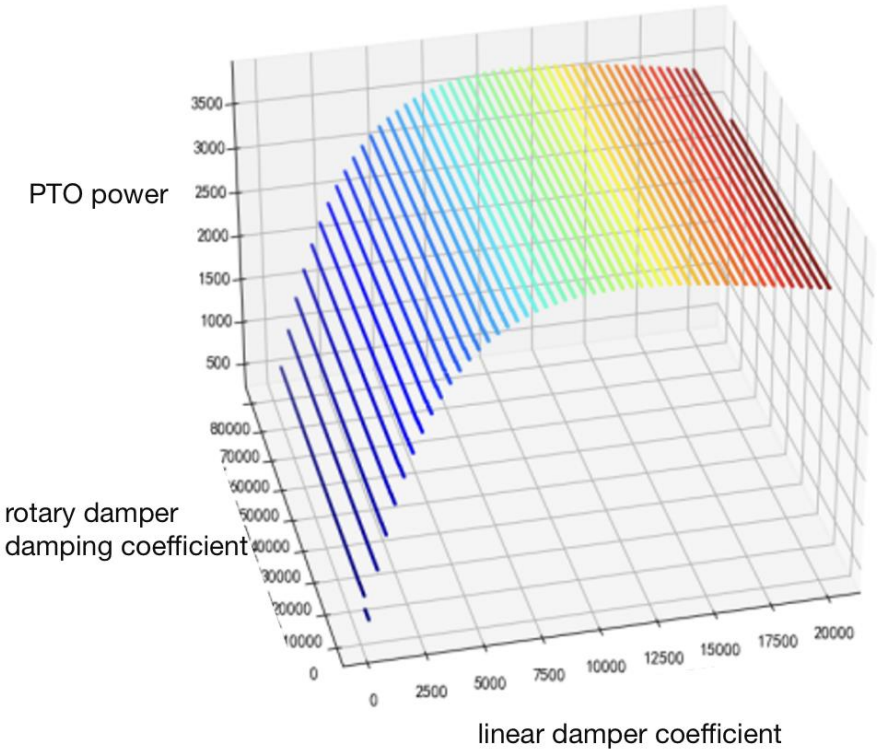


Figure 10. Scatter plot of linear damper coefficient and rotary damper coefficient versus PTO device power

Optimal damping coefficient for the linear damper: 11,200 N·s/m
Optimal damping coefficient for the rotational damper: 400 N·s/m
Best power: 373,253.6

4. Conclusion

In this section, the research focused on establishing a mathematical model for a wave energy device undergoing heaving motion. The study began with a force analysis, establishing the force equations for both the float and the oscillator. Subsequently, Newton's second law was applied to solve for the relevant accelerations. Through differential equations, the velocity was further derived, and displacement equations were established, thereby solving for the trajectories of displacement over time and velocity over time. This part of the research provides a foundation for the performance analysis of wave energy devices under heaving motion.

Future research can build upon this model to further explore and optimize the design of wave energy devices, enhancing their efficiency and stability in practical applications. Although this model provides a mathematical basis for the heaving motion of wave energy devices, there may be limitations and shortcomings in practical applications. For instance, the model may not have considered all possible real-world environmental factors, such as ocean currents and wind speeds, which could affect the actual performance of the device. Additionally, the model overlooked the mass of the shaft, base, intermediate layers, and Power Take Off (PTO), as well as various types of friction when it was established, which might lead to deviations between predicted results and actual conditions in certain cases. Future research can further optimize the model by including these factors.

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