Facing Environmental Uncertainties: A Submersible Path Prediction Model Based on Kalman Filter

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Abstract. Covering two-thirds of the Earth's surface, the oceans are abundant in natural resources, presenting significant opportunities for exploration and exploitation. This underlines the importance of enhancing the positioning accuracy and safety of submersibles, which serve as essential instruments for oceanic exploration and study. Facing the complexities of the marine environment and the inherent challenges of exploration, this paper focuses on the development of an advanced submersible positioning model. By integrating a thorough analysis of ocean physics and dynamics with the unique environmental features of the Ionian Sea, a tributary of the Mediterranean Sea, this model is established. At its core, the Kalman filter algorithm is employed, enabling precise prediction and estimation of a submersible's real-time position within challenging marine settings. This capability allows for the submersible's navigational direction and speed to be automatically adjusted based on predicted positional changes, effectively addressing environmental challenges and ensuring both accuracy in positioning and navigational safety. Such advancements offer crucial technical support for ocean exploration and significantly improve the autonomous navigation of submersibles in uncharted and intricate marine environments.

Keywords: Submersible positioning model, Ionian Sea, Kalman filter algorithm, difference equations.

1. Introduction

In the current era, marked by the rapid advancement of science and technology, marine exploration has escalated in frequency, demanding enhanced capabilities in marine engineering technologies. The precision in positioning submersibles, pivotal tools for marine exploration, is critically linked to the effectiveness and efficiency of various activities, including marine resource exploration [1], seabed topographic mapping, biodiversity studies, and underwater archaeology. Thus, the enhancement of submersibles' positioning accuracy and their predictive capabilities emerges as a key concern within marine research. Traditional technologies for submersible positioning confront numerous challenges, such as the dynamic and complex marine environment, the high density and conductivity of seawater, signal interference, and the intense pressure found in deep-sea conditions. These challenges can result in inaccurate and delayed positioning data, consequently impeding the mission's efficiency and safety. This paper underscores the necessity for novel technological solutions capable of real-time, precise submersible position prediction, aiming to significantly improve the efficiency and safety of oceanic explorations.

The Kalman filter algorithm stands as a formidable instrument for system state estimation, boasting extensive utility across various domains, such as navigation systems[2] ,drones, self-driving vehicles [3] ,signal processing[4] ,and robotics[5] .The flexibility of this algorithm allows for innovative adaptations or integrations with other methodologies and domains of knowledge to meet specific operational demands. Instances include decentralized Kalman filtering for multi-UAV cooperative positioning, which facilitates navigation information correction amidst significant interference[6] ,and the enhancement of attitude accuracy for small UAVs during extensive maneuvers or sustained flight through extended Kalman filtering [7] .Owing to its proficiency in amalgamating historical and real-time sensor data, this paper leverages the Kalman filter algorithm in conjunction with fluid physics and mechanics. This integration facilitates the real-time estimation and prediction of a submersible's

position through precise mathematical modeling, significantly elevating the accuracy and dependability of positioning assessments. By simulating conditions within an actual marine environment, the effectiveness and stability of this approach are validated, offering a novel technological avenue for augmenting future oceanic exploratory missions.

The focus of this paper is the Ionian Sea, which is an important tributary sea of the Mediterranean Sea, as shown in Figure 1.

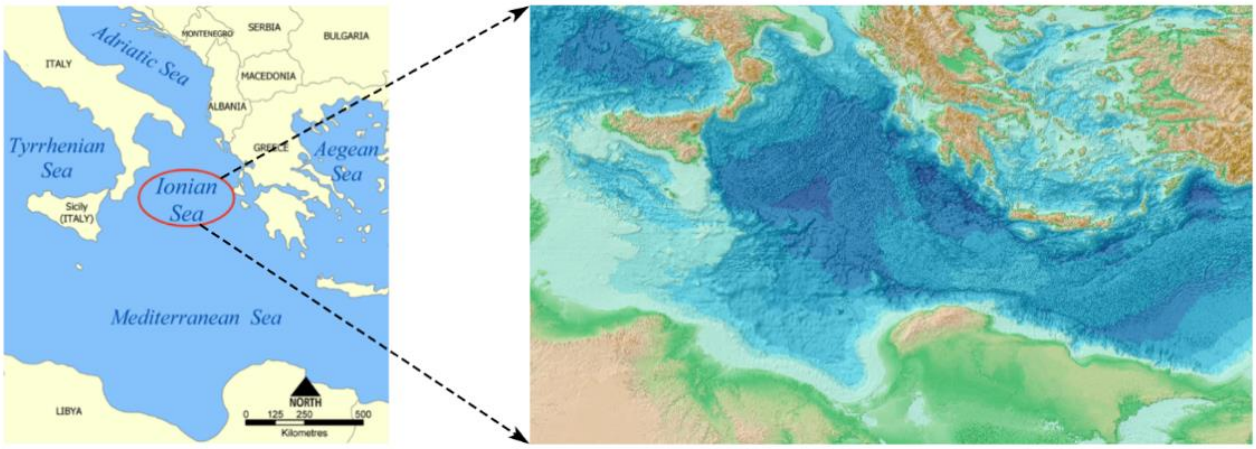


Figure 1. Geographic position of the Ionian Sea

By collecting seafloor topographic data in this sea, this paper utilizes ArcGIS Pro, an advanced GIS software, to create a detailed digital elevation map of the seafloor. This map not only provided us with an accurate view of the complex topography of the sea but also helped us to gain a deeper understanding of the environmental conditions in the region. This information was essential for evaluating the various environmental factors that may affect the navigation of the submersible, thus enabling us to build an accurate three-dimensional force analysis model based on the actual seabed environment. To further improve the accuracy of the positioning of the submersible in the complex seabed terrain, a Kalman filter algorithm is used to establish a dynamic submersible positioning model. By setting the initial motion state of the submersible, the model adopts a continuous prediction-update iteration process to effectively predict the position of the submersible over time.

2. Model Preparation and Construction

To accurately model the underwater positioning of a submersible, it is crucial to understand every detail of the underwater environment. In this process, this paper has collected and analyzed the climate, ocean currents, density, and temperature distribution of seawater[8] ,as well as the topography of the seabed in the Ionian Sea. Then the underwater forces of the submersible are systematically analyzed using physics and the characteristics of the marine environment.

2.1. Data collection

The environment around the submersible includes the following factors.

(1) Seawater density significantly influences the buoyancy and propulsion efficacy of the submersible, thereby directly affecting its operational depth and performance. The principle is straightforward: the higher the seawater density, the greater the buoyancy exerted on the submersible. This factor not only determines the maximum achievable depth of the submersible by affecting its ability to counteract the increased underwater pressure but also impacts its propulsive capabilities. Specifically, as seawater density rises, navigating the submersible becomes more challenging due to the enhanced resistance, which in turn affects its speed and agility. Thus, understanding and accommodating seawater density variations are crucial for optimizing submersible operations in varying marine environments.

(2) Seawater temperature: seawater temperature affects the density of seawater; seawater temperature will have an impact on the internal electronic equipment of the submersible and the environment of the cabin, especially at extreme temperatures.

(3) Seafloor topography: on an uneven and irregular seafloor, the submersible needs to have good stability and coping ability to avoid dangerous situations such as tilting, side-slipping or getting stuck.

All data sources are shown in Table 1.

Table 1. Data and Database Websites

Data	Database Websites
ocean currents	https://earth.nullschool.net/
Seafloor Geography	https://freegisdata.rtwilson.com/
Maps	https://www.wikipedia.org/ https://download.gebco.net/
Topographic Maps	https://www.openstreetmap.org/

2.2. Data Visualization

In this paper, the latitude $35°\sim36°$ North and longitude $17°\sim18°$ East were selected as the study area. And the Ionian Sea was visualized using ArcGIS Professional (ArcGIS Pro) software.

Professional (ArcGIS Pro) software to create a digital elevation topographic map of the seabed of the Ionian Sea, and Origin software to create a 3D stacked map, from bottom to top, the planar geographic map, three-dimensional elevation map, contour map, gradient three-dimensional topographic map, as shown in Figs. 2 and 3.

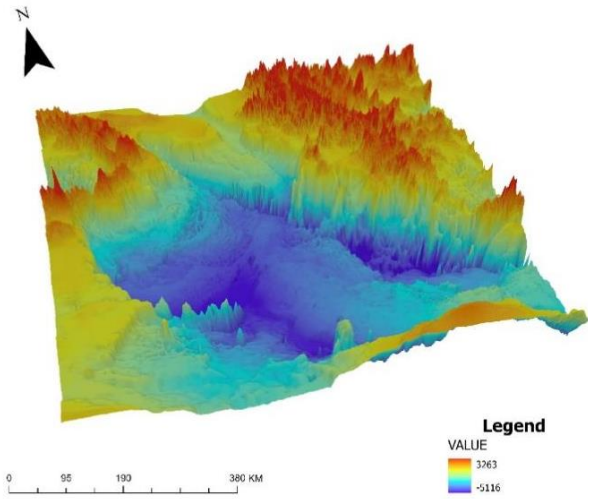


Figure 2. Ionian Sea Elevation Model

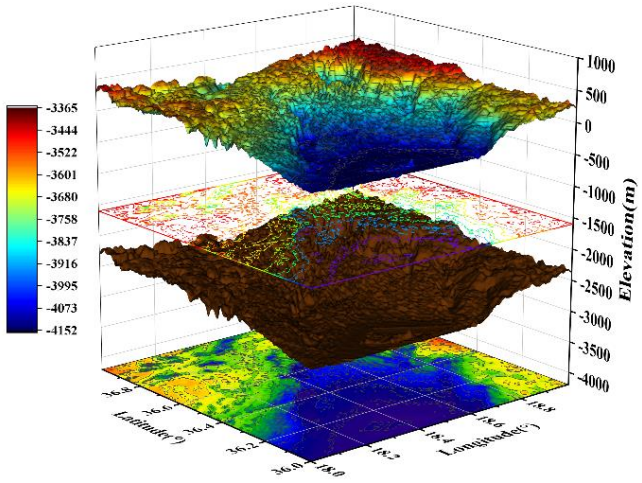


Figure 3. 3D Stacking Diagram

As can be seen in Figs. 2 and 3, the Ionian Sea is mostly over 3,000 m deep, with the deepest part of the eastern part reaching over 5,000 m, and the seafloor topography is complex.

2.3. Dynamic modeling

Given the basic situation in the sea, our team stands in the perspective of fluid physical mechanics and dynamics, based on this task, the model is further assumed as follows to simplify the problem.

Assumption 1: The submersible is a cylinder. The height is h , and the radius is r .

Assumption 2: The ocean current V_C and depth $|Z|$ change less and less, and have the following relations.

$$V_C = \frac{2}{|Z|+1} \ m/s \tag{1}$$

Assumption 3: Under normal conditions, the submersible can be controlled by the engine and the water discharge system to have constant acceleration, i.e., to do an approximate uniform acceleration.

Assumption 4: The initial position is random, and the direction of engine thrust is consistent with the direction of ocean current.

Based on the above assumptions, a brief force analysis of the submersible is shown in Figure 4.

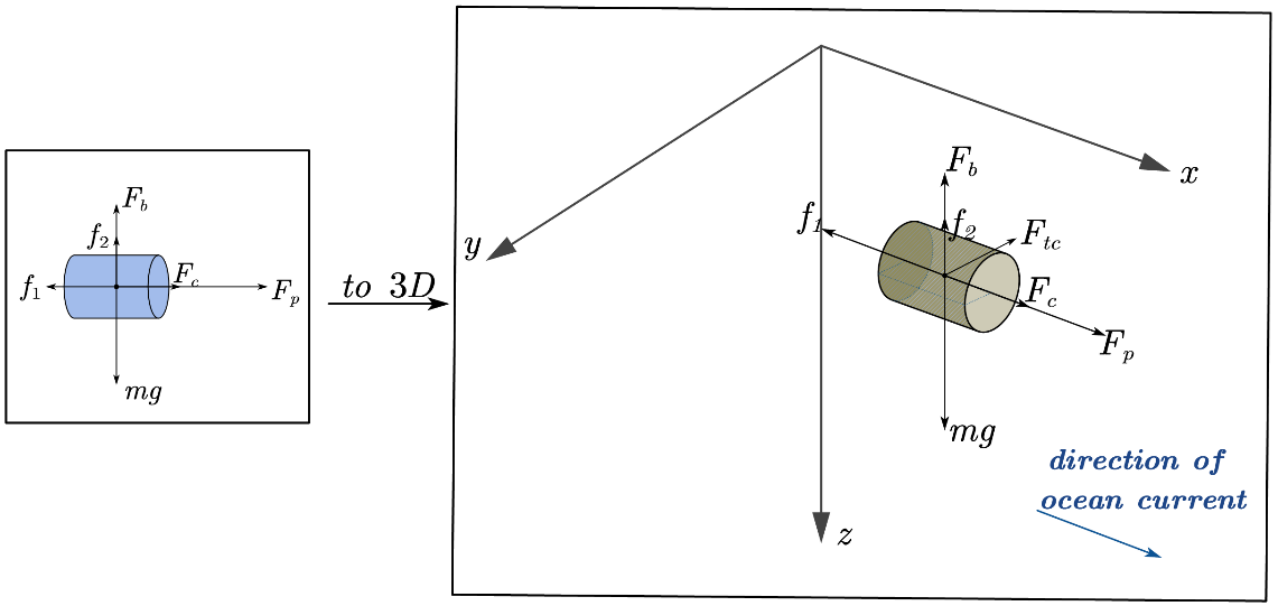


Figure 4. Force analysis of underwater submersibles

The seawater increases with the increase of ocean depth, and the relationship between density and depth can be found in the relevant literature as follows. $\rho = \sqrt{1.024H} + 100$

Let the position state variable $\vec{x} = (x, y, z, v_1, v_2, v_3)$ be, which where x, y, z denotes the three-dimensional coordinates of the submersible, and $v_1\ v_2, v_3$ denotes the velocity of the submersible in each dimension, then according to Newton's second law $F = ma$ and the force analysis of Figure 4 can be listed as follows.

$$\begin{cases} \dfrac{F_p + F_c - f_x}{m} = a_x = \dfrac{dv_1}{dt} \\ \dfrac{F_{tc}}{m} = a_y = \dfrac{dv_2}{dt} \\ \dfrac{mg - f_z - F_b}{m} = a_z = \dfrac{dv_3}{dt} \end{cases} \tag{2}$$

$$f_i = \frac{1}{2}\rho C S |v_i|^2 \tag{3}$$

Where F_p is the engine thrust, F_c is the current thrust, pointing to the X direction, and consistent with the direction of the engine thrust. m is the mass of the submersible, which changes with the draught and discharge, and f_x is the resistance to motion in the X direction. F_b denotes the turbulent

force in the sea, f_z is the resistance to motion in the Z direction, the damping coefficient, and the waterfront area.

The motion of the submarine can be approximated as a uniformly accelerated straight-line motion within a relatively small Δt area. Therefore, according to the kinematic equation, in are latively small Δt, there is the following differential relation.

$$\begin{cases} x_t = x_{t-1} + v_{1,t-1}\Delta t + \frac{1}{2}a_{1t}\Delta t^2 \ , \ v_{1t} = v_{1,t-1} + a_{1t}\Delta t \\ y_t = y_{t-1} + v_{2,t-1}\Delta t + \frac{1}{2}a_{2t}\Delta t^2 \ , \ v_{2t} = v_{1,t-1} + a_{1t}\Delta t \\ z_t = z_{t-1} + v_{3,t-1}\Delta t + \frac{1}{2}a_{3t}\Delta t^2 \ , \ v_{3t} = v_{3,t-1} + a_{3t}\Delta t \end{cases} \tag{4}$$

3. Kalman Filter Optimization Strategies for Dynamic Submersible Positioning

3.1. Modeling based on the Kalman Filter Algorithm

The team will model the submarine from the following two scenarios.

- Submersible with the normal power system.

The effects of drag, buoyancy, currents, etc. can be filtered, i.e., uniformly accelerated motion. Normal state prediction.

- In the case of damage to the power system and the inlet and outlet systems.

In this case, only the initial velocity, turbulence, ocean currents, drag, buoyancy, etc. before the damage are affected, and cannot be ignored. The damage state prediction is carried out.

3.1.1 Introduction of the model

Based on the derivation of the differential equation, it can be found that the linear relationship between the front and rear positions of the submarine within a relatively small area Δt is obvious. Based on this finding, to accurately solve the submersible positioning problem, this paper decides to use the Kalman filter algorithm.

Kalman filtering is an efficient recursive state estimation algorithm, which can estimate the actual state of the dynamic system from incomplete measurement data under the condition of noise interference. In the iterative process of Kalman filtering, the state estimation for the current moment is not only based on the state estimation and dynamic model of the previous moment but also may include the control inputs and current observation data, which makes the state estimation more accurate and reliable. The iterative core process includes the following steps.

1) **Prediction:** Based on the previous state estimation, the state at the current moment is predicted using the dynamic model of the system and the control inputs (if any).

2) **Update:** Using the observed data at the current moment, calculate the Kalman gain by comparing the difference between the predicted value and the observed value, and adjust the predicted value by the Kalman gain to get a more accurate state estimation at the current moment.

3) **Recursive iteration:** The above two steps are carried out recursively, each iteration uses the state estimate of the previous moment to predict the state of the current moment, and the well updates the state estimate by the observation data of the current moment. In this way, the Kalman filter gradually improves the estimation accuracy of the state of the system during the iterations.

The following will be elaborated in detail with the specific situation of this problem.

3.1.2 Modeling

Step1: Preparation.

Establish the dynamic model. It can be matrixed from Eq. (5) as follows.

$$\begin{pmatrix} x_t \\ y_t \\ z_t \\ v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_{t-1} \\ y_{t-1} \\ z_{t-1} \\ v_{1,t-1} \\ v_{2,t-1} \\ v_{3,t-1} \end{pmatrix} + \begin{pmatrix} \frac{\Delta t^2}{2} & 0 & 0 \\ 0 & \frac{\Delta t^2}{2} & 0 \\ 0 & 0 & \frac{\Delta t^2}{2} \\ \Delta t & 0 & 0 \\ 0 & \Delta t & 0 \\ 0 & 0 & \Delta t \end{pmatrix} \begin{pmatrix} a_{1t} \\ a_{2t} \\ a_{3t} \end{pmatrix} \tag{5}$$

Where F denotes the state transfer matrix, B denotes the input matrix, and $\vec{u}$ denotes the control input vector.

From the above, the paper can establish the dynamic transfer equation: $\vec{x}_t = F\vec{x}_{t-1} + B\vec{u}$. Let $L_t = (x,y,z)_t^T$ denote the position of the submersible at t.

The following measurement equation can be constructed, where ε_t denotes the measurement noise and H denotes the observation equation.

$$L_t = (x,y,z)_t^T + \varepsilon_t = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} x_t^T + \varepsilon_t = Hx_t^T + \varepsilon_t \tag{6}$$

To further simplify the model, the initial covariance matrix is $P = I$, the covariance matrix of measurement noise is $R = I$, and the covariance matrix of process noise is $Q = 0.1I$, where I denotes the unit matrix.

Step2: Iterative recursion

This step consists of two main steps: the prediction step (Predict) and the update step (Update) (which is defined as described in the introduction to the model), which alternate at each moment of the system, ultimately forming an iterative recursive process of Kalman filter. To better visualize this process, a flow chart of the algorithm for this process is produced in this paper, as shown in Figure 5.

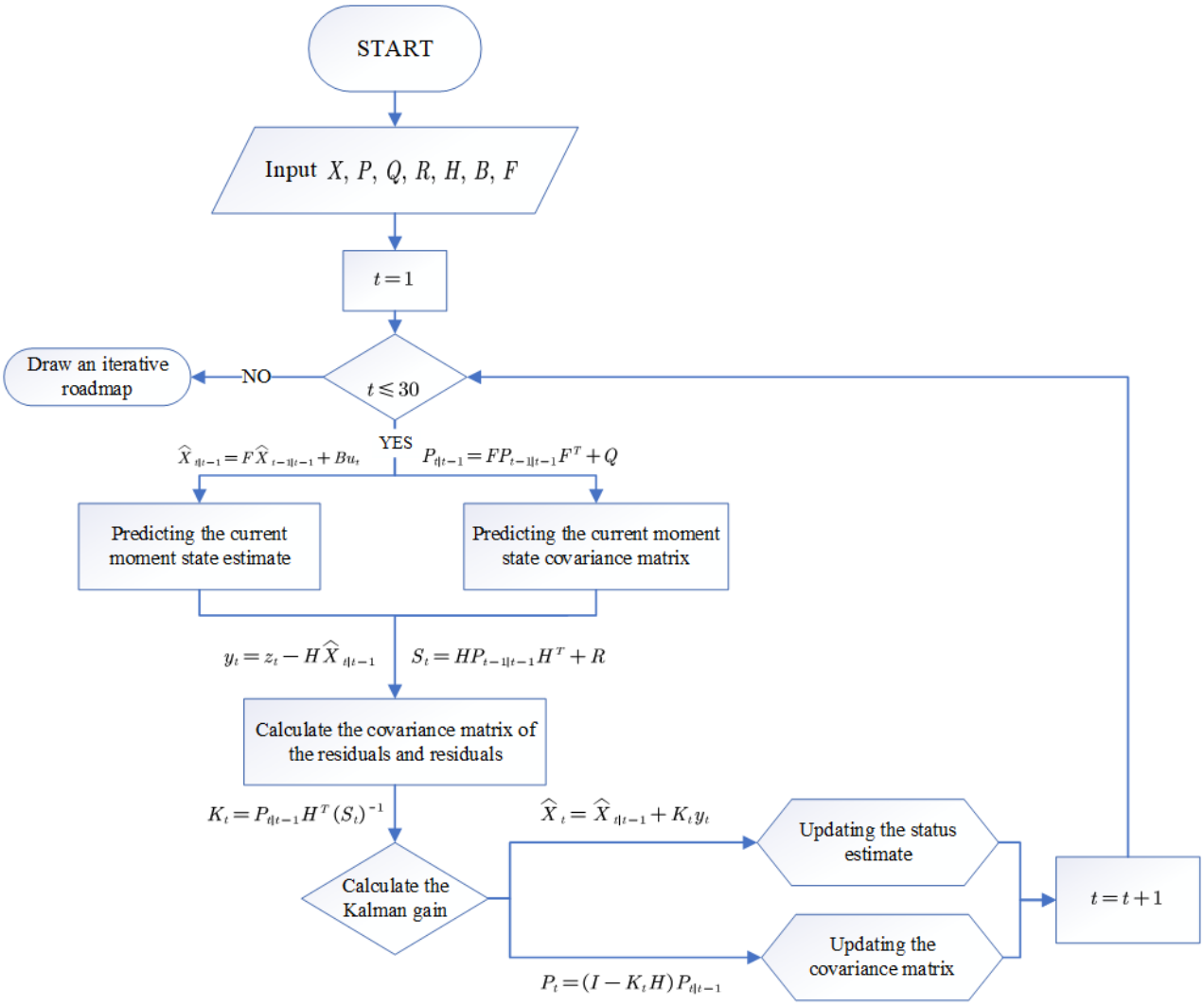


Figure 5. Flow chart of Kalman linear filtering

Where $\hat{\hat{x}}_{t|t-1}$ is the estimated value of the state of the submersible at the time t, $P_{t|t-1}$ is the covariance matrix of the state of the submersible at the time t, R is the covariance matrix of the process noise to characterize the uncertainty of the system dynamics model, K is the Kalman benefit, y_t is the residual, and S_t denotes the covariance matrix of the residual. $\hat{\hat{x}}_t$ denotes the estimated value of the diver state at time t, and P_t is the covariance matrix of the diver state at time t. The covariance matrix of the diver state at time t is the covariance matrix of the process noise.

3.2. results

Using Python to run the Kalman filter algorithm, the final results are shown in Figure 6.

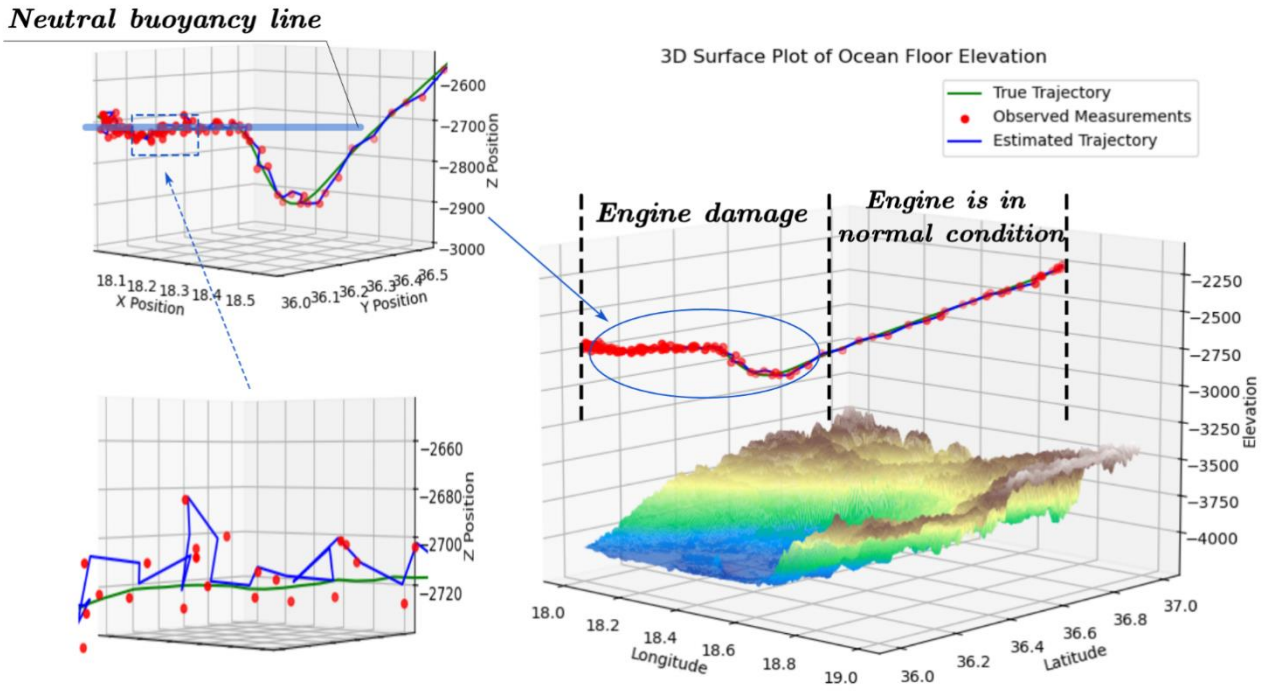


Figure 6. Trajectories predicted by Kalman filter

In the figure 6, the green solid line is the simulated real trajectory, the blue solid line is the predicted path of the submersible, and the red dots represent the predicted path nodes. When the initial launching position of the submersible is given, the predicted path of its position over time can be obtained. the paper simulates the uniformly accelerated motion under normal power, which is approximated as a linear motion after considering the process error. Meanwhile, after the power is damaged, the inertia effect of the initial velocity, the influence of the ocean current, and the drag force will become more significant to its motion state. However, after a certain period, the damaged submersible will return to the Neutral buoyancy surface, and the horizontal resistance will cause it to decelerate and finally move randomly. It can be found that the prediction results of the Kalman filter are still good despite the large fluctuation after the random motion.

After several simulation experiments, we conclude that the mean deviation between the predicted and actual positions of the submersible is no more than 1km, and the variance is very small. It can be concluded that the model is stable and can locate the position of the submersible more accurately.

4. Conclusions

Amidst economic growth and scientific advancements, the utilization of scientific methodologies to safeguard undersea exploration endeavors has emerged as both a prevalent and pivotal necessity. Against this backdrop, this paper has formulated a precise localization model leveraging the Kalman filtering technique. The initial step involved gathering essential environmental data, such as currents and seawater density within the Ionian Sea, followed by the creation of an accurate seafloor Digital Elevation Model (DEM) via ArcGIS Pro software, thereby replicating the seabed landscape. Subsequently, this study melded hydrodynamic principles and physical mechanics to scrutinize the forces acting upon the submersible within the intricate marine milieu. The culmination of this research was the establishment of a dynamic force model and the application of the Kalman filter algorithm, enabling the precise prediction of the submersible's trajectory over time, significantly enhancing the safety of seabed exploratory activities.

This investigation not only unfolds a novel research idea and framework but also validates its applicability in crucial sectors such as marine search and rescue. By fusing sophisticated GIS technology with the Kalman filter algorithm, this paper has not only heightened the precision of

submersible positioning but also fortified technical support for marine rescue operations. Consequently, this research paves the way for profound exploration and practical engagement in related domains, highlighting the vast potential of marine science and technology in advancing human well-being.

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