Research on multibeam line laying scheme based on geometric analysis and optimization modeling

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Abstract. The multibeam full-coverage bathymetry technique has greatly improved the accuracy and efficiency of marine mapping and has been widely used in many fields. In order to measure the seafloor topography in the sea area with known topographic features, this paper designs the optimal survey line arrangement scheme. It is finally concluded that the fitting surface function of the sea depth is first fitted using the least squares method, after which the gradient of the sea depth data at each point is calculated, and the hierarchical clustering method is used to determine the optimal clusters for clustering. The surface formed by the set of points with similar gradients is approximated as a flat slope with a fixed slope angle, and based on the optimization model constructed, survey lines are laid along the isobath direction for each approximated slope after clustering.

Keywords: Geometric Analysis, Preedy Algorithms, Cluster Analysis, Search Algorithms, Optimization Models.

1. Introduction

With the development and utilization of lakes and oceans in China, seabed topography has become an important basic data for marine scientific research, marine engineering and military applications. Carrying out seabed topography survey is the basic important work of ocean mapping, the core of which is the measurement of water depth. Multibeam bathymetry is one of the main means of seabed topography measurement nowadays, but the real seabed topography has a large undulation, which brings serious challenges to multibeam bathymetry[1].

After a long period of development, bathymetry technology has undergone changes from manual to electronic and then to single-beam technology. The traditional single-beam point-line measurement mode, although low-cost and simple in principle, suffers from the defects of low measurement efficiency and limited accuracy, which led to the further development of the multibeam faceted full-coverage measurement mode, which greatly improves the measurement accuracy and efficiency[2]. This mode is based on the multibeam theory, and when implementing the seabed measurement, the topography can be measured by the echo of the transducer transmitting beams to form the bathymetric topographic map of the sea area. The system operates with a collection of network, sensing equipment, hydroacoustic and navigation and positioning and other technical functions, together to complete the seabed mapping work. The system acoustic wave is sent and received to and from the target sound through the transducer array at a fixed point, and then different sensing equipment is used to calculate the specific position of the beam detection point respectively, so as to obtain the bathymetry data that maintains a perpendicular angle with the heading[3].

Nowadays, researchers at home and abroad have carried out more in-depth studies on multibeam bathymetry technology, and have proposed a variety of methods to help coarse detection and rejection when pre-processing the data extracted from the transducer, such as statistical feature-based filtering, trend surface filtering, and so on[4]. The improvement of these methods effectively improves the reliability of multibeam bathymetric computation data and the accuracy of measurement.

For the sea area with known area, central sea depth and slope angle, this paper researches the theory of multibeam line bathymetry based on the optimization model of geometric analysis and greedy algorithm, establishes the optimal model of line deploymen.[5] The model uses the least squares method to obtain the fitted surface function of the depth of the sea area and then uses the K-

means clustering method to cluster the gradient of each point in the sea area. Based on the established optimization model, the survey line layout scheme that meets the requirements is obtained[6].

2. Problem description and modeling

2.1. Description of the issue

The model needs to solve the following problems: First, it is known that a rectangular sea area with a length of 2 nautical miles from north to south and a width of 4 nautical miles from east to west, the depth of the sea water at the center of the sea area is 110 m, the depth is deep in the west and shallow in the east, the slope is 1.5 degrees, and the opening angle of the multibeam transducer is 120 degrees. Design a set of measurement lines with the shortest measurement length, which can completely cover the whole sea area to be measured, and the overlap rate between neighboring strips meets the requirement of 10%~20%. Secondly, a single-beam bathymetric data of a rectangular sea area is known, and a multibeam line layout scheme is designed, requiring that the scanning strips of the line cover the sea area to be measured as much as possible, and the overlap rate of the neighboring strips is less than 20% as much as possible, and the total length of the line is as short as possible. The total length, the percentage of sea area missed and the total length of lines with an overlap rate of more than 20% were also calculated according to the line scheme involved[7].

In order to make the model have a wider scope of application and simplify the constraint indicators, this paper makes the following idealized assumptions:

(1) assuming that the topography of the sea area given in the title is absolutely ideal and the slope can be kept constant;

(2) the assumption that the ship is an ideal point of mass, can travel along the measurement line measurement, not affected by wind, sea water conditions and other external interference elements;

2.2. Line length optimization model based on greedy algorithm[8]

2.2.1 Solving for coverage width and overlap ratio

In the Code for Hydrographic Surveying (GB12327-1988) and Technical Requirements for Measurement of Multibeam Bathymetric Systems (JT/T 790-2010), it is pointed out that the survey lines in the survey area should be in parallel relationship, and the direction of the lines should be the same as the direction of the isobaths. In this way, the seabed coverage can be maximized and the effective swath width can be kept within a certain interval[9].

Multibeam bathymetric systems use transducers to transmit acoustic signals through the fan array (Fig 1.), each time tens or hundreds of independent broadband beams into the seafloor, the signal is transmitted by the seafloor reflection and then received by the receiver. Using the characteristics of acoustic wave transmission, the water depth data is calculated for a certain width of the sea area centered on the survey line. Where, if there is a slope angle in the sea area, it is the width of coverage along the slope.

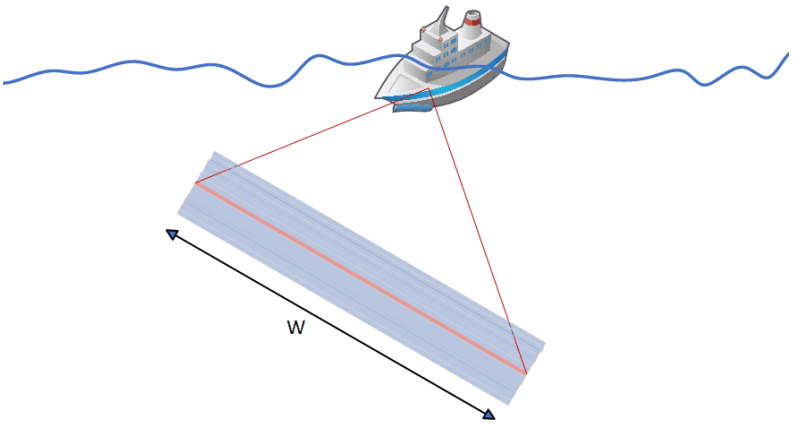


Fig. 1 Schematic diagram of multibeam bathymetry

The sector array of each launch can be regarded as an isosceles triangle with the location of the launch signal as the vertex, so the center point of the sea area (i.e. the initial position of the ship's measurement line) can be taken as the origin, and a right-angle coordinate system can be established with a straight line perpendicular to the sea level as the y-axis and a straight line perpendicular to the measurement line as the x-axis as shown in Fig 2. below:

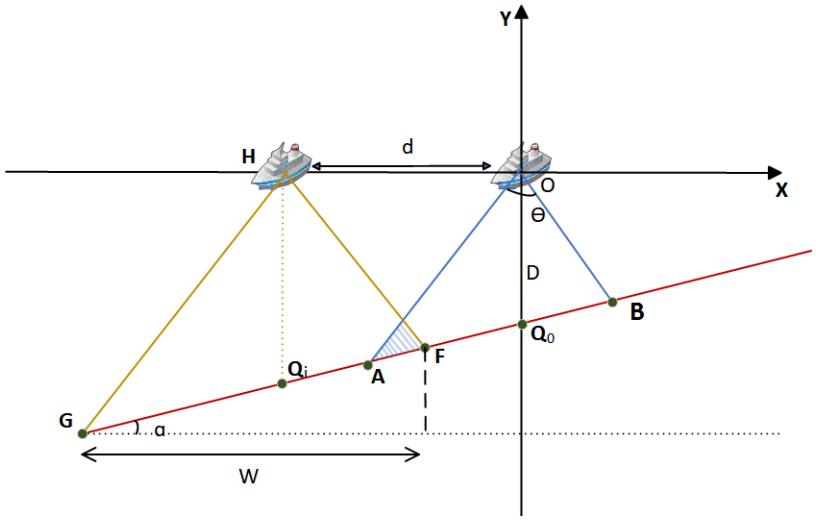


Fig. 2 Schematic diagram of building system

The Fig 2 draws a schematic diagram of the scanning area formed by the multiple beams emitted by the ship as it passes through the center point O of the sea area and as the ship is 200m away from the center point of the sea area (point H), each emission can be simplified to the x-axis and the origin of each point at a distance of 200m as a vertex to form a triangle with an apex angle of 120 degrees, with the bottom edge on a slope, and the required width of coverage is the bottom edge of each triangle.

First, the depth of seawater HQ at different survey lines needs to be calculated, assuming that $Q_i(x_i, y_i)$ is the coordinates of the seafloor at different survey lines, as can be seen from the title: $x_i = 200t_i (t_i = -4, -3, \cdots, 3, 4)$ So the depth of seawater can be expressed as:

$$HQ_i = -y_i = x_i \tan\alpha - 110 \tag{1}$$

In triangle OAC and triangle OBC, by the sine theorem:

$$\begin{cases} \dfrac{OC}{\sin(30° - \alpha)} = \dfrac{AC}{\sin(\theta/2)} \\ \dfrac{OC}{\sin(30° + \alpha)} = \dfrac{BC}{\sin(\theta/2)} \end{cases} \tag{2}$$

where OC is the depth of seawater y_i found above, substituted into equation (2). The coverage width W should in reality be the width of the strip seen looking down on the sea level, and since the sea is sloping at this point, there is a translation between this and the actual calculated coverage width: $W = (AC + CB) \times \cos\alpha$ .Finally the expression for the coverage width can be derived as follows:

$$W_i = \cos\alpha \times \sin\frac{\theta}{2} \times (200t_i \tan\alpha - 110) \times \left(\frac{1}{\sin(30° - \alpha)} + \frac{1}{\sin(30° + \alpha)}\right) \tag{3}$$

The overlap rate between neighboring strips is defined as $\eta = 1 - d / W_{i-1}$, $d(d = 200)$ is the spacing between two neighboring stripes, and W_{i-1} is the coverage width of the first measurement line along the slope, which can be obtained by substituting the above obtained stripe coverage width:

$$\eta = 1 - \frac{200}{\sin\frac{\theta}{2} \times (200 t_{i-1} \tan\alpha - 110) \times (\frac{1}{\sin(30° - \alpha)} + \frac{1}{\sin(30° + \alpha)})} \tag{4}$$

2.2.2 Optimal planning model for line placement

When the seabed depth is deeper, the effective measurement width is greater. When the overlap between all neighboring strips is $\eta = 10\%$, the lines are sparse in deeper waters. Therefore, the ideal situation is that the overlap between adjacent strips of all lines can reach a minimum of 10%, so that the theoretical minimum number of lines is required. In practice, it may not be possible to scan the entire sea area by forcing $\eta = 10\%$, so it is necessary to adjust the strip overlap between some of the lines to meet the requirements and to minimize the total length of the lines[10].

Firstly, we analyze the profile of the sea area, and take the straight line perpendicular to the survey line and on the sea level as the X-axis, and the survey line where the ship is located at the midline point of the sea area as the origin to build the system. Let the coordinates of the first westernmost survey line be $A(x_0, 0)$, and the spacing between the two neighboring survey lines be d_i, as shown in Fig 3 below:

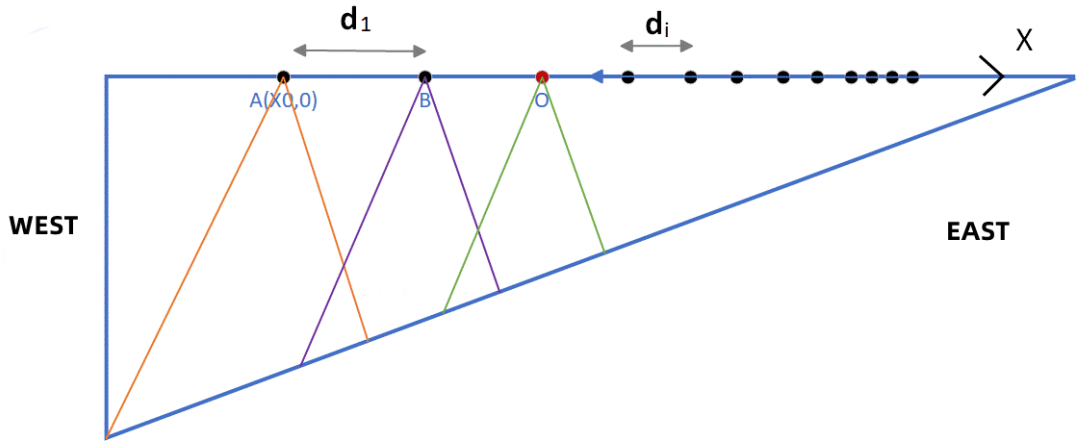


Fig. 3 Schematic of the profile of the sea area

Let a total of n survey lines need to be laid, the overlap between the i strip and the $i+1$ strip is η_i, and the objective planning function is established for this problem:

$$\min \sum_{i=1}^{n} \frac{\eta_i}{i} \tag{5}$$

The constraint is that the sum of the total spacing of adjacent strips is equal to the north-south breadth of the sea area of 2 nautical miles (1 nautical mile = 1,852 m):

$$\sum_{i=1}^{n} d_i = 7408 - (3704 - x_0) \tag{6}$$

Also based on the definition and requirement of the overlap rate given in the question, an objective planning model can be developed to solve for the minimum number of measurement lines:

$$\mathrm{H}(\eta_i,\mathrm{i})=\min\sum_{i=1}^{n}\frac{\eta_i}{i}$$

$$s.t\begin{cases}\sum_{i=1}^{n}d_i=7408-(3704-x_0)\\ 10\%\le\eta_i\le 20\%\\ W_i=\cos\alpha\times\sin\frac{\theta}{2}\times(x_i\tan\alpha-110)\times(\frac{1}{\sin(30°-\alpha)}+\frac{1}{\sin(30°+\alpha)})\\ \eta_i=1-\frac{d_i}{W_{i-1}}\end{cases}\tag{7}$$

2.3. An optimization model for sea area detection based on clustering algorithm

Based on the seawater depth detection data from the sea area measured by the single beam, a suitable surface is fitted to it using the least squares method to minimize the squared error between the observed data points and the fitted surface. The objective of the least squares method at this point is to minimize the following loss function:

$$L(\beta)=\sum_{i=1}^{n}(z_i-f(x_i,y_i,\beta))^2\tag{8}$$

Where β represents the parameter to be fitted to the surface, and the goal is to find the optimal value of that parameter to minimize $L(\beta)$. Suppose the fitting surface:

$$\begin{aligned}f(x,y,\beta)=\beta_{00}+\beta_{10}x+\beta_{01}y+\beta_{11}xy+\beta_{20}x^2+\\ \beta_{02}y^2+\beta_{21}x^2y+\beta_{12}xy^2+\beta_{03}y^3\end{aligned}\tag{9}$$

This is a polynomial surface model where $\beta_{00},\beta_{01},\beta_{10},\beta_{11},\beta_{20},\beta_{02},\beta_{21},\beta_{12},\beta_{03}$ is the parameter to be fitted, and in order to minimize $L(\beta)$, β needs to be optimized, which can be achieved by solving the following system of gradients of zero:

$$\frac{\partial L}{\partial\beta_{00}}=0,\quad\frac{\partial L}{\partial\beta_{01}}=0,\quad\frac{\partial L}{\partial\beta_{10}}=0,\quad\frac{\partial L}{\partial\beta_{11}}=0,\quad\cdots\tag{10}$$

This will result in the optimal solution for $\beta_{00},\beta_{01},\beta_{10},\beta_{11},\beta_{20},\beta_{02},\beta_{21},\beta_{12},\beta_{03}$, thus fitting the least squares surface.

The gradient of the seawater depth at each point is calculated, and the curve of the sea area has been fitted, which is a binary function $z=f(x,y)$ .Obviously, the function has a first-order continuous partial derivative on the plane region D, then a vector:

$$\left\{\frac{\partial f}{\partial x},\frac{\partial f}{\partial y}\right\}=f_x(x,y)\bar{i}+f_y(x,y)\bar{j}\tag{11}$$

can be determined for each point $P(x,y)$, namely:

$$gradf(x,y)=\nabla f(x,y)=\left\{\frac{\partial f}{\partial x},\frac{\partial f}{\partial y}\right\}=f_x(x,y)\bar{i}+f_y(x,y)\bar{j}\tag{12}$$

After the gradient is calculated, the sea area with similar gradient is classified into a class by K-means clustering, which can be regarded as a slope surface with known slope Angle. The model established in problem 3 is used to design survey line layout schemes for different sea areas respectively. Each area is arranged parallel to its isobath line, and the appropriate overlap rate is

selected to minimize the total length of the survey line, while covering the sea area as comprehensively as possible, and the survey line layout plan is drawn.

3. Result and conclusion

3.1. Result display

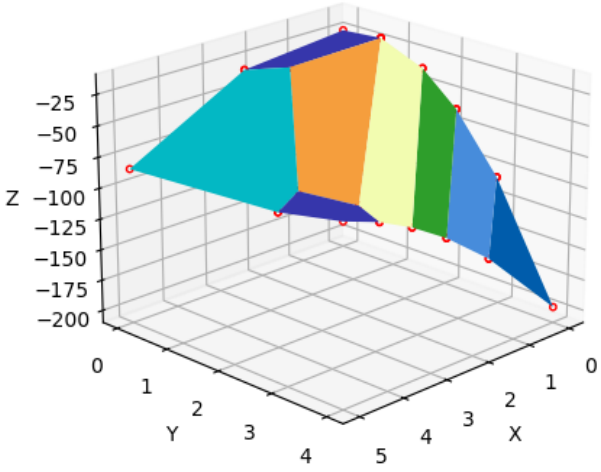


Fig. 4 Folded surface map after clustering of maritime terrain

The gradient of each point can be calculated from the seawater depth of each point. Firstly, hierarchical clustering is carried out, and after analysis, the topography of the sea area is determined to be grouped into 7 categories. As can be seen from Fig 4, there is a discontinuous sea area with similar gradient, so the sea area can be converted into 8 slope areas with different slope angles, and then the greedy algorithm is used to arrange the measurement lines, as shown in the Fig 5 below:

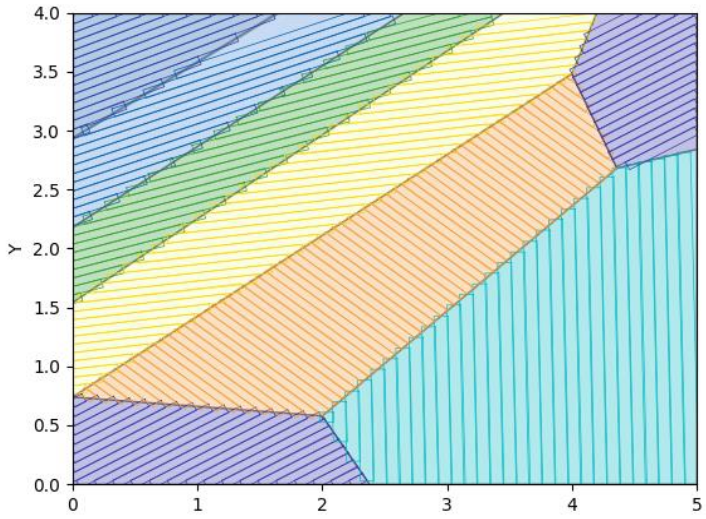


Fig. 5 Schematic diagram of the sea-wide survey line layout scheme

3.2. Result analysis

After calculation, it can be seen that the overlap rate of the survey line is basically between 10% and 20%, and the part of the overlap rate exceeding 20% only accounts for 0.4% of the total length of the survey line, and the omitted area only accounts for 2.538% of the area to be surveyed, which basically meets the requirements of the task of arranging the survey line for complex sea areas.

From the data point of view, the model proposed in this paper can better solve the task of complex sea layout line survey, and the computational complexity of the model is low.

4. Conclusion

Submarine topography provides important research data for submarine scientific research and marine engineering, but the complex undulating terrain of the seabed brings severe challenges to the mainstream multi-beam sounding methods. In this paper, a model of survey line layout scheme under complex sea terrain is constructed by fitting the places with similar gradients into a plane and arranging survey lines with greedy algorithm. The experimental results show that the survey lines arranged by the model basically achieve full coverage of the sea area and the overlap rate is not too large, and the computational complexity of the model is low, which has certain practical application value.

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