Research on multi-stage rocket debris localization and recovery based on three-sphere localization

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Abstract. Advances in modern rocket technology have created challenges in the field of spaceflight with regard to the localization and recovery of rocket debris. In particular, in multi-stage rocket structures, transonic sonic booms are generated after the separation of the next stage or booster, which increases the difficulty of wreckage recovery. To solve this problem, a mathematical model based on multi-sensor data and three-sphere localization technique is proposed for accurately determining the location of the sonic boom point. The theoretical equations were constructed from monitoring equipment data and transformed into an optimization problem for numerical solution, and the longitude, latitude, altitude and time of the sonic boom birth were successfully determined. In addition, the localization and differentiation methods of multiple wrecks are studied, and the alignment and repeated verification strategies are used to improve the localization accuracy and reliability. Finally, the model validation is carried out by adding random perturbations, and the results show that the new model can still effectively locate the wreckage under a small range of perturbations, which provides an important support for the development of rocket wreckage recovery technology.

Keywords: Rocket Debris Localization, Sonic Boom Points, Nonlinear Least Squares Method, Multi-sensor, Three-sphere Localization.

1. Introduction

With the development of space technology, rocket debris recovery has become a challenge.Positioning technologies such as UWB, GPS, BeiDou, and INS have received attention. This paper reviews the research progress of these technologies, focusing on their achievements in improving positioning accuracy, stability and multi-source information fusion. Some researches have proposed improved UWB positioning methods to enhance indoor positioning accuracy; in the area of rocket debris handling, some researches have also explored the application of small solid rocket motors and paid attention to debris disposal and risk control; and some researches have proposed rocket debris fallout calculation models and prediction methods, which provide theoretical support for precise recovery [1]. In terms of multi-source information fusion, research has achieved the fusion of multiple positioning information, improved prediction accuracy, and proposed new algorithms for map construction and positioning integrity monitoring; finally, research has also been conducted to achieve high-precision positioning in indoor environments, providing new ideas for indoor navigation and location services. The above methods have shortcomings such as high cost and complexity of multidisciplinary intersection as well as adaptability in different environments. In this paper, we optimise the model by increasing the number of detection devices and adding the interference of random errors In order to accurately locate the rocket debris, we first need to accurately analyse the location and time of the sonic boom of a single debris. A sonic boom is a short wave generated by an object travelling at speeds greater than the speed of sound, which can provide us with important positional information. After determining the sonic boom information for a single wreck, the article needs to further identify multiple wrecks and calculate where and when they produced sonic booms in the air. This process requires validation of random errors to check the accuracy of the model and thus improve the overall positioning accuracy [2]-[6].

In order to further improve the accuracy and reliability of the positioning, an error term can be introduced and corrected by multiple sampling and statistical analysis. Specifically, we can use mathematical methods such as least squares and Kalman filtering to optimise our model. Error

analysis not only improves the accuracy of the data, but also helps us identify and correct possible systematic biases [7]-[10].

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When choosing a mathematical model and optimisation algorithm, we need to base our choice on the specific problem. For example, if the motion trajectory of the wreckage is more complex, we may need to use a three-dimensional motion model to describe its motion trajectory. In addition, considering the accuracy and reliability of the data, we also need to constantly update and improve the data processing methods to ensure that the final results obtained have a high degree of confidence.

In conclusion, by accurately predicting and controlling rocket orbits, adopting advanced recovery and positioning technologies and continuously improving data processing and analysis methods, we can achieve the safe and efficient recovery of rocket debris. This will not only help to reduce the impact of space debris on the Earth, but also provide more resources and security for future space missions. Through technological innovation and comprehensive responses, we are confident of overcoming the challenges we currently face and contributing to the great cause of human space exploration.

2. Model Preparation

2.1. Model Assumptions

(1) Assume that the propagation speed of the vibration wave is fixed at 340m/s.

(2) Assume that the curvature of the ground can be ignored when calculating the distance between two points.

(3) The value of distance per degree of latitude is approximated as 111.263 km, and the value of distance per degree of longitude is approximated as 97.304 km.

(4) It is assumed that the objects under study are all multi-stage rockets.

Assuming that the arrangement of monitoring devices obeys the three-sphere localization model.

2.2. Symbolic Description

Symbolic Description is shown in table 1.

Table 1. Symbolic Description

Serial number	Symbolic	Symbols indicate meaning
1	(X,Y,Z)	Coordinates of Sonic Boom Location
2	(Xi,Yi,Zi)	Coordinates of monitoring equipment
3	T_0 (s)	Time of explosion at sonic boom site
4	Li (m)	Distance of monitoring equipment from the point of sonic boom
5	V(m/s)	Speed of sound traveling through air

2.3. Single rocket wreckage modeling and solution

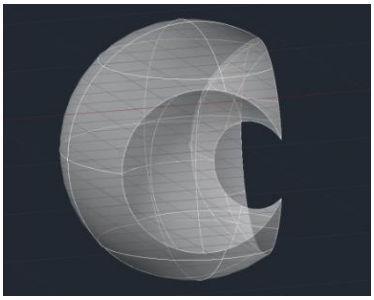


Figure 1. Schematic diagram of three-ball sum

We can use the three-sphere localization model to draw three spheres in space with the three monitoring devices as the center of the sphere and the distance from the sonic boom location to the monitoring devices as the radius. When two spheres intersect, a circle is formed; when three spheres intersect, two points are formed(The three-ball sum is shown in Figure 1). Ultimately, the fourth sphere is used to determine the correct location of the sonic boom point. The accuracy of this method depends on the location and precision of the arrangement of the monitoring equipment. Therefore, according to the principle of the three-sphere localization model, a minimum of four monitoring devices need to be arranged to achieve the precise positioning of the position of a single wreck when a sonic boom occurs. So that the location of the sonic boom point coordinates(x,y,z), the coordinates of the monitoring device one(x_1,y_1,z_1), the coordinates of the monitoring device two (x_2,y_2,z_2), device three(x_3,y_3,z_3), device four(x_4 ,y_4,z_4); the distance between the device one and the sonic boom point is L_1, the distance between the device two and the sonic boom point is L_2, the distance between the device three and the sonic boom point is L_3 and the distance between the device The distance between device four and the sonic boom point is L_4, then there are

$$(x_1-x)^2+(y_1-y)^2+(z_1-z)^2=L_1^2 \quad (1)$$

$$(x_2-x)^2+(y_2-y)^2+(z_2-z)^2=L_2^2 \quad (2)$$

$$(x_3-x)^2+(y_3-y)^2+(z_3-z)^2=L_3^2 \quad (3)$$

Substitute the data into (1), (2), (3) to solve the function to get x, y, z (at this point there should be two sets of solutions)

As the rocket recovery project requires high accuracy, so in order to minimize the loss and more accurate positioning, we also need a fourth monitoring device to do the positioning test, the resulting solution is brought into the (4) in the test, the equation is established then that is the sonic boom point.

$$(x_4-x)^2+(y_4-y)^2+(z_4-z)^2=L_4^2 \quad (4)$$

So, let's start with data organization of the given data: (the sonic boom does not occur at the 0th second, set the device A as the zero point coordinates, and transform the whole sphere into a plane). Distance of the wreckage from the sonic boom point is shown in table 2.

Table 2. Distance of the wreckage from the sonic boom point

Installations	Longitude(°)	Latitude(°)	Altitude(m)	Distance from the sonic boom
A	0	0	824	34260.78-340t_0
B	52446.856	28038.276	727	38154.8-340t_0
C	45830.184	64643.803	742	63926.8-340t_0
D	973.04	69094.323	850	88054.9-340t_0
E	27537.03	69094.323	786	40270.62-340t_0
F	21990.704	79775.571	678	90736.14-340t_0
G	-18876.976	-9234.829	575	55428.16-340t_0

So with four equations,four unknowns, no equation is redundant, i.e., there should be a finite number of solutions to the equation. Bring the data from table 2 into (1),(2),(3),(4) to solve.

However, in practice, we find that there may be no solution in the actual arithmetic process, this is due to the fact that there is always a bias in the process of data recording and calculation, in order to solve the error in this regard, we transform the above problem into a constrained problem of inequality:

Define the relative distance difference between the sonic boom location and the ith observation point as

$$\sqrt{(x_i-x)^2+(y_i-y)^2+(z_i-z)^2}-(L_i-c)^2 \quad (5)$$

$$c = 340\mathrm{T0} \tag{6}$$

Define the objective function as minimizing the sum of all relative distance differences:

$$min \sum_{i=1}^{7} \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} - \ (L_i - c)^2 \tag{7}$$

The constraints are:
-100≤x≤100 ,-100≤y≤100, 0≤z≤10, -100≤c≤500
The correlation coefficient is shown in table 3:

Table 3. Correlation coefficient table

	X(km)	Y(km)	Z(km)	L(km)
A	0	0	0.824	34.2607
B	52.4468	28.0382	0.727	38.1548
C	45.8301	64.6438	0.742	63.9268
D	0.97304	69.0943	0.850	88.0549
E	27.5370	45.9516	0.786	40.2706
F	21.9907	79.7755	0.678	90.7361
G	-18.8769	-9.2348	0.575	55.4281

Solving the optimized model, the final result is obtained Longitude 110.584 Dimension 27.224 Height 787.120 Time 172.394. as shown in Fig. 2.

In summary, the position and time at which the wreckage experienced a sonic boom are.(110.584, 27.224, 787.120) and 172.394s

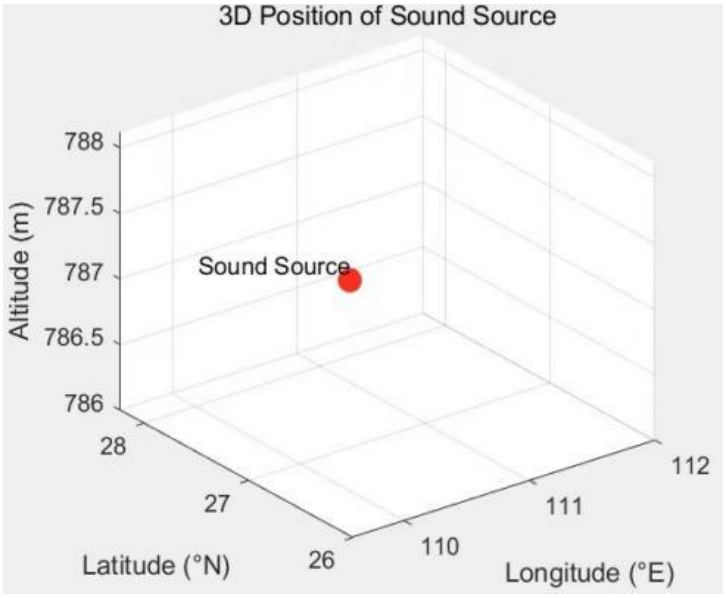


Figure 2. Schematic representation of the sonic boom location in three dimensions

2.4. Multiple rocket debris modeling and solutions

Define the relative distance difference square for a particular sonic boom point, relative to the ith observation point as:

$$min \sum_{i=1}^{7} \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} - (L_i - c)^2 \tag{8}$$

$$c = 340\mathrm{T0} \tag{9}$$

Take over all objective functions of the following optimization model:

$$min \sum_{i,\ j=1,\ 2,\ 3,\ 4}^{i,\ j} \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - y)^2} - (L_i - c)^2 \quad (10)$$

Establish the equation of the sphere and solve it in a 3D coordinate system:

$$(x_1\text{-}x)^2+(y_1\text{-}y)^2+(z_1\text{-}z)^2=L_1^2 \quad (11)$$

$$(x_2\text{-}x)^2+(y_2\text{-}y)^2+(z_2\text{-}z)^2=L_2^2 \quad (12)$$

$$(x_3\text{-}x)^2+(y_3\text{-}y)^2+(z_3\text{-}z)^2=L_3^2 \quad (13)$$

$$(x_4\text{-}x)^2+(y_4\text{-}y)^2+(z_4\text{-}z)^2=L_4^2 \quad (14)$$

That is, by controlling the value of L_{ij} in each device to take a distance, and then the distance to reach the best location of all the devices, and then by determining whether the result of the objective function is appropriate to determine whether it is a sound outbreak birth location.

The different combinations are analyzed using numerical traversal. The specific steps are as follows:

1. Selection of suitable monitoring data: first, suitable records need to be selected from the monitoring data provided. These records should be able to accurately reflect the position and time information of the wreckage for subsequent analysis.

2. Control the distance between monitoring devices: set a suitable distance L_{ij} between each monitoring device. this distance can be set according to the actual situation to ensure that it can cover the possible locations of sound outbreaks.

3. Calculate the location of the wreckage where the sonic boom may occur: from the records of each monitoring device, take out the distance of the corresponding time point and substitute these distances into the three-sphere localization model. This gives the location of the wreckage where the sonic boom is likely to occur.

4. Traverse different combinations: By traversing different combinations, i.e., different combinations of monitoring data and equipment distances, we can get multiple possible locations and times of sonic booms.

5. Determine the most appropriate sound outbreak location: For each combination, an objective function can be used to evaluate whether the conditions for sound outbreaks are met. If the results of the objective function are as expected, it means that the combination may be the most appropriate place and time for the sound burst.

In this process, it is necessary to ensure that the selected monitoring data accurately reflect the positional and temporal information of the wreckage, as well as the proper understanding and application of the three-sphere localization model. Through accurate data processing and analysis, combined with the guidance of the theoretical model, we can effectively solve problem three and obtain an accurate location and time of the sound outbreak. This method makes full use of the monitoring data provided and combines the advantages of the theoretical model to provide reliable theoretical support and practical guidance for problem solving.

The results obtained after bringing the data into the model are:

First set of explosions:	110.812°E, 28.172°N, 737m, time: 87.265s
Second set of explosions:	110.842°E, 27.638°N, 748m, time: 134.610s
Third group of explosions:	110.858°E, 27.771°N, 726m, time: 176.643s
Blast Group IV:	114.733°E, 19.759°N, 624m, time: 213.875s

The position of the wreckage in three-bit space is shown in Figure 3.

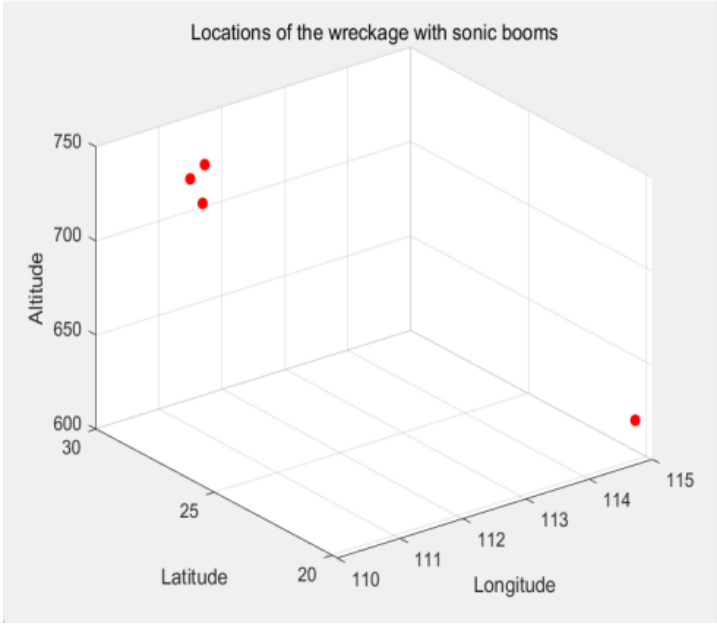


Figure 3. Schematic representation of the wreckage in three-dimensional space

2.5. Solution of time error on the localization of rocket debris

In this case, we include a random error with a time error of ±0.5, which means that our observations may be biased in time by up to ±0.5 seconds. To account for this temporal error in the model, we can express it as a parameter, e.g., (△t), indicating the range of temporal errors in the observations. Then, in the model, we can express the time of each observation as (t_0±△t), where (t0) is the true time of the observation. This kind of error is common in real scenarios and may be caused by factors such as the accuracy of the measurement equipment and environmental conditions. In order to reflect the impact of this error on the localization results, we need to consider this time error in the model. Next, we can use the same least squares solver as the solution in 3.2 to solve this model with time error. By taking the time error into account, we can get more accurate localization results, which can better reflect the characteristics of the observed data in real situations. Comparing the above results with the current results, we can analyze the effect of time error on the positioning results. If the error is found to be too large, we can consider increasing the number of detectors or utilizing techniques such as Kalman filtering to reduce the error.

Increasing the number of detectors can provide more observations and thus increase the accuracy of localization. Kalman filtering, on the other hand, is a commonly used state estimation technique that can utilize the dynamic model of the system and the observation data to estimate the system state and correct the prediction results to reduce the error. These methods can help us better deal with the errors in the observation data in practical applications, so as to improve the accuracy and reliability of positioning. Meanwhile, the optimization solution is carried out by using the formula in 3.2.

The results obtained after bringing the data into the model are:

First set of explosions:	107.930°E, 20.466°N, 694m, time: 85.167s
Second set of explosions:	110.823°E, 27.638°N, 753m, time: 134.513s
Third group of explosions:	110.809°E, 27.768°N, 734m, time: 176.442s
Blast Group IV:	122.493°E, 1 .612°N, 727m, time: 207.791s

The position of the wreckage in three-bit space is shown in Figure 4.

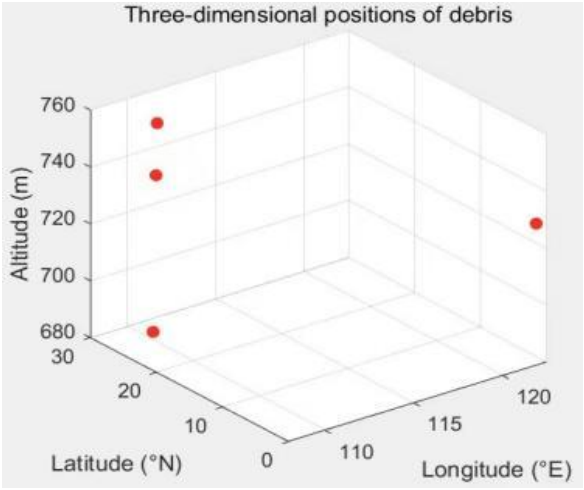


Figure 4. Schematic representation of the wreckage in three-dimensional space

It is assumed that there is some error in the timing of the sonic boom received by the device, which may differ from the original timing by 0 to 0.5 seconds. We repeat the operation using the model in 3.2 to get the classification results for the second time. Theoretically, these results should deviate slightly from those in 3.2, which would demonstrate that there is some bias in the modeling and that a corrective model is needed.

In practice, we will find that even small errors can lead to inconsistent classification results. This inconsistency may stem from the model's sensitivity to the data, as well as measurement errors that are difficult to avoid in a real-world environment. Therefore, we need to further improve the model by taking these error factors into account to increase its accuracy and stability.

2.6. Model improvements

In order to improve the accuracy of the localization model, we can take measures to ensure that the model can accurately reflect the actual situation of acoustic wave propagation.

First of all, it is crucial to study in depth the change rule of sound wave propagation speed in different media. Through experiments and theoretical analysis, we can establish the propagation model of sound waves in different media, so as to more accurately predict the propagation path and propagation speed of sound waves to reduce the error.

Secondly, utilizing more accurate locators is also the key to improving positioning accuracy. The development of modern positioning technology enables us to utilize high-precision devices such as satellite positioning systems and inertial navigation systems to obtain the position information of the detector, further reducing the positioning error.

Finally, the effect of ground curvature on acoustic wave propagation needs to be considered in practical application. The earth is not flat, and the propagation path of sound waves will be curved by the influence of ground curvature. Therefore, the ground curvature needs to be corrected in the positioning model, and the corrected propagation path is incorporated into the model to improve the applicability and accuracy of the model at different distances.

The measures mentioned above provide an effective way to improve the accuracy of the localization model. In-depth study of the changing law of propagation velocity of sound waves in different media and the establishment of propagation model can reduce the error and increase the accuracy of the model.

3. Conclusions

This sonic boom monitoring point-based approach not only has significant potential for application in space debris recovery, but also has a number of compelling possibilities for future directions. First, its real-time localization capability and relatively low cost make it particularly effective in dealing with emergencies and complex environments, where the ability to quickly and accurately locate

debris contributes to increased safety and efficiency. In the future, as technology continues to advance and intelligent systems are applied, for example, in combination with artificial intelligence and big data analytics, monitoring accuracy and response speed can be further improved, enabling more efficient debris recovery and management. In addition, global standardization and promotion of regulations will provide a solid operational foundation for its application, ensuring its wide application and sustainable development in different countries and regions. Together, these aspects contribute to the widespread application and future development of this technology in the field of space debris management.

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