Optimization Model of Multibeam Line Placement Based on Geometric Relationship and Planning Solution

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Abstract. This article focuses on the issue of measuring line layout and control in seabed bathymetry using multi-beam bathymetric technology, aiming to optimize the measuring line layout and control of this technology. Through geometric relationships and planning solutions, the efficiency and accuracy of seabed bathymetry are improved. Three mathematical models are presented in this paper, firstly, the coverage width and overlap rate of multibeam bathymetry on a two-dimensional vertical section for the case of an inclined seafloor and the traveling direction of the survey vessel is parallel to the inclined plane and the sea level. Secondly, the mathematical model of coverage width and overlap rate of multibeam bathymetry in three-dimensional space is constructed by expanding it to the case that the survey vessel can travel in any direction. Finally, taking the specific case of an inclined seabed with arbitrary driving direction as the object, the direction, and spacing of the survey lines are solved by the planning model to realize the full coverage of the surveyed area and ensure that the overlap rate is within a reasonable range. The results of this research provide a scientific survey line deployment and control program for seabed bathymetry, which helps to improve the accuracy and efficiency of seabed topographic mapping. The simulation results show that it is most effective to lay the survey line when it is parallel to the seabed slope.

Keywords: Seafloor Bathymetry, Multibeam Bathymetry, Line Placement, Spatial Geometry Analysis, Planning Models.

1. Introductory

Nowadays, beam bathymetry is the main method for surveying seabed topography. The use of beam bathymetry system can simultaneously transmit and receive multiple beams to carry out underwater terrain measurement, in order to real-time display of the seabed terrain conditions and the accuracy of the measurement in the real-time[1]. Bathymetry is one of the main purposes of seabed topographic survey, multibeam bathymetry is a widely used underwater measurement technology, which is more efficient than single-beam bathymetry technology, and it has the characteristics of full coverage, high accuracy and high efficiency[2]. It is characterized by full coverage, high accuracy and high efficiency. Single-beam bathymetry mostly adopts single-point continuous measurement method, while multibeam bathymetry system realizes the key leap from "point-line" measurement to "line-plane" measurement, and its technological advancement is particularly important in the field of underwater inspection, which requires high precision and high efficiency. this technological progress is of epochal significance to the field of underwater inspection, especially in the field of immersed tube tunnel engineering which requires high-precision measurement[3]. Li Ming and others considered the application of multibeam bathymetry system in water conservancy projects to improve the management level of reservoirs by analyzing the trend of topographic changes in reservoirs[4]. However, it mainly considered the hardware facilities of the multibeam bathymetry system, and did not consider the survey line deployment and control of multibeam bathymetry, which is extremely important in practical applications.

In this paper, we use geometric analysis and planning model for multibeam model system establishment of seabed slope, consider the multibeam coverage model in three-dimensional space under different directions, then optimize the survey line laying scheme under different sea conditions, which gives the survey line laying scheme of the sea to be surveyed with practical value.

2. Line placement for multibeam bathymetry on seafloor slopes

Data for this article comes from http://www.mcm.edu.cn/html_cn[5]. Selecting important nodes for classification of sea level data at various points.

2.1. Mathematical modeling of multibeam bathymetric strip coverage widths

Firstly, the case of a survey ship measuring at sea level with a parallel survey line and the seafloor topography as a slope with a certain angle of inclination is considered. The plane that is perpendicular to the direction of the measurement line is taken as a cut-off plane, and the geometric relationship is used to establish a mathematical model for solving the seawater depth and the coverage width of the multibeam transducer on the cut-off plane, as well as the overlap rate between its strips and neighboring strips.

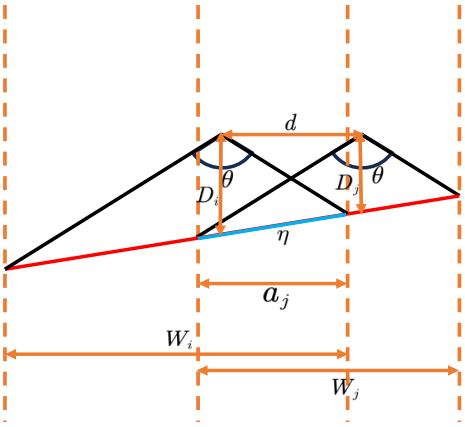


Figure 1. Relationship between coverage width, line spacing and overlap rate on an inclined surface

According to the definition of overlap rate in the case of parallel survey lines and flat seabed terrain, the overlap rate on the section perpendicular to the direction of the survey line is the ratio of the width of the overlapping part to the smaller value of the width of the strip coverage of the two neighboring survey lines, so that the range of overlap rate can be taken as 100%, such as shown in Fig. 1, the survey line i and the survey line j for the two parallel neighboring survey lines, the definition of its overlap rate can be expressed as follows

$$\eta = \frac{a_j}{\min\{W_i, W_j\}} \tag{1}$$

Where η is the overlap rate between neighboring strips, a_j is the width of the overlap between strip j and the previous neighboring strip, W_i is the strip coverage width of the measurement line i, and W_j is the strip coverage width of the measurement line j .

Consider the relationship between the width of coverage of the multibeam sounding and the angle between the projection of the normal vector in the seabed slope on the horizontal plane and the direction of the survey line and the distance of the survey vessel from the center of the sea when the survey vessel is surveying at sea level with the center of the sea as the starting point.

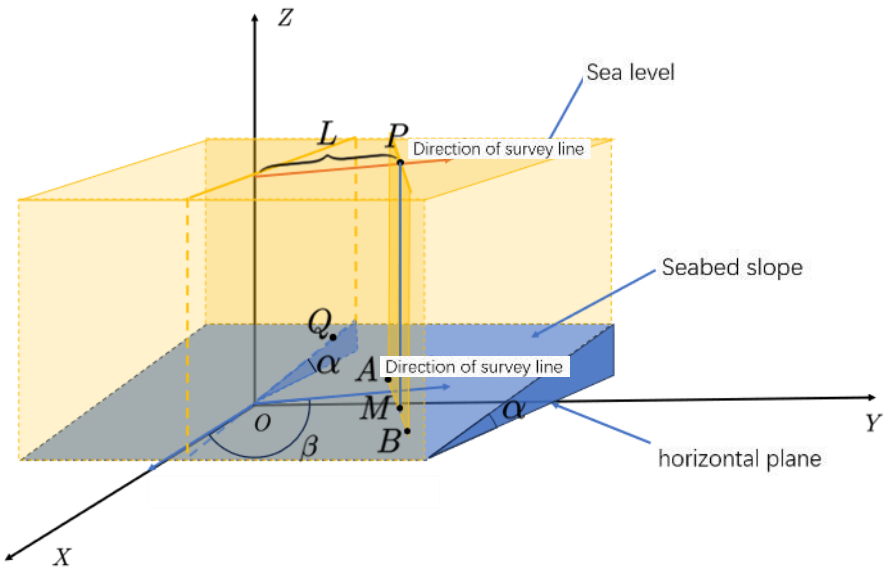


Figure 2. Establishment of space right-angle coordinate system

Presuming a depth of 120 meters in the sea area.

The establishment of spatial Cartesian coordinate system shown in Figure 2, in the seabed slope on the seabed depth of 120m slope line to find any point as the origin of the spatial Cartesian coordinate system O, the normal vector of the slope surface in the horizontal plane at the projection direction is described as the direction of the x-axis, while the horizontal plane that is perpendicular to the upward as the Z-axis direction of the horizontal plane, the slope line of the 120m slope direction to the right as the direction of the y-axis.

The coordinates of the location in the survey vessel can be expressed as $P(x_p, y_p, z_p)$, and the intersection points of the survey vessel transmitting the beam along with the maximum opening angle at the slope surface are defined as A and B , respectively.

1. The direction vector of the line of intersection of the submarine slope with the XOZ section $\overrightarrow{OQ}$ is

$$\overrightarrow{OQ} = (-\cos\alpha, 0, \sin\alpha) \tag{2}$$

The direction vector of the Y-axis $\vec{n}$ that paralleled to the seabed slope can be written as

$$\vec{n} = (0, 1, 0) \tag{3}$$

Then the submarine slope equation can be written as

$$z = -\tan\alpha \cdot x \tag{4}$$

2. The coordinates of the points on the plane along the z-axis are the depth of the sea water at the center point of the sea area,

$$z_p = D_0 \tag{5}$$

Let the intersection of the vertical line from the survey vessel P to the horizontal plane and the slope of the seabed be M, then the coordinates of the point M along the z-axis are

$$z_M = -\tan\alpha \cdot x_M = -\tan\alpha \cdot x_P \tag{6}$$

Then measure the depth of seawater on the ship P

$$D_p = |PM| = D_0 + \tan\alpha \cdot x_P \tag{7}$$

3. The angle between the direction of the survey line and the normal vector of the seabed slope projected on the horizontal plane is β , then the angle between the direction of the survey line and the direction of the x-axis is β , and the direction vector of the survey line is

$$\vec{p} = (\cos\beta, \sin\beta, 0) \tag{8}$$

Then, when the distance between the survey ship at point P and the center sea area is equal to L, the coordinates of point P can be expressed as

$$P\ (L\cos\beta, L\sin\beta, D_0) \tag{9}$$

The corresponding strip coverage width of the measurement line W can be expressed as

$$W = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} \tag{10}$$

Where the coordinates of points A and B, are

$$\begin{cases} t_A = \dfrac{D_0 + L\tan\alpha\cos\beta}{\cos\frac{\theta}{2} + \tan\alpha\sin\frac{\theta}{2}\cdot\sin\beta} \\ x_A = -\sin\dfrac{\theta}{2}\cdot\sin\beta\cdot t_A + L\cos\beta \\ y_A = \sin\dfrac{\theta}{2}\cdot\cos\beta\cdot t_A + L\sin\beta \\ z_A = -\cos\dfrac{\theta}{2}\cdot t_A + D_0 \end{cases} \tag{11}$$

$$\begin{cases} t_B = \dfrac{D_0 + L\tan\alpha\cos\beta}{\cos\frac{\theta}{2} - \tan\alpha\sin\frac{\theta}{2}\cdot\sin\beta} \\ x_B = \sin\dfrac{\theta}{2}\cdot\sin\beta\cdot t_B + L\cos\beta \\ y_B = -\sin\dfrac{\theta}{2}\cdot\cos\beta\cdot t_B + L\sin\beta \\ z_B = -\cos\dfrac{\theta}{2}\cdot t_B + D_0 \end{cases} \tag{12}$$

2.2. Line placement on submarine slopes

On the basis of the above, for the rectangular sea area with known range, the design of the survey line program requires that the survey line has the shortest length and can cover the entire sea area, the overlap rate of the strips of adjacent survey lines meets the range of 10% to 20%, and in order to facilitate the measurement of the depth of the seabed, the survey lines should all be straight lines.[6] In order to facilitate seabed depth measurement, the survey lines should be straight lines.

The survey line is divided into two cases according to the direction of the lateral line, the first case is that the direction of the survey line is not parallel to the projection direction of the normal vector of the seabed slope in the horizontal plane, and the second case is that the direction of the survey line is parallel to the projection direction of the normal vector of the seabed slope in the horizontal plane, and the first case is analyzed for the case in which the direction of the survey line is not parallel to the projection direction of the normal vector of the seabed slope in the horizontal plane.

1. The first case:

The center of the rectangular sea area is used as the origin, the direction of projection of the normal direction of the seabed slope on the horizontal plane (due west) is used as the positive direction of the x-axis, and the due north direction is used as the positive direction of the y-axis.

(1) According to the geometrical relationship, it can be proved that the two lines l_1 and l_2 formed by the straight line l_0 where the survey line is located and the point of projection of the intersection

of the maximum open-angle transmitter beam of the survey vessel with the slope at each position on the sea level are at a common point, hereinafter referred to as point N as the common point.

If and only if $L=-\frac{D_0}{\tan\alpha\cos\beta}$, $t_1=t_2=0$, then the coordinates of point N can be written as

$$N\left(-\frac{D_0}{\tan\alpha},-\frac{D_0}{\tan\alpha}\tan\beta\right) \tag{13}$$

It was found that when the depth of the sea water at the center point of the sea area D_0 and the angle of the seabed slope α are determined, the intersection point N of the three lines will always be on the co-equal straight line formed by the co-equal points l : $x=-\frac{D_0}{\tan\alpha\tan\beta}$.

(2) The vector $\vec{p}$ is the unit vector in the direction of the survey line $\vec{p}=(\cos\beta,\sin\beta)$, and the direction vectors of the lines l_1 and l_2 are $\overrightarrow{p_1'}=(x_1-x_1',y_1-y_1')$ and $\overrightarrow{p_2'}=(x_2-x_2',y_2-y_2')$, respectively, where x_1 and x_1' are the transverse coordinates of two points on the line l_1 , y_1 and y_1' are the longitudinal coordinates of two points on the line l_1 , x_2 and x_2' are the transverse coordinates of two points on the line l_2 , and y_2 and y_2' are the longitudinal coordinates of two points on the line l_2 , and the projected point of the intersection of the maximum-open-angle transmitter beam and the slope on the sea level that corresponds to the measurement vessel as the measurement vessel moves a unit distance in the direction of the survey line. The movement vector is

$$\begin{aligned}\overrightarrow{p_1}&=(\mu_1,\phi_1)=\left(\frac{\cos\beta\cos\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\tan\alpha\sin\beta},\frac{\tan\alpha\sin\frac{\theta}{2}+\sin\beta\cos\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\tan\alpha\sin\beta}\right)\\ \overrightarrow{p_2}&=(\mu_2,\phi_2)=\left(\frac{\cos\beta\cos\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\tan\alpha\sin\beta},\frac{\sin\beta\cos\frac{\theta}{2}-\tan\alpha\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\tan\alpha\sin\beta}\right)\end{aligned} \tag{14}$$

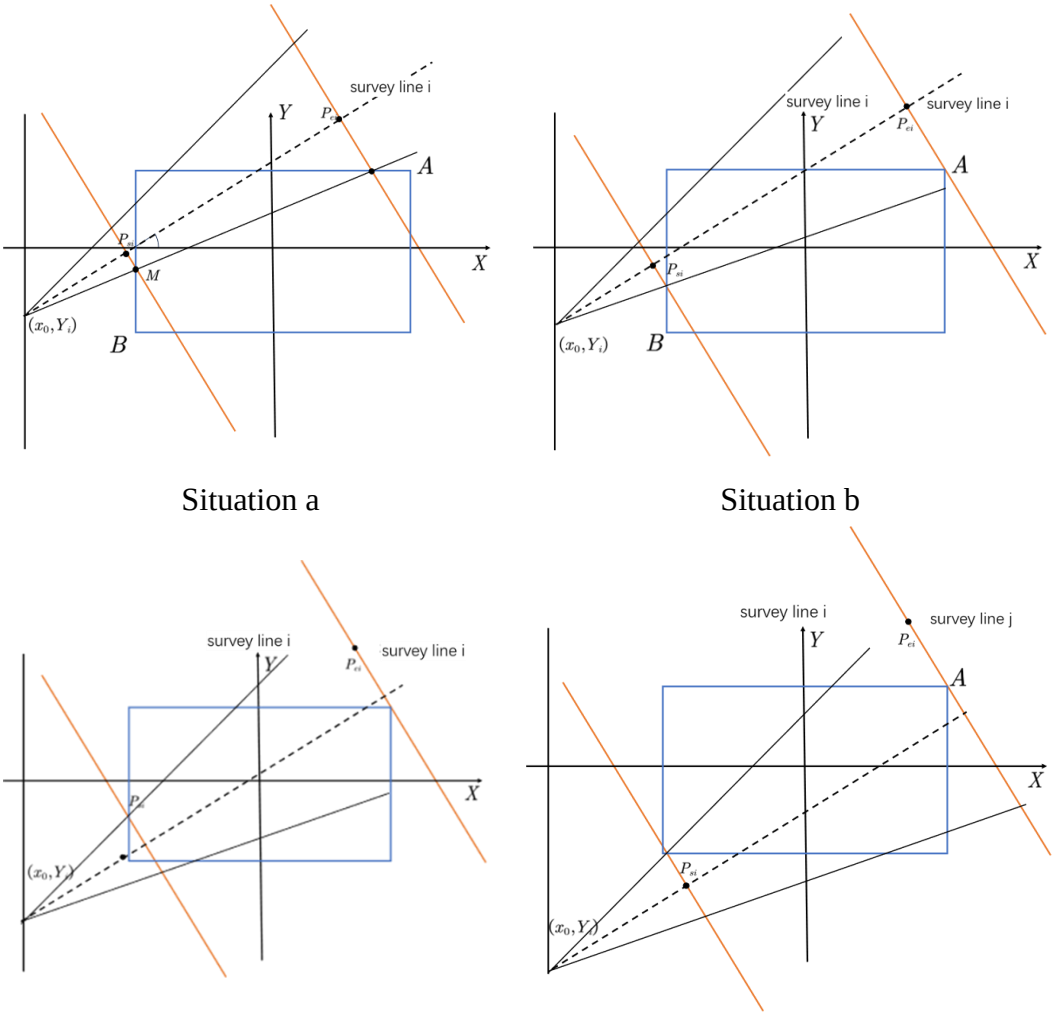
where (μ_1,ϕ_1) is a direction vector along the line l_1 and (μ_2,ϕ_2) is a direction vector along the line l_2 .

The line l_1 and the points on the line l_2 can also be expressed as

$$\begin{aligned}l_1:\ (x_1,y_1)&=(x_0,y_i)+k_i(\mu_1,\phi_1)\\ l_2:\ (x_2,y_2)&=(x_0,y_i)+k_i(\mu_2,\phi_2)\end{aligned} \tag{15}$$

where k_i is the distance from the corresponding measurement point on the line of measurement to the common point for the point on the line.

(3) In order to achieve full coverage, for each measurement line, its boundary points should be selected so that the area covered by the angle range is covered, specifically, the boundary points of the line are selected as shown in Figure 3, which is divided into the following four cases



Situation a Situation b

Situation c Situation d

Figure 3. Schematic diagram of the selection of boundary points of the survey line

(4) In order to design a line program that meets the shortest measurement length of the survey line and can cover the entire sea area, the overlap rate of the strips of the adjacent survey line meets the range of 10% to 20%, and in the case of a flat seabed plane, an overlap rate of 10% is selected, which can effectively reduce the overlap rate when covering, and a planning model is established to solve the problem.

The angle between the direction of the survey line and the projection of the normal vector of the seabed slope in the horizontal plane β is used as the first decision variable.

The co-occurring longitudinal coordinates on a co-occurring straight line l of the i line in a set of parallel lines whose direction is at the same angle as the projection of the normal vector of the seabed slope in the horizontal plane y_i is used as the second decision variable.

The goal is to design a line program that meets the constraints of the shortest measurement length, for the ith line its length equation can be expressed as the distance between the start point and the cutoff point of the line, i.e.

$$|P_{si}P_{ei}| = \sqrt{(k_{si}\cos\beta - k_{ei}\cos\beta)^2 + (k_{si}\sin\beta - k_{ei}\sin\beta)^2} \tag{16}$$

Then the objective function is the sum of the lengths of all the measured lines

$$\min\left\{\sum_{i=1}^{N}\sqrt{(k_{si}\cos\beta - k_{ei}\cos\beta)^2 + (k_{si}\sin\beta - k_{ei}\sin\beta)^2}\right\} \tag{17}$$

Where N is the number of measurement lines.

The constraints are as follows:

In order to ensure that the measuring range of the line designed by can completely cover the whole area, it is necessary to ensure that the measuring range of the first line includes the upper left vertex A, and the measuring range of the last line includes the lower right vertex A' and , i.e., the slopes of the lines connecting the vertex A and the co-points corresponding to the first line are between the slopes of the two projections of the projected points of the intersection between the transmitter beam and the slope at the sea level and the maximum open angle of the lines at the positions of the first line of the survey line. between the slopes of the two straight lines $l_{1,1}$ and $l_{1,2}$, and between the slopes of the two straight lines $l_{N,1}$ and $l_{N,2}$ formed by the projections of the points of intersection of the maximum-open-angle transmitter beam and the slope on the sea level at each position of the straight line where the first line of measurement is located, and between the slopes of the line between the apex A' and the point of co-occurrence of the last line of measurement.

$$\begin{cases} s_{1,1} \geqslant \dfrac{-2-x_0}{1-Y_1} \geqslant s_{1,2} \\ s_{N,1} \geqslant \dfrac{2-x_0}{-1-Y_N} \geqslant s_{N,2} \end{cases} \tag{18}$$

Where $s_{1,1}$ denotes the slope of the line $l_{1,1}$ corresponding to the first line of measurement, $s_{1,2}$ denotes the slope of the line $l_{1,2}$ corresponding to the first line of measurement, $s_{N,1}$ denotes the slope of the line $l_{N,1}$ corresponding to the first line of measurement, and $s_{N,2}$ denotes the slope of the line $l_{N,2}$ corresponding to the first line of measurement.

Then the constraints are

$$s.t. \begin{cases} \eta = \dfrac{a_i}{\min\{W_i, W_{i-1}\}} = 10\% \quad i \geqslant 2 \\ a_i = \sqrt{(x_{i1} - x_{i-1,2})^2 + (y_{i1} - y_{i-1,2})^2} \\ a_i = \sqrt{(x_{i1} - x_{i-1,2})^2 + (y_{i1} - y_{i-1,2})^2} \\ k_{1,1} \geqslant \dfrac{-2-x_0}{1-Y_1} \geqslant k_{1,2} \\ k_{N,1} \geqslant \dfrac{2-x_0}{-1-Y_N} \geqslant k_{N,2} \\ i = 1, 2, ...N \end{cases} \quad i = 1, 2, 3, ...N \tag{19}$$

2. The second case model

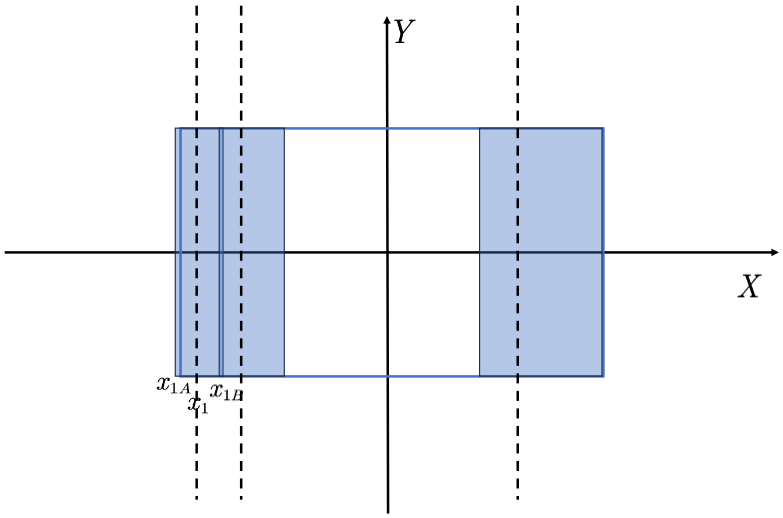


Figure 4. Establishment of the right-angle coordinate system for the second case

In the second case, a right-angle coordinate system is established as shown in Figure 4, considering the sea area within a specific range, with the center point of the rectangular sea area as the origin, the direction of the projection of the normal direction of the seabed slope on the horizontal plane (due west) as the positive direction of the x-axis, and the due north direction as the positive direction of the y-axis.

(1) The decision variables are as follows:

a) Use whether or not the ith line of measurement is a measurement line as the first decision variable

$$b_i = \begin{cases} 0 & The\ ith\ survey\ line\ is\ not\ chosen\ as\ a\ measurement\ line. \\ 1 & The\ ith\ survey\ line\ is\ chosen\ as\ a\ measurement\ line. \end{cases} \tag{20}$$

b) Use the horizontal coordinates of the point on the first measurement line x_1 as the second decision variable

c) The width of the interval between the ith and the latter strip d_i is used as the third decision variable

(2) The objective function is as follows

When the survey boat is moving parallel to the y-axis, the length of each line is equal, then the shortest length is measured when the number of lines is minimum, i.e.

$$\min\left\{\sum_{i=1}^{N} b_i\right\} \tag{21}$$

where N is the maximum number of bands.

(3) The constraints are as follows

(a) Line selection

To ensure that every line of measurement before the ith line of measurement is used as a measurement line, i.e., the

$$b_{i-1} \geqslant b_i \quad i \geqslant 2 \tag{22}$$

The horizontal coordinates of the point on the ith survey line are

$$x_i = \left(x_1 + \sum_{j=1}^{i-1} d_j\right) a_i \tag{23}$$

b) Overlap rate

The overlap between neighboring bands is at 10%.

c） full coverage

The line is designed to cover the entire area ensuring that the first line includes the left most boundary, i.e.

$$x_{1A} \leqslant -2 \leqslant x_{1B} \tag{24}$$

The measurement range of the last survey line includes the rightmost boundary of the

$$x_{NA} \leqslant 2 \leqslant x_{NB} \tag{25}$$

Then the constraints are

$$s.t.\begin{cases} b_{i-1} \geqslant b_i \quad i \geqslant 2 \\ \eta = \dfrac{a_i}{\min\{W_i, W_{i-1}\}} \quad i \geqslant 2 \\ x_i = \left(x_1 + \sum_{j=1}^{i-1} d_j\right) a_i \\ x_{iA} = \dfrac{D_0 - \tan\left(\frac{\pi}{2} - \frac{\theta}{2}\right) l_i}{\tan\alpha - \tan\left(\frac{\pi}{2} - \frac{\theta}{2}\right)} \qquad i = 1, 2, 3, ...N \\ x_{iB} = \dfrac{D_0 - \tan\left(\frac{\pi}{2} + \frac{\theta}{2}\right) l_i}{\tan\alpha - \tan\left(\frac{\pi}{2} + \frac{\theta}{2}\right)} \\ a_i = x_{i-1B} - x_{iA} \\ W_i = x_{iB} - x_{iA} \\ x_{1A} \leqslant -2 \leqslant x_{1B} \\ x_{NA} \leqslant 2 \leqslant x_{NB} \end{cases} \tag{26}$$

3. Resulting rectangular sea area

Consider the depth of the sea at the center of a rectangular sea area 2 nautical miles long from north to south and 4 nautical miles wide from east to west. It is 110 m, deep in the west and shallow in the east, with a slope of 1.5° , and a multibeam transducer opening angle of 120°.

Table.1. Results of Case I

Measuring line direction angle β/°	Number of lines/bar	Total length of survey line/m
1.31048934474678	69	187091.203699326

As shown in Table 1 for the optimal results of case 1, when the angle between the survey line and the projection of the normal direction of the slope on the sea level is 1.31048934474678° , the number of survey lines is 69, and the total length of the survey line is minimum 187091.2037m.

Table 2. Results of case II

Measuring line direction angle β/°	Number of lines/bar	Total length of survey line/m
90	34	125936

As shown in Table 2 for the optimal results of Case 1, when the angle between the survey line and the projection of the slope normal to the sea level is 90° , the number of survey lines is 34, and the total length of the survey line is minimized to 125,936m.

Comparing the results of case I and case II, it is found that in case II when the angle between the survey line and the projection of the slope normal on the sea level is 90° , the number of survey lines is 34 and the total length of the survey line is minimized to 125936m.

4. Conclusions

In this paper, for the survey of multibeam system in seabed terrain, a survey line program design model based on geometric relationship and planning model is established. The results show that the deployment of the survey line parallel to the seabed slope can effectively improve the survey efficiency of the vessel along the survey line, which is of great significance to the survey of seabed terrain.

In the actual seabed terrain survey, the seabed terrain is rugged. Therefore, the seabed terrain should be analyzed by heat map, the seabed terrain should be divided into regions, and the deployment and control of the survey line program should be carried out separately for the seabed terrain with different slopes, which is conducive to obtaining predictive data with higher accuracy and making reasonable decisions.

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