

Analysis of the Lamprey Ecosystem Based on the Lotka-Volterra Model

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Abstract. The lamprey (Vertebrinidae) is the sole survivor of the oldest vertebrate known. Many studies have shown that sex differentiation in sea lamprey is influenced by external environmental factors, making it dependent on and also influencing the surrounding ecosystem. This article focuses on studying the impact of this sexual variation on the species itself and the surrounding ecosystem. Using differential equations, based on reality, this article constructed a cellular automaton model and a variant Lotka-Volterra model incorporating three species. This article comprehensively applied software programming tools such as MATLAB to solve these models. By building the model, obtained the general operation of lamprey ecosystem and the reproduction and evolution of lamprey population. This article primary aim is to investigate the sex ratio of species and its dependence on and impact on local environmental conditions. This article aim to understand how organisms within this ecosystem interact and adapt to environmental changes. Furthermore, this article seek to understand how sex ratio is influenced during upstream spawning migrations, and whether this dynamic change in sex ratio has ecological adaptive significance.

Keywords: Lotka-Volterra model, Sea lamprey, Cellular automaton model.

1. Introduction

The lamprey plays a special role in the food chain it inhabits. Living in lakes or oceans, it serves as a food source, is regarded as a parasite, and is also a part of the traditional diet of residents. It has a significant impact on the ecosystem. Simultaneously, the lamprey stands out as an extraordinary species, with its sex ratio significantly influenced by external environmental factors, potentially deviating from the norm. This phenomenon is referred to as adaptive sex ratio variation. When food supply is scarce, the growth rate of marine lampreys is relatively low, and the proportion of males increases to approximately 78%. Conversely, when the food supply is abundant, the growth rate of marine lampreys is higher, and the proportion of males decreases to around 56% [1]. This adaptive sex ratio variation not only relies on the environment but also has implications for the surrounding ecosystem. In previous research reports, some established an average daily growth model including streams and lakes as contributing variables [2]. Some have shown that the sex of lamprey is related to a specific gene, which is expressed more in male testes and also exists in undifferentiated larvae and females with a lower expression level. This indicates that the male-specific expression is regulated and changed by specific genes, and this regulation can be affected by the living environment [3]. However, among the current research achievements, studies on the environmental influence on the gender ratio of the lamprey population, as well as the adaptability of the lamprey population with different gender ratios to the environment and their impact on the environment are still very rare. This article includes two models, the Lotka-Volterra model based on differential equations [4-6]. Introducing the interaction between predators and prey. We also utilized a cellular automaton model to analyze the changes in the gender ratio of the lamprey population under different environments and the impact of the lamprey population with variable gender ratios on their surrounding environment [7-10].

2. The basic fundamental of Lotka-Volterra Model and Cellular Automaton Model

2.1. The structure of Lotka-Volterra model

In the 1920s, Volterra and Lotka independently proposed the now-called Lotka-Volterra system (LV system) to study the mutual evolutionary relationships and chemical reactions between biological populations. When prey and predators exist independently as Figure 1 shows, the presence of predators causes a decrease in the growth rate of the prey, and this decrease is proportional to the number of predators. Conversely, the prey causes a decrease in the death rate of the predators, and this decrease is proportional to the number of prey. Based on these interactions, the Lotka-Volterra model is established. The general form of the Lotka-Volterra model is given as:

$$\begin{cases} \frac{dU}{dt} = \alpha U - \gamma UV \\ \frac{dV}{dt} = -\beta V + e\gamma UV \end{cases} \quad (1)$$

where α is The natural growth rate of prey, β is the natural mortality rate of prey, U is The population size of prey, V is The population size of predators, γ is The proportion of prey captured by predators per unit of time, e is the conversion coefficient between the number of prey captured and the number of newborn predators.

Considering that male and female lampreys have different predation and fertility conditions, the proportion of female lampreys increases when food is plentiful, and the proportion of male lampreys increases when food is scarce. When the proportion of females is high, the natural growth rate of the lamprey population increases correspondingly, and vice versa. The sex ratio of lampreys has a direct impact on the natural growth rate of lampreys. We derive the natural growth rate of sea lamprey according to the law in reality:

$$\alpha = 0.01 * ra + 0.015 \quad (2)$$

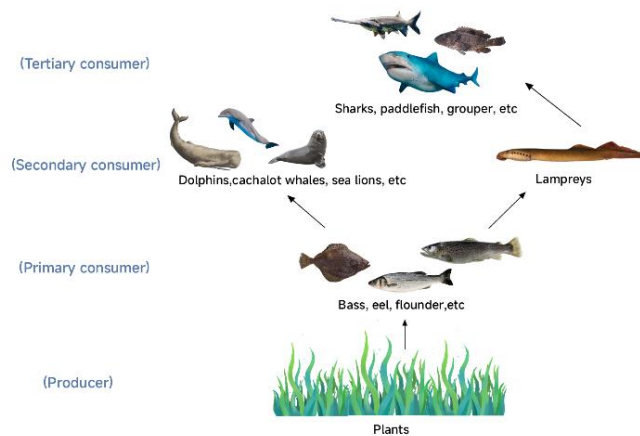


Figure 1. Food web of the lamprey ecosystem

2.2. Basic introduction of Cellular Automaton Model

The cellular automaton Cellular Automata (CA) was proposed in the early 1950s by the father of computers von Neumann (J.von Neumann) to simulate the self-replication function of living systems. Since then, Stephen Wolfram has studied the cellular automaton theory. For example, he made an in-depth study of the models produced by all 256 rules of one-dimensional elementary cellular machines and divided the cellular automata into four types: stationary, periodic, chaotic, and complex. Cellular

automata employ a discrete spatial layout and discrete time intervals to divide cells into finite species states. The evolution of individual cellular states is only related to its current state and the state of one of its local neighbors. Cellular automata use computer modeling and simulation methods to study the macroscopic behavior and laws of a complex system composed of a large number of individual parallel units, similar to biological cells (cells). L-system, lattice gas model, lattice gas-Boltzmann method, traffic flow model, and so on are the embodiment of cellular automata, which have important theoretical significance and practical application value.

The cellular automata contain the following elements:

Cell: When the simulation is a grid with a memory storage state function, which is the most basic component of cellular automata. It has the following three characteristics: The most basic unit of the cellular automata. The function functions as a memory storage state. All the cellular states are continuously updated according to the cellular rules.

State: Each cell has its state, and the state changes over time.

Cell space: a lattice of cells distributed in space, which can be 1,2, or n dimensions.

Neighbor: a surrounding cell that can affect the state at the next moment. Generally speaking, there are three neighbor patterns, This paper adopts the 2-D neighbor pattern shown in Figure 2.

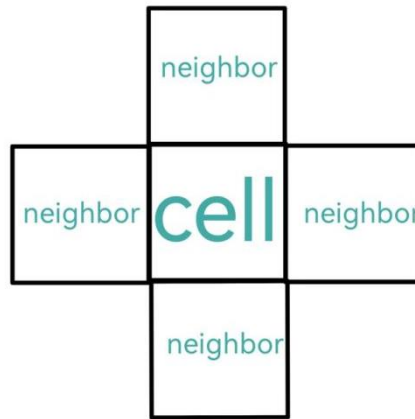


Figure 2. Von. Neumann

3. Results

3.1. The establishment of simulation model

3.1.1 Lotka-Volterra model application and parameter design

When the lamprey acts as a predator, the equation form needs to be changed as:

$$\begin{cases} \frac{dU}{dt} = \eta U - \gamma UV \\ \frac{dV}{dt} = -\beta V + \alpha UV \\ \alpha = 0.01 * ra + 0.015 \end{cases} \quad (3)$$

where U as the number of prey, V as the number of lamprey, η as the natural growth rate of prey, γ as the probability that the prey is prey, α as the natural growth rate of lamprey, β represents the natural lamprey mortality rate.

Let's think further that a complete ecosystem includes not only a single prey-predator relationship but also many complex relationships of competition, parasitism, predation, and predation. To make this relationship more fully, we design the network as Figure 3 shows.

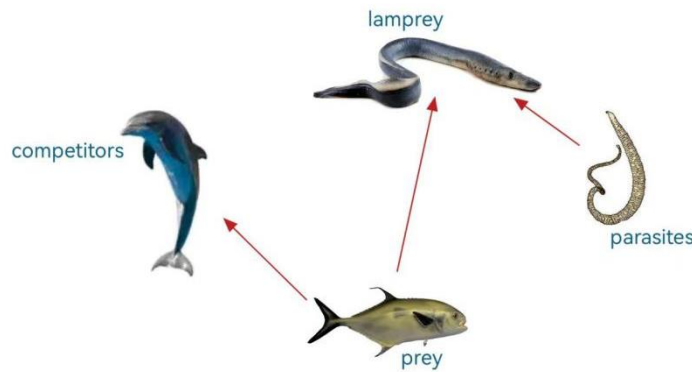


Figure 3. A lamprey ecological network system containing three other species

The network includes lamprey populations that can self-regulate sex ratios, predators, lamprey prey, parasites that harbor lamprey, and competitors that compete with lamprey. The Lotka-Volterra model is reconstructed as follows:

$$\begin{cases} \frac{dF}{dt} = r_f F \left(1 - \frac{F}{k_f}\right) - d_f F + FP_0 k_f a \\ \frac{dM}{dt} = r_m M \left(1 - \frac{M}{k_m}\right) - d_M M + MP_0 k_m a \\ \frac{dP_0}{dt} = r_{P_0} P_0 \left(1 - \frac{P_0}{k_{P_1}}\right) - P_0 d_{P_1} \\ \frac{dP_2}{dt} = r_{P_2} P_2 \left(1 - \frac{P_2}{k_{P_2}}\right) - P_2 d_{P_2} + \alpha P_0 \left(0.3 + \frac{1}{F + M}\right) \\ \frac{dP_3}{dt} = r_{P_3} P_3 \left(1 - \frac{P_3}{k_{P_3}}\right) - P_3 d_{P_3} + k_3 (F + M) P_3 \end{cases} \quad (4)$$

where $F=30$ represents the number of female sea lampreys, $M=70$ represents the number of male sea lampreys, $P_0=10$ is the number of prey species, $P_2=15$ indicates the number of sea lamprey competitors, $P_3=20$ is the number of parasites, $r_f=0.2$ represent the reproductive rate of female sea lampreys, $r_m=0.1$ represent the reproductive rate of male sea lampreys, $r_{P_0}=0.05$ represent The natural growth rate of the prey, $r_{P_2}=0.04$ represent the natural growth rate of sea lamprey competitors, $r_{P_3}=0.03$ represent reproductive rate of parasites, $k_f=100$ is female environmental capacity, $k_m=100$ is male environmental capacity, $k_{P_1}=500$ is prey environmental capacity, $k_{P_2}=200$ is competitors environmental capacity, $k_{P_3}=300$ is parasites environmental capacity, $d_f=0.06$ indicates mortality of female sea lampreys, $d_M=0.02$ indicates Mortality of male sea lampreys, $d_{P_1}=0.01$ indicates Mortality of prey, $d_{P_2}=0.015$ indicates Mortality of competitors, $d_{P_3}=0.03$ indicates Mortality of parasites.

3.1.2 Cellular Automaton Model application and parameter design

This article use cellular automaton models to simulate the process of gender ratio changes in the lamprey population, identify possible trends in gender ratio changes under different environmental conditions, and analyze the reproduction rate, survival rate, and genetic diversity of the lamprey population with variable gender ratios under different resources and conditions. Thereby, we can understand how the sex ratio is influenced during upstream spawning migrations.

The cellular automaton model consists of a set of cells, each with a state that can represent the gender, age, location, etc as Figure 4 shows. of the sea lamprey. The state of the cell is updated at each time step according to a set of local rules, which can represent behaviors such as growth, reproduction, migration, competition, and predation of the sea lamprey. We use a cellular automaton

model to simulate the process of sex ratio changes in the lamprey population. As Figure 4 shows, the simulated data can intuitively and conveniently analyze the reproduction rate, survival rate, and genetic diversity of the lamprey population.

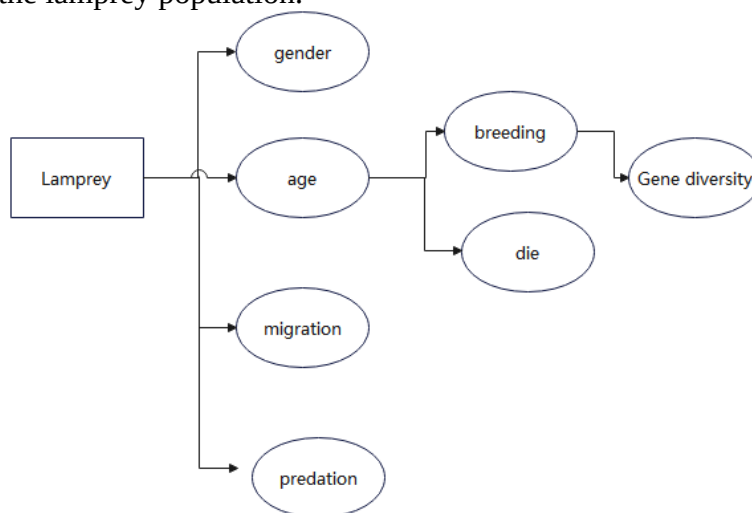


Figure 4. Individual attributes of lampreys

3.2. Analysis of experimental results

3.2.1 Lotka-Volterra Model analysis of ecological networks containing sea lamprey

When the Lotka-Volterra Model involves only the lamprey and its prey, changes in lamprey and prey populations at different sex ratios are shown in Figure 5 and Figure 6. It can be seen that when the initial female proportion increases, the lamprey number rises faster in the short term, but the population decline rate is also higher after the later food resources become less.

By assigning reasonable values to the model Lotka-Volterra model we have built, we can obtain the graphs showing the changes in the four populations of lamprey, the prey of lamprey, the competitors of lamprey, and parasites of lamprey over time, as shown in Figures 7, 8 and 9. We can see that lamprey numbers grow to a certain number, prey and competitors numbers decrease, and parasites increase more. Eventually their numbers will all reach a long-term, constant, stable value. Meanwhile, the proportion of female lamprey increased when prey increased and females decreased as competitors and parasites increased.

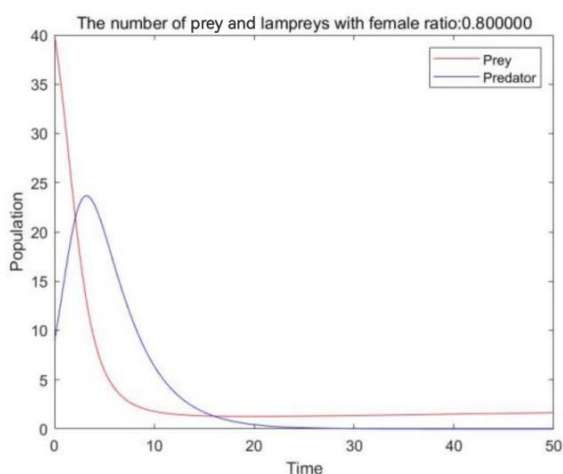


Figure 5. sex ratio=0.8

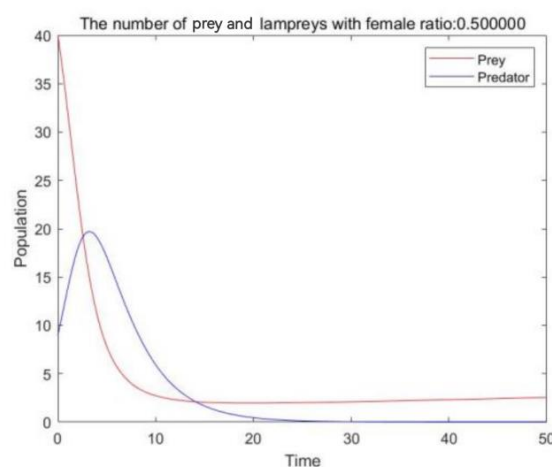


Figure 6. sex ratio=0.5

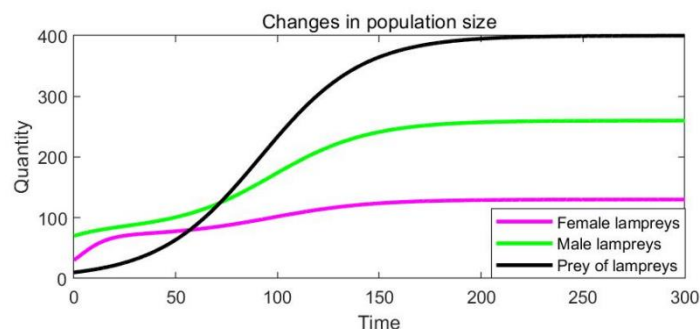


Figure 7. Changes in lamprey and preys abundance

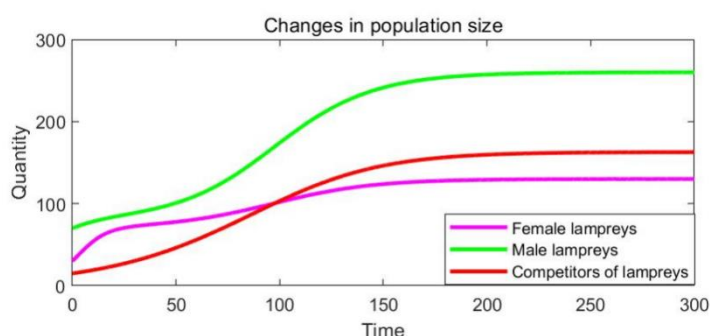


Figure 8. Changes in lamprey and competitors abundance

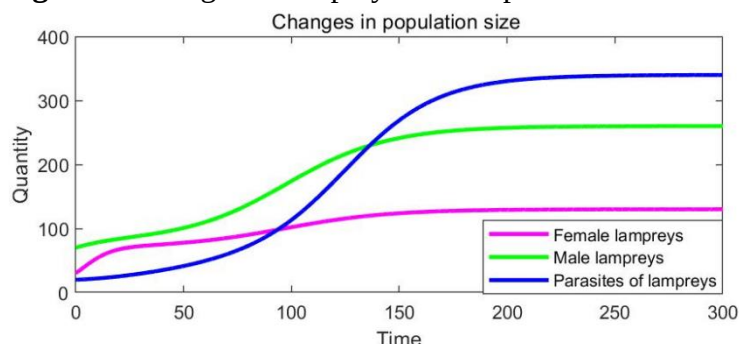


Figure 9. Changes in lamprey and parasites abundance

3.2.2 Simulation and Analysis of Cellular Automaton Models

Here, a stationary cellular automaton model is established. Each individual is assigned attributes such as gender (male, female), age (0-7 years), survival status (alive, dead), migration location, and gene frequency (0.4-0.8 for males, 0.2-0.6 for females). A population is randomly generated using a random number function. The above parameters are set after consulting relevant data and conform to objective laws.

In this model, we assume an initial condition where the available food resources are insufficient to meet the survival needs of the entire lamprey population. As can be seen from Figure 10, the scarcity of resources leads to a sharp decline in the total population size, which later stabilizes and reaches a relatively stable state, satisfying the environmental carrying capacity under these conditions.

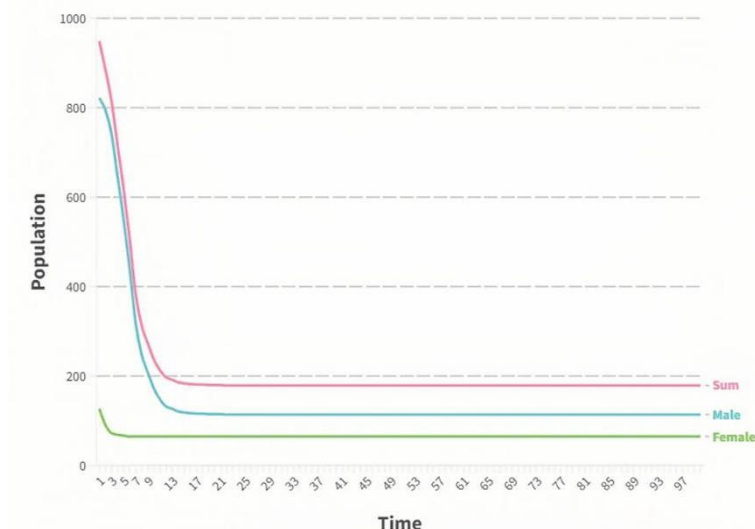


Figure 10. When food resources are insufficient, when food resources are insufficient, a change in sea lamprey populations

Through Figure 11, we can observe that initially, despite the decline in the number of female, male, and total lampreys, the proportion of males increases. This indicates that male lampreys have a higher tolerance for harsh environments compared to females. Under conditions of resource scarcity, more larval lampreys develop into males, increasing the probability of the entire population surviving the critical period. However, a higher proportion of males implies a decrease in the reproductive capacity of the entire population.

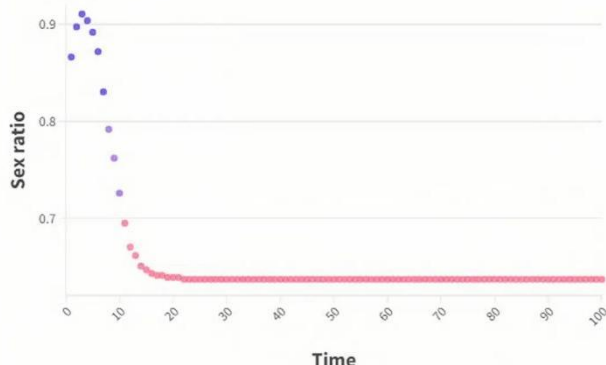


Figure 11. Scatter plot of male proportion (where "sex ratio" refers to male proportion) in the lamprey population over time when food resources are insufficient.

When the proportion of male lampreys is higher, genetic diversity is better. Gene density refers to the ratio of the total number of genes to the total number of individuals in the population. It is used to reflect genetic diversity, and a higher gene density indicates richer genetic diversity. Figure 12 and Figure 13 show the relationship between male proportion and gene density in the population. they show two different results obtained from solving the model, but the trends are the same. This not only verifies the randomness of the simulation using this model but also confirms the accuracy of the conclusions.

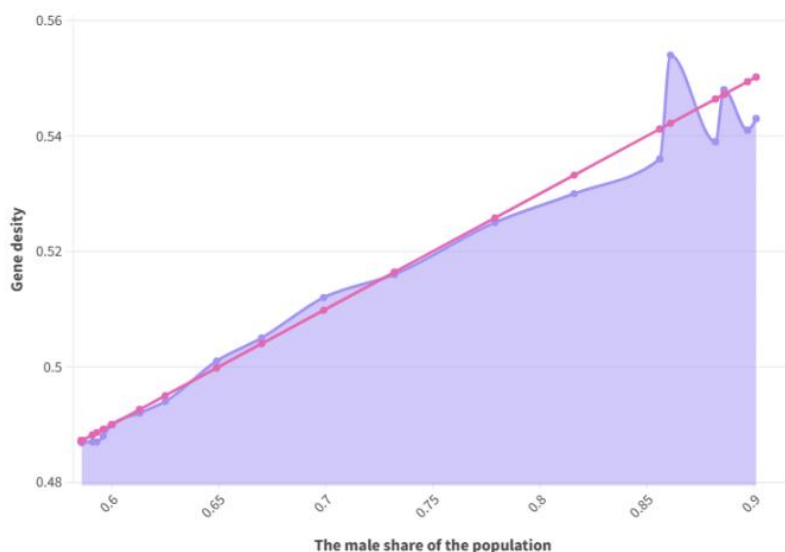


Figure 12. Plot of gene diversity versus male ratio (Results 1)

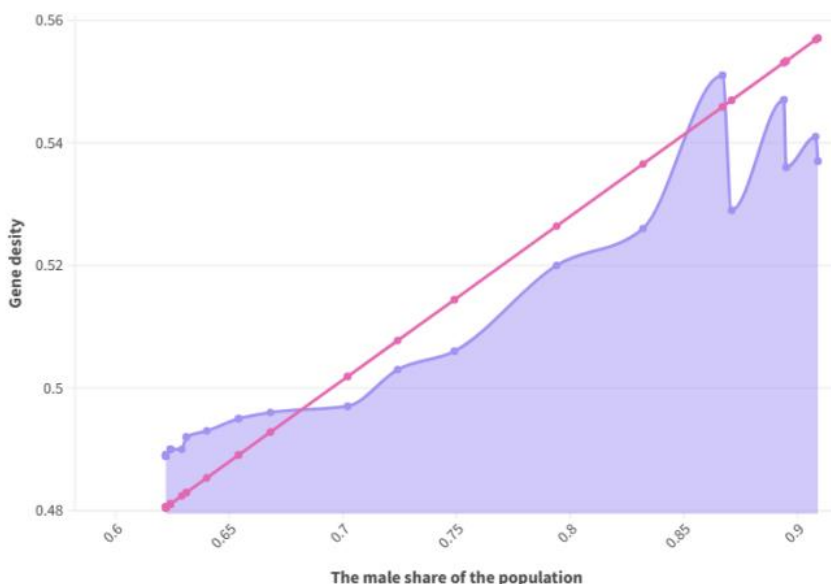


Figure 13. Plot of gene diversity versus male ratio (Results 2)

4. Conclusions

By building the model, we obtained the general operation of lamprey ecosystem and the reproduction and evolution of lamprey population. In our model, the equation and parameter settings of the Lotka-Volterra model are supported by real data, fully considering the real situation in nature, and making assumptions about the ability of lampreys to change the sex ratio. After refinement, the Lotka-Volterra model contains three parasite variants, fully considering the relationship between prey, predator, competitor and parasite, providing a more reasonable equation with reasonable model results. Our cellular automata model comprehensively and fully considers the important life processes of lampreys in real life, using a stochastic model to ensure that each lamprey individual has its personality and follows basic laws. The model can simulate other species by changing key parameters, making it expandable. The simulated results follow the laws of the real world. In this model, lampreys can change their sex ratio in extreme environments to successfully survive dangerous periods, which is consistent with the historical facts of lamprey population development.

But our model still has some shortcomings. In our model, the consideration of the ecological environment is not comprehensive enough, and many important influencing factors in nature are not taken into account in the model. The simulated population is relatively small, and the impact of unexpected events in nature is not considered. The mechanism for sex change in lampreys is not fully addressed in the model, lacking a certain degree of flexibility. In the future, we expect to improve the environmental factors in the model by tracking and developing more reliable models.

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