

The Dynamics of Charged Particle Navigating Through an Electromagnetic Field and Landau Levels

Weikai Sun *

School of Applied physics, Shanghai University of Electric Power, Shanghai, China

* Corresponding Author Email: 20212432@shiep.edu.cn

Abstract. This text begins by applying a theoretical framework and mathematical derivation to ascertain the magnetic induction intensity generated by electrons within an electromagnetic field. Following this foundational analysis, it explores the specific scenario of electrons undergoing uniform linear motion within the electromagnetic field, calculating the corresponding magnetic induction intensity for this motion. The narrative progresses to consider the broader implications of charged particle motion in electromagnetic fields, culminating in a discussion of Landau levels. By employing calculation and derivation within the context of a symmetric gauge, it determines the magnitude of these Landau levels. The findings reveal that the electric displacement flux created by a uniformly moving charge on a circular plane displays a symmetric distribution. Additionally, it highlights a consistent relationship between the changes in displacement current and magnetic induction intensity, both of which vary according to their spatial positioning. Importantly, the results pertaining to Landau levels align with theoretical expectations, offering a coherent and insightful exploration of the physical principles at play.

Keywords: Charged particles, Electromagnetic field, Landau levels, Biot-Savart Law.

1. Introduction

In the past hundred years, related research on the movement of charged particles in electromagnetic fields has been at the forefront of the times and has achieved good results. In 1922, British physicist Aston won the Nobel Prize for his invention and use of mass spectrometers to conduct related scientific research and achieved great results. In 1932, American physicist Lawrence Orlando used electromagnetic theory to develop a cyclotron. In 1976, John Modi of Stanford University invented the free electron laser and so on [1]. However, for students who are new to the field of electromagnetic fields and groups with weak physical knowledge systems, there are still insurmountable difficulties in understanding this theory. In recent years, with the development of information technology and the improvement of computer numerical simulation capabilities, this problem has been solved.

In classical electrodynamics, the electromagnetic field is described by the electric field strength $\vec{E}$ and magnetic induction intensity $\vec{B}$, which are directly measurable physical quantities. For convenience, electromagnetic potentials, namely scalar potential ϕ and vector potential, are also introduced in electrodynamics [2]. In classical electrodynamics, since the objective attributes of the electromagnetic field are represented by the measurable physical quantities $\vec{E}$ and $\vec{B}$, and different gauges correspond to the same $\vec{E}$ and $\vec{B}$. If potentials are used to describe the electromagnetic field, the objective laws should be independent of the specific choice of potentials. When potentials undergo gauge transformations, all physical quantities and laws should remain invariant; this invariance is known as gauge invariance [3]. Merely by magnetic induction intensity, the magnetic vector potential cannot be completely determined, hence the necessity to introduce gauge transformations. In practical solution processes, symmetric gauge and asymmetric gauge (Landau gauge) are often introduced.

Therefore, building on the foundation, this paper primarily investigates the Landau levels under symmetric gauge and obtains the specific Landau levels under symmetric gauge through mathematical calculations.

2. Theoretical Model and Derivation

2.1. The Magnetic Field of a Moving Charge

The magnetic field generated by a moving charge is understood through the electromagnetic field theory. According to this theory, the motion of a charged particle induces a changing electric field. The varying electric field, in turn, produces a magnetic field. As the charge moves, there is the presence of an electric current, which further excites the magnetic field. To figure out the magnetic field created by a moving charge [4], the physical model considered herein involves a point charge q initiating motion from a point P and proceeding uniformly along the PO direction at a constant velocity v . The associated circular magnetic field is centered at O , with the distance from P to O denoted as z , and a circular region with radius a formed perpendicular to the velocity vector v .

Expressing the total charge in the volume dQ as $qdN = qnSvdt$, where $dV = Svdt$ represents the volume containing charge and $dN = n dV = nSvdt$ is the number of charges, the concept of a current element $Idl = qnSvdl$ is introduced. Utilizing the Biot-Savart Law, the resulting magnetic induction B is expressed as $B = \left(\frac{\mu_0 q}{4\pi r^3}\right)(v \times r)$, where r represents the vector from the current element to the observation point, and μ_0 denotes the permeability of free space.

2.2. The Electromagnetic Field of Uniform Linear Motion

In classical electromagnetics, the electric field produced by a uniformly moving charged particle can be described using Coulomb's Law. Coulomb's Law expresses the magnitude F of the electric field force produced by a charge q_1 on another charge q_2 at a distance r . Here, k is Coulomb's constant with a value of $8.9875 \times 10^9 \text{ N m}^2/\text{C}^2$. This formula depicts the interaction force between two-point charges. However, for continuously distributed charges, integration is employed [5].

Assuming a charged straight conductor with linear charge density λ and a charge element $dq = \lambda \cdot dx$, one can integrate this charge element to obtain the expression for the electric field intensity $E = \int \frac{k \cdot \lambda \cdot dx}{r^2}$. Here, r represents the distance from the observation point to the charge element on the conductor. If the conductor is infinitely long, r can be expressed as the perpendicular distance from the observation point to the conductor [6].

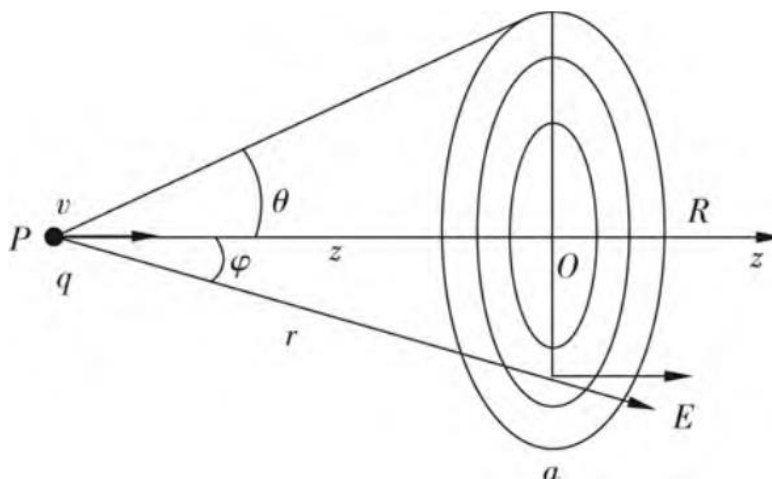


Fig. 1 Illustration of the flux of a charged particle.

The physical model of a charged particle undergoing uniform rectilinear motion, with a constant current magnitude, can be considered as a single current-carrying straight conductor, generating a circular magnetic field in space. The representation of this model is depicted in Fig. 1. Specifically, a point charge q moves along the PO direction with a constant velocity v , starting from a point P and reaching point O . The distance PO is denoted as z , and a circle is constructed with a radius a centered at point O , such that the circular plane is perpendicular to the velocity v .

The electric flux passing through a circular loop of radius R on the circular plane is [7]

$$d\Phi = EdS \cos\varphi = \frac{qz}{2\varepsilon_0} \frac{rdR}{(z^2 + R^2)^{\frac{3}{2}}} \quad (1)$$

The area is $dS = 2\pi R dR$. The magnitude of electric field strength is

$$\frac{q}{4\pi\varepsilon_0 r^2} = \frac{q}{4\pi\varepsilon_0 (z^2 + R^2)}; \cos\Phi = \frac{z}{r} \quad (2)$$

By rating the preceding equation, it is readily to obtain

$$\int d\Phi E = \frac{qz}{2\varepsilon_0} * \int_0^a \frac{d(z^2 + R^2)}{2(z^2 + R^2)^{\frac{3}{2}}}. \quad (3)$$

The electric flux passing through the circle can be obtained as

$$\Phi E = \frac{q}{2\varepsilon_0} * \left(\frac{z}{|z|} - \frac{z}{\sqrt{z^2 + a^2}} \right). \quad (4)$$

Combining the aforementioned analysis, using the definition of electric displacement $D = \varepsilon_0 E$, the electric displacement flux can be obtained as

$$\Phi D = \varepsilon_0 \Phi E = \frac{q}{2} \left(\frac{z}{|z|} - \frac{z}{\sqrt{z^2 + a^2}} \right). \quad (5)$$

So, the displacement current is

$$ID = \frac{d\Phi_D}{dt} = \frac{q}{2} * \frac{a^2 v}{(z^2 + a^2)^{\frac{3}{2}}}. \quad (6)$$

According to Ampère's circuital law, the integral of the magnetic field along a closed loop is equal to the total current passing through the loop. Therefore, the magnetic field's line integral $\oint H \cdot dl = I + ID$, and the displacement current $I = 0$. Therefore,

$$H = \frac{ID}{2\pi a} \quad (7)$$

Thus, the magnetic field

$$B = \mu^0 H = \mu^0 \left(\frac{ID}{2\pi a} \right) = \mu^0 \left(\frac{\mu^0 q v}{4\pi} \right) * \frac{a v}{(z^2 + a^2)^{\frac{3}{2}}}. \quad (8).$$

2.3. Landau Levels Under the Symmetric Gauge

The laws of motion of the electromagnetic field can be written in the Hamiltonian form. As known from the laws of electrodynamics, an electron with mass m and charge $q = -e$ (a non-relativistic particle) moving in an electromagnetic field described by the potentials (ϕ, A) , its equation of motion can be given by the Hamiltonian proposed by Pauli, which includes the Pauli term for spin

$$H = \frac{1}{2m} * [\sigma \cdot (p - qA)]^2 + q\phi = \frac{1}{2m} * [\sigma \cdot (p + eA)]^2 - e\phi. \quad (9)$$

Combining the equation with the following identity,

$$(\sigma \cdot A)(\sigma \cdot B) = A \cdot B + i\sigma(A \times B), \quad (10)$$

it leads to the conclusion that [8]

$$H = \frac{1}{2m} (p + eA)^2 - \mu_s \cdot B - e\phi. \quad (11)$$

In the Eq. (11), the spin magnetic moment $\mu_s = -\frac{eh}{2m}\sigma$. Here, the term $-\mu_s \cdot B$ represents the potential energy of a magnetic dipole in an external magnetic field. Notably, since the direction of the spin magnetic moment is always opposite to the magnetic field generated by the spin of the electron, the author does not consider the negative sign in front of the B in the potential energy term.

Furthermore, because the magnetization current caused by the magnetic moment is not considered in the calculation of the electric displacement field, the B here refers to $B = B\hat{k}$, the magnetic field in the direction of k . Therefore,

$$H = \frac{1}{2m}(p + eA)^2. \quad (12)$$

The Hamiltonian of the electron is expressed as

$$H = \frac{1}{2m} \left[\left(px - \frac{eB}{2}y \right)^2 + \left(py + \frac{eB}{2}x \right)^2 + pz^2 \right], \quad (13)$$

which simplifies to

$$H = \frac{1}{2m}(\hat{p}x^2 + \hat{p}y^2) + \frac{e^2B^2}{8m}(x^2 + y^2) + \frac{eB}{2m}\hat{L}_z + \frac{\hat{p}z^2}{2m}. \quad (14)$$

Here, $\hat{L}_z$ is the z -component of the orbital angular momentum operator, defined as

$$\hat{L}_z = x\hat{p}y - y\hat{p}x = -i\hbar \left(x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x} \right) = -i\hbar \frac{\partial}{\partial \phi}, \quad (15)$$

which corresponds to the derivative with respect to the azimuthal angle ϕ in cylindrical coordinates. For the cylindrical coordinates (ρ, ϕ, z) , the Hamiltonian is expressed as

$$H = -\frac{\hbar^2}{2m}\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial}{\partial \rho}\right) + \frac{1}{2}m\omega_L\rho^2 + \frac{1}{2m\rho^2}L_z^2 - \omega L_z + \frac{1}{2m}pz^2, \quad (16)$$

where $L_z = -i\hbar \left(x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x} \right)$ is the z -component of the angular momentum, $\omega = eB/(2m)$ is the cyclotron frequency, and $\hbar$ is the reduced Planck constant. The potential energy term associated with the magnetic dipole is $\mu_s \cdot B$, but since the magnetic moment $\mu_s = -e\hbar/(2m)\sigma$ and B only has a component in the z -direction, this term simplifies to $-\omega L_z$, hence it does not appear explicitly in the Hamiltonian. This allows for the Schrödinger equation to be solved for the wave function $\psi(\rho, \phi, z)$, which is given by $\psi(\rho, \phi, z) = e^{ipz\frac{z}{\hbar}}e^{im\phi}R(\rho)$, where $m = 0, \pm 1, \pm 2$, etc.

The radial part of the wave function $R(\rho)$ satisfies the differential equation

$$\frac{d^2R}{d\rho^2} + \frac{1}{\rho}\frac{dR}{d\rho} + \left[\frac{2m_\mu\varepsilon}{\hbar^2} - \left(\frac{m_\mu\omega_L}{\hbar} \rho \right)^2 - \frac{m^2}{\rho^2} \right] R = 0 \quad (17)$$

Defining the dimensionless variables $\xi = \alpha\rho$, where $\alpha = \sqrt{\frac{m_\mu\omega_L}{\hbar}}$, the author obtains the differential equation as

$$\frac{d^2R}{d\xi^2} + \frac{1}{\xi}\frac{dR}{d\xi} + \left[\lambda - \xi^2 - \frac{m^2}{\xi^2} \right] R = 0. \quad (18)$$

The solution to this equation can be expressed as $R(\xi) = \xi^{|m|}e^{-\frac{\xi^2}{2}}x(\xi)$, where $x(\xi)$ satisfies the differential equation

$$\frac{d^2x}{d\xi^2} + \left(\frac{2|m|+1}{\xi} - 2\xi \right) \frac{dx}{d\xi} + [\lambda - 2(|m|+1)]x = 0. \quad (19)$$

For $\xi = \xi^2$, it is found that

$$\xi \frac{d^2x}{d\xi^2} + \frac{[|m|+1-\xi]dx}{d\xi} + \left[\lambda - \frac{|m|+1}{2} \right] x = 0. \quad (20)$$

Solving this equation, one finds the quantization condition for λ , which is

$$\alpha = -\left(\lambda - \frac{|m|+1}{2} \right), \gamma = |m| + 1 \quad (21)$$

The eigenvalues are given by

$$\alpha = -\left(\lambda - \frac{|m| + 1}{2}\right) = -n_\rho, n_\rho = 0, 1, 2, \dots \quad (22)$$

This determines the energy levels (Landau levels) as [9]:

$$\varepsilon = \frac{1}{2} \hbar \omega_L, \lambda = \frac{1}{2} \hbar \omega_L \left(\frac{n}{2} + \frac{1}{2}\right). \quad (23)$$

3. Applications

Electron Landau levels stability coefficient and its application to strongly magnetized neutron stars. First, by introducing the stability coefficient of electron Landau levels, including gn and the Dirac- δ function, the expression for the relativistic electron pressure pe under super-strong magnetic fields was obtained. Then, the quantum electrodynamics effects of super-strong magnetic fields inside neutron stars were studied. Finally, the ideal neutron-proton-electron (n-p-e) matter system is considered as representative of the crystalline material of the crust and the outer core of neutron stars. The effects of super-strong magnetic fields on the equation of state were discussed. The study found that in the lattice material below the "neutron drip density" of neutron stars, a phase transition occurs when the nuclide (A, Z) transforms into nuclide (A', Z'), and super-strong magnetic fields cause the phase transition to occur at lower material densities. The overall pressure of the ideal n-p-e matter increases with the strength of the magnetic field [10].

Using linear spin-wave theory and quantum Monte Carlo methods, the pseudo-Landau levels of hexagonal lattice quantum antiferromagnets under curved strain have been studied. Through linear spin-wave theory, it was found that magnetic pseudo-Landau levels appear at the high-energy end of the magnon spectrum, with the spacing between levels being directly proportional to the square root of the level index. Both linear spin-wave theory and quantum Monte Carlo methods show that with the same dimensions, as the strain intensity gradually increases, the local magnetization strength gradually weakens. Under the same strain intensity, the antiferromagnetic order continuously weakens in the y-direction, because the Heisenberg chains at the upper boundary decouple into isolated vertical chains, leading to the destruction of magnetic order near the upper boundary. The quantum Monte Carlo method provides a more precise evolution of antiferromagnetic order: at a specific strain intensity, the vertical correlation at the upper boundary remains unchanged, the horizontal correlation increases, thus affecting the magnetization strength, causing the local magnetization at the upper boundary to show an upward trend. These are all studies of Landau levels in the forefront areas of research.

4. Conclusion

This paper first utilizes theoretical models and derivations to study the trajectories of charged particles in an electromagnetic field, calculating the magnetic induction intensity generated by the motion of these charged particles. It then proceeds to simulate the electromagnetic field produced when charges move at a constant speed in a circular magnetic field from a distance towards the center. Lastly, it delves into the motion of charged particles in an electromagnetic field, calculating the specific Landau levels under symmetric gauge. These results all align well with expectations. By deriving the motion of charged particles in an electromagnetic field, as well as Landau levels, this paper aids in understanding and addressing the issues related to the motion of charged particles in electromagnetic fields. However, it does not adequately explain the specific details of Landau levels, such as those under asymmetric gauge. The trajectory of charged particles moving in an electric field and the Landau levels will find applications in future fields and have promising development prospects in areas like graphene and lithium batteries.

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