

Study on the Impact of Sex Ratio Changes in Lamprey on the Ecosystem and Its Stability

Bin Shen^{1,*}, Ruiqi Li²

¹College of Information Science and Engineering, HoHai University, Nanjing, China, 213200

²School of Electrical and Power Engineering, HoHai University, Nanjing, China, 213200

*Corresponding author: 2262410111@hhu.edu.cn

Abstract. Different species adopt different survival strategies. For lampreys, adaptive changes in sex ratio are crucial for their survival and reproduction. Then, we aim to find out how the change of lampreys' sex ratio affect the ecosystem. Firstly, by expanding the Lotka-Volterra equations to three species and species competition models, we find that an increase in the male-female ratio of lampreys under different food resources leads to higher peaks of population, more pronounced fluctuations, and greater threats to their prey. Secondly, applying a method to calculate stability index with species' biomass, we discussed the impact of lampreys' sex ratio changes on ecosystem stability with species that are ranked by PageRank algorithm. When male-female ratio increases, the ecosystem tends to become unstable. In conclusion, adaptive changes in lampreys' sex ratio has a great effect on their survival and reproduction. Therefore, studying the effects of lamprey sex ratio changes on ecosystems will contribute significantly to addressing the current issues.

Keywords: Improved Differential Equation Model; PageRank algorithm; S-B-P Model.

1. Introduction

In recent years, the reappearance of lampreys in lakes has caused devastating damage to lake trout and other economically important fish species [1][2]. Lampreys being contributors to the intricate complexity of ecosystems, the regulatory mechanisms of them have significant impacts on themselves, other species, and the entire ecosystem. This study mainly focuses on the effects of sex ratio changes of lampreys on other species and ecosystem stability.

When studying the competition between lampreys and other species, we consider using the Lotka-Volterra equations [3]. However, since its application is limited to only two species which can't reflect the impact of sex ratio on the results of species' population. Therefore, we refer to the research by Hilal N, Wang SY, and others [4][5] and extend the Lotka-Volterra equations to a system of three species differential equations, distinguishing between male and female lampreys.

By constructing predator-prey relationships in the food web [6], we learn how to analyze the stability of complex food webs [7] and build an ecosystem composed of species. We learn the principles and formulas of the PageRank algorithm analyzed by Sharma PS et al. [8] to rank the importance of species in the food web. Understanding the sustainability of ecosystems largely depends on understanding ecological stability. Although there are various methods for assessing ecosystem stability, stability indices are still widely used in micro and macroecological studies [9]. Parparov proposed a method for calculating the stability index based on lakes' ecology and employed it to quantify the stability of the Lake Kinneret phytoplankton community [10]. We use this method combined with important species ranked by PageRank algorithm to calculate the stability index to evaluate changes of ecosystem stability.

2. The basic fundamental of differential equation model and S-B-P Model

2.1. The structure of improved predator-prey model

The Lotka-Volterra equations, also known as the predator-prey equations, consist of two first-order nonlinear differential equations. They are commonly used to describe the dynamics of predator

and prey populations interacting in an ecosystem, depicting the fluctuations in the population sizes of both species. The classic Lotka-Volterra equations are shown as equation (1):

$$\begin{cases} \frac{dx_1}{dt} = x_1 \times [(r_1 - e) - \lambda_1 \times x_2] \\ \frac{dx_2}{dt} = x_2 \times [-(r_2 + e) + \lambda_2 \times x_1] \end{cases} \quad (1)$$

We make some changes to the above differential equations, dividing the predators into male predators and female predators, and establishing a predator-prey model regarding the relationship between male predators, female predators, and prey, as shown in equation (2):

$$\begin{cases} \frac{dx_1}{dt} = x_1 \times [(r_1 - e) - \lambda_2 \times x_2 - \lambda_3 \times x_3] \\ \frac{dx_2}{dt} = x_2 \times [-(r_2 + e) + \lambda_1 \times x_1] \\ \frac{dx_3}{dt} = x_3 \times [-(r_3 + e) + \lambda_1 \times x_1] \end{cases} \quad (2)$$

where x_1, x_2, x_3 respectively represent the numbers of prey, female predators, and male predators; r_1, r_2, r_3 respectively indicate the growth rate of prey, the self-extinction rate of female predators, and the self-extinction rate of male predators; $\lambda_1, \lambda_2, \lambda_3$ respectively denote the prey's provisioning capacity to predators, the ability of female predators to capture prey, and the ability of male predators to capture prey; e represents human impact, such as fishing.

2.2. The structure of improved population competition model

When there are two or more populations within an ecosystem, they may engage in competitive interactions. Population competition model is often used to describe the quantitative relationships between competing populations within an ecosystem. The classical population competition model is represented by equation (3):

$$\begin{cases} \frac{dx_1}{dt} = r_1 \times x_1 \times \left(1 - \frac{x_1}{N_1} - \sigma_1 \frac{x_2}{N_2}\right) \\ \frac{dx_2}{dt} = r_2 \times x_2 \times \left(1 - \frac{x_2}{N_2} - \sigma_2 \frac{x_1}{N_1}\right) \end{cases} \quad (3)$$

We make some changes to the above differential equations, dividing the predators into male predators and female predators, and establishing a population competition model regarding the relationship between male predators, female predators, and competitors, as shown in equation (4):

$$\begin{cases} \frac{dx_4}{dt} = r_4 \times x_4 \times \left(1 - \frac{x_4}{N_4} - \sigma_2 \times \frac{x_2}{N_2} - \sigma_3 \times \frac{x_3}{N_3}\right) \\ \frac{dx_2}{dt} = r_2' \times x_2 \times \left(1 - \frac{x_2}{N_2} - \sigma_4 \times \frac{x_4}{N_4}\right) \\ \frac{dx_3}{dt} = r_3' \times x_3 \times \left(1 - \frac{x_3}{N_3} - \sigma_4 \times \frac{x_4}{N_4}\right) \end{cases} \quad (4)$$

where x_4, x_2, x_3 respectively denote the numbers of competitors, female predators, and male predators; r_4, r_2', r_3' respectively indicate the growth rates of competitors, female predators, and

male predators; N_4, N_2, N_3 respectively indicate the carrying capacities of competitors, female predators, and male predators in the ecosystem; $\sigma_4, \sigma_2, \sigma_3$ respectively represent the ratio by which one population's consumption of food per unit affects the food supply for another population, relative to its own consumption of food per unit.

2.3. The structure of S-B-P Model

2.3.1 SI Criteria for Evaluating Ecosystem Stability

The evaluation of ecological system stability requires comprehensive consideration of multiple factors such as species diversity, functionality, and structural characteristics, with numerous methods available. Here, we adopt the evaluation criteria based on Euclidean distance proposed by scholars. The definition of the Stability Index (SI) for ecosystem is as follows:

$$SI = \frac{1}{1 + EuD_i} \quad (5)$$

where EuD_i represents the standardized Euclidean distance for the i -th species population from its current state to its reference state. The reference state of a population denotes the value corresponding to the state of the ecosystem when it is relatively stable ($SI = 1$). When SI approaches 1, the stability of the ecosystem is higher. The calculation method for standardized Euclidean distance is as follows:

$$EuD_i = ABS\left(\frac{\sqrt{B_i} - \sqrt{Ref(B_i)}}{STD(B_i)}\right) \quad (6)$$

where ABS represents the absolute value; B_i indicates the biomass of the current state of the i -th species population; $Ref(B_i)$ denotes the biomass of the reference state of the i -th species population; $STD(B_i)$ indicates the standard deviation of the biomass of the current state for the i -th species population. Here, the selected indicator value for the current state is biomass.

Of course, a complete ecosystem contains multiple communities, each composed of several species populations. Therefore, knowing the stability index of just one species population does not adequately reflect the stability index of the entire ecosystem. The revised calculation method for the Euclidean distance EuD_{New} and the stability index SI_{New} is as follows:

$$EuD_{New} = \sqrt{\sum_i (EuD_i)^2} \quad (7)$$

$$SI_{New} = \frac{1}{1 + EuD_{New}} \quad (8)$$

2.3.2 Finding core populations based on the PageRank algorithm

The population size in ecosystems is extremely large, making it impractical to calculate the Euclidean distance for all population biomasses based on the above criteria. Therefore, it is essential to identify the core populations that have the greatest impact on the ecosystem. Food chains and food webs effectively illustrate the interactions between populations at different trophic levels. Applying the PageRank algorithm to them can identify core populations.

The PageRank algorithm is a ranking algorithm used by the Google search engine for search results. It views the internet as a directed graph, with each webpage being a vertex of the graph, and hyperlinks between webpages being edges of the graph. The algorithm proceeds as follows:

Construct the adjacency matrix $B = (b_{ij})_{N \times N}$. N is the number of vertices. The method of assigning values to edge b_{ij} is defined as follows:

$$b_{ij} = \begin{cases} 0, & \text{there is a hyperlink from webpage } a \text{ to } b \\ 1, & \text{there is no hyperlink from webpage } a \text{ to } b \end{cases} \quad i, j = 1, 2, \dots, N \quad (9)$$

Calculate the transition probability matrix of the Markov Chain, where:

$$a_{ij} = \frac{1-d}{N} + d \frac{b_{ij}}{r_i}, \quad i, j = 1, 2, \dots, N \quad (10)$$

According to the fundamental properties of Markov chains, for a regular Markov chain, there exists a stationary distribution $x = [x_1, x_2, \dots, x_N]^T$, where:

$$A^T x = x, \quad \sum_{i=1}^N x_i = 1 \quad (11)$$

In the equation, vector x indicates the probability distribution of visiting each webpage as the number of transitions tends to infinity, which Google defines as the PageRank value of each webpage. A larger value of x_i indicates more hyperlinks related to the i -th webpage.

2.3.3 Establishment of the S-B-P model

Based on the above two models, we integrate them organically to obtain the S-B-P Model. This model aims to utilize known food webs to analyze the impact of changes in a certain population on other core populations within the ecosystem, and thus analyze their influence on the overall stability of the ecosystem.

3. Results

3.1. Simulation and visualization based on the improved predator-prey model

For the lamprey population, some parameters in the improved predator-prey model are typically related as follows: $r_2 > r_3$, moreover, when food is abundant, the value of r_2 is smaller than when food is scarce; when food is abundant, the value of r_3 is greater than when food is scarce; $\lambda_2 < \lambda_3$. Based on the above relationships, we select the parameters as shown in Table 1 for simulation.

Table 1. Equation parameters

Food availability	r_1	r_2	r_3	λ_1	λ_2	λ_3	e
Low	1	0.44	0.4	0.02	0.1	0.2	0.1
High	1	0.42	0.41	0.02	0.1	0.2	0.1

The visualization results are shown in Figure 1:

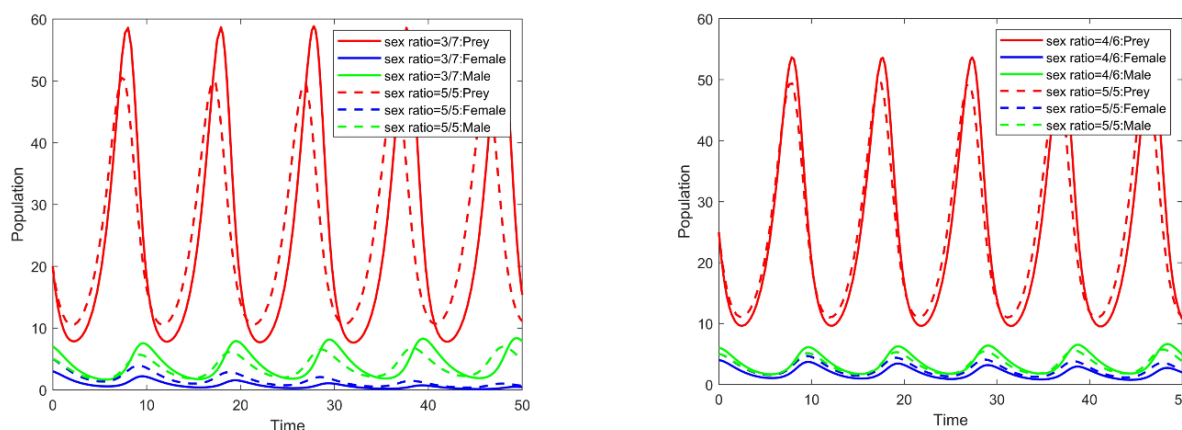


Figure 1 The simulated plot of the altered predator-prey model

From Figure 1, it can be concluded that in both ecosystems, whether it's an environment with abundant or scarce food supply, changes in the sex ratio of lampreys will result in variations in the prey population. Compared to a situation where the sex ratio remains constant (with a 1:1 ratio of females to males), the times when the prey population reaches its extremes are consistently delayed. The fluctuations in the sex ratio of lampreys have a more pronounced impact on ecosystems with limited food availability. In this ecosystem, the variations in prey population are more drastic.

3.2. Simulation and visualization based on the improved population competition model

When the lamprey dominates in competition, we select the values as shown in Table 2 for simulation.

Table 2. Simulation values of σ_i

σ_4	σ_2	σ_3
0.5	1	2

The visualization results are shown in Figure 2:

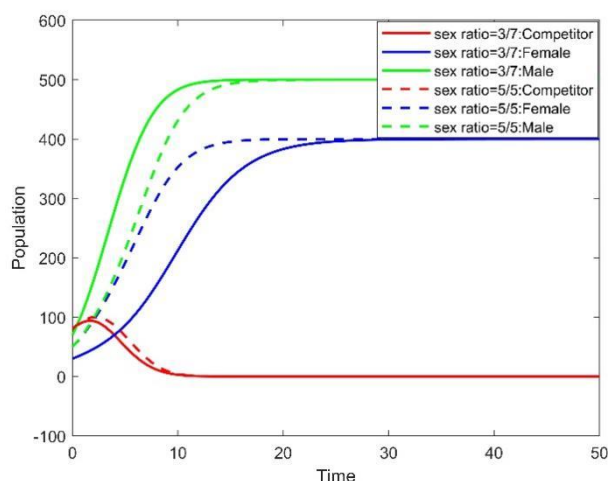


Figure 2. Simulation result when lampreys are in a competitive advantage situation

In this situation, as indicated by the competitor's response curve, represented by the red line in the graph, changes in the sex ratio of lampreys result in a more rapid decline in the number of competitors. This imposes greater competitive pressure on the competitor.

When the lamprey is at a disadvantage in competition, we select the values as shown in Table 3 for simulation.

Table 3. Simulation values of σ_i

σ_4	σ_2	σ_3
1.2	0.6	0.8

The visualization results are shown in Figure 3:

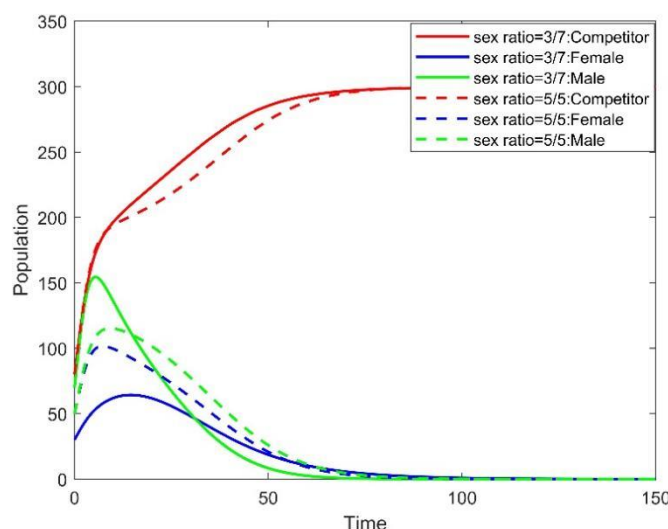


Figure 3. Simulation result when lampreys are in a competitive disadvantage situation

In this situation, as indicated by the competitor's response curve, represented by the red line in the graph, changes in the sex ratio of lampreys result in a faster increase in the number of competitors. This further strengthens their advantage in the competition between the two.

However, further increasing the value of σ_4 leads to the following result, as shown in Figure 4:

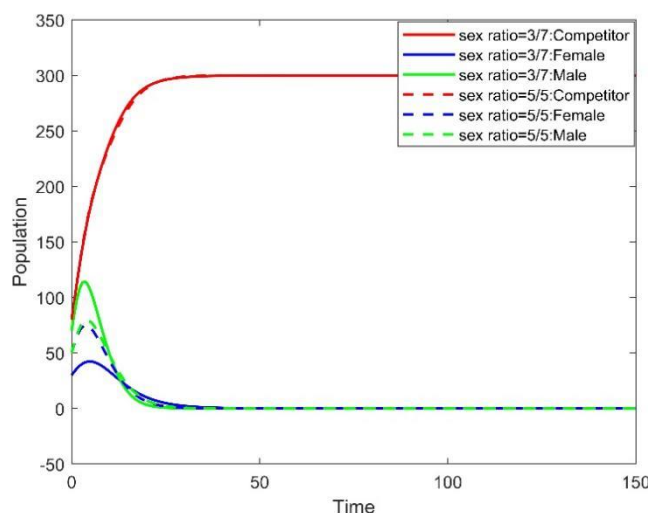


Figure 4. Simulation result when lampreys are in a competitive disadvantage situation

When the competitor holds an absolute dominant position in the competition with the lam-prey population, changes in the sex ratio of lampreys have minimal impact on this competitor. It is challenging to undermine its status and dominance in the competition.

3.3. Example and result analysis of the S-B-P Model

We analogize the food web to a network of web pages, substituting populations for web pages and interspecies relationships for hyperlinks between web pages, thus obtaining the network diagram of the food we. By transforming the food web into a weighted directed graph, we utilize the PageRank

algorithm to obtain the corresponding stationary distribution and PageRank values of populations. A higher PageRank value indicates a tighter connection between the population and other populations in the food web, identifying the core populations sought after.

Based on experience, we simulate a food web of a lake ecosystem, as shown in the diagram below:

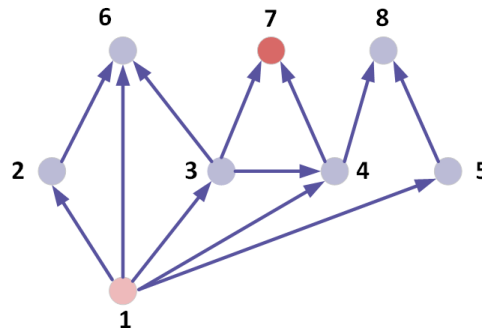


Figure 5. Simulated food web of a lake ecosystem

The numbers in Figure 5 represent different species populations. 1 denotes producers, such as algae in the lake; 2,3,4,5 represent intermediate trophic levels; 6,7,8 are top consumers, with 7 indicating the lamprey. For the lamprey, it preys/parasitizes on 3 and 4, and competes with 6 and 8. Using the PageRank algorithm, we obtain the stationary distribution of this food web:

$$x = [0.018, 0.035, 0.035, 0.073, 0.035, 0.224, 0.372, 0.208] \quad (12)$$

From equation (13), we can deduce that excluding the PageRank value of the seventh lamprey itself, the remaining populations are ranked from high to low as 6, 8, 4, 2/3/5, and 1. We select the top three, namely 6, 8, and 4, as the core populations to analyze the impact of species 7 on the stability of the ecosystem.

Once the populations are determined, we need to simulate data. By consulting the literature, line graphs depicting the biomass changes over time for six population types in the lake were provided. Upon observation, it is noted that the biomass of each population tends to exhibit a rapid increase followed by a rapid decrease over a certain period of time. We select one of these line graphs and smooth out the curves of local fluctuations, revealing a distribution similar to a normal distribution, as shown in Figure 6:

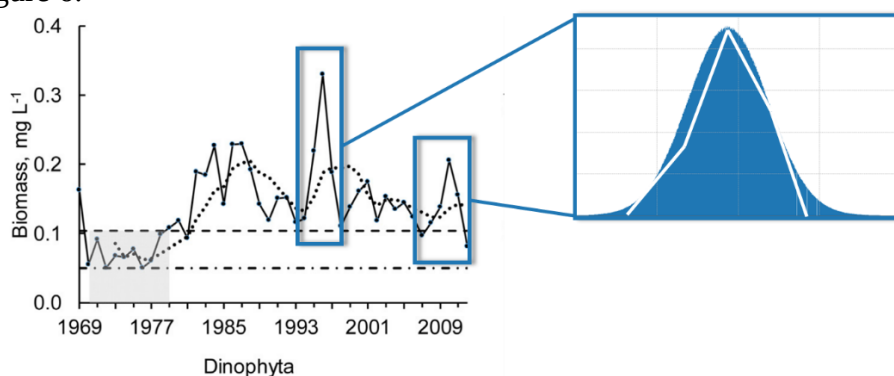


Figure 6. Approximation of Biomass Normal Distribution

Therefore, we can assume that the biomass of populations in the lake ecosystem approximately follows a normal distribution. Let the biomass of core populations 4, 6, and 8 be denoted as P, Q, R . In the food web, populations at higher trophic levels have fewer individuals and greater fluctuations. Based on this characteristic, we make the following normal distribution simulation:

$$\begin{aligned}P &\sim N(5, 0.5) \\Q &\sim N(3, 1.0) \\R &\sim N(2, 0.8)\end{aligned}\tag{13}$$

where the unit of μ (mean value) is $mg \cdot L^{-1}$.

We use MATLAB to generate random numbers following a normal distribution. For each of the three populations, we generate 50 data points to simulate 50 lakes. According to equations (5), (6), (7) and (8), we calculate SI_{New} . From the first question, changes in the sex ratio of the lamprey population will result in changes in population size. If the population increases, the prey population 4 will definitely decrease in the short term; if the population decreases, the consumption of 3 by the lamprey will decrease, resulting in more 3 being preyed upon by 6, leading to an increase in the number of 6 in the short term. Therefore, the number of core species will certainly change. We manipulate the simulated data by increasing or decreasing it accordingly, i.e. adjusting each data point to ± 0.5 . Calculate $SI_{Newzeng}$ and $SI_{Newjian}$ according to the calculation method of SI_{New} . The final selection of the list of 5 data sets is as follows:

Table. 4. Simulate the stability index of the ecosystem

No.	SI_{New}	$SI_{Newzeng}$	$SI_{Newjian}$
1	0.6710	0.3001	0.3522
2	0.6489	0.4078	0.5347
3	0.8231	0.3216	0.3799
4	0.9207	0.3329	0.4003
5	0.7477	0.3740	0.4727

According to Table 4, it can be observed that for each group, both $SI_{Newzeng}$ and $SI_{Newjian}$ are less than SI_{New} , indicating that after experiencing disturbances in biomass, the stability index of the entire ecosystem decreases. From this, we conclude that changes in the sex ratio of the lamprey will lead to a decrease in the stability of the ecosystem.

4. Conclusions

In this essay, we explore the impact of changes in lamprey sex ratios on the ecosystem and its stability. Firstly, we develop Lotka-Volterra equations and employ it to forecast quantities of both lampreys and their preys. In both ecologically abundant and scarce food systems, changes in the sex ratio of lampreys lead to variations in the quantity of their prey. Then, improved population competition model is applied. When the lamprey holds a dominant position in competition, it exerts greater competitive pressure on rivals. However, when the lamprey is in a disadvantaged position, alterations in sex ratio may further strengthen the competitor's advantage over it. Last, we calculate SI of top three species ranked by PageRank algorithm to evaluate the ecosystem's stability and find that species changes in its gender ratios also affect the biomass of core populations, resulting in a decrease to the ecosystem stability.

References

- [1] Siefkes M J, Johnson N S, Muir A M. A renewed philosophy about supplemental sea lamprey controls[J]. Journal of Great Lakes Research, 2021, 47: S742-S752.
- [2] Fissette S D, Buchinger T J, Wagner C M, et al. Progress towards integrating an understanding of chemical ecology into sea lamprey control[J]. Journal of Great Lakes Research, 2021, 47: S660-S672.

- [3] Das S, Gupta P K. A mathematical model on fractional Lotka–Volterra equations[J]. Journal of theoretical biology, 2011, 277(1): 1-6.
- [4] Hilal N. Mathematical and numerical simulations of the Lotka-Volterra Equations[J]. 2022.Ma Kunlong. Short term distributed load forecasting method based on big data [D]. Changsha: Hunan University, 2014.
- [5] Wang S Y, Chen W M, Wu X L. Competition analysis on industry populations based on a three-dimensional lotka–volterra model[J]. Discrete Dynamics in Nature and Society, 2021, 2021.
- [6] Pringle R M, Hutchinson M C. Resolving food-web structure[J]. Annual Review of Ecology, Evolution, and Systematics, 2020, 51: 55-80.
- [7] Tian W, Zhang H, Wang Z, et al. Analysis on the stability of plankton in a food web with empirical organism body mass distribution[J]. Environmental Science and Pollution Research, 2023, 30(8): 21327-21343.
- [8] Sharma P S, Yadav D, Garg P. A systematic review on page ranking algorithms[J]. International Journal of Information Technology, 2020, 12: 329-337.
- [9] Lü D, Lü Y, Gao G, et al. A landscape persistence-based methodological framework for assessing ecological stability[J]. Environmental Science and Ecotechnology, 2024, 17: 100300.
- [10] Parparov A, Gal G, Zohary T. Quantifying the ecological stability of a phytoplankton community: the Lake Kinneret case study[J]. Ecological indicators, 2015, 56: 134-144.