Study on Sex Ratio of Lampreys Based on Logistic Regression Model

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Abstract. The sex ratio of lampreys is subject to fluctuations influenced by external environmental factors, and alterations in this ratio may exert a significant impact on the stability of the surrounding ecosystem. To assess the position of lampreys within the food chain and comprehend the consequences of sex ratio variations on other species and ecological balance, this passage constructed an ecosystem dynamics model, a sex ratio variation model, and a logistic model. Additionally, this passage integrated food chain action factors and environmental variables for comprehensive analysis. The findings indicate that sex ratio fluctuations in lampreys have profound implications for their reproduction rate, population density, genetic diversity, and the overall stability of the ecosystem. If one sex significantly outnumbers the other, competition will increase and reproductive rates may decline. Resources can be overused, affecting the population as a whole. If the proportion of males in lampreys increases, the number of species they prey on May decrease, affecting the balance of the food chain. If the number of a particular sex is significantly too high or too low, it can lead to a reduction in genetic diversity, thereby increasing the genetic risk to the population. Our research contributes to a better understanding of the process of lampreys sex ratio change and provides a systematic analytical framework for predicting ecosystem dynamics, which can maintain ecosystem stability and improve the sustainable management and protection of the natural environment.

Keywords: Ecosystem Dynamics Modeling, Sex Ratio Change Modeling, Logistic model, Sex Ratio Shifts

1. Introduction

Existing studies have shown that the sex ratio of Lamprey varies due to the external environment and deviates from the traditional 1:1 sex ratio [1-2]. Lamprey becomes male or female depending on how fast they grow during the larval stage. The growth rate of the larvae is affected by the food supply [3]. In environments with less food available, growth rates are lower, and the proportion of male Lamprey can reach about 78%. In environments where food is more readily available, the proportion of males is about 56% of the population (Data from 2024 MCM Question A: Resource Availability and Sex Ratio). Changes in Lamprey's sex ratio could alter the ecological balance. If these changes lead to fluctuations in population density, they can have an impact on the entire food chain in the ecosystem. In addition, changes in the sex ratio may affect the functional role of eels within a larger ecological framework.

After establishing ecosystem dynamics, sex ratio change, and Logistic models, this passage conducted a thorough examination of lampreys' position within the food chain. Furthermore, this passage analyzed the impact of sex ratio fluctuations in lampreys on other species and the regulation of ecological balance. Our findings indicate that alterations in sex ratio have the potential to influence lamprey reproduction rates, population density, genetic diversity, and ultimately, the stability of the entire ecosystem [4].

2. The fundamentals of Model

2.1. Logistic regression model

The Logistic regression model was proposed by the statistician David Cox. Its essence is to fit the data into the Logistic model to predict the possibility of an event. Because the dependent variable is dichotomous (or multi-classified), it can represent the probability that an event will occur or not occur.

If the binary variable y represents the likelihood of an event, $y = 1$ indicates the event occurrence and $y = 0$ represents the incident did not happen. If there are p deterministic variable $x_1, x_2, ..., x_p$ s with influence on the value of the variable y, Simple linear regression models were considered

$$y_i = x_i^T \beta + \varepsilon_i \tag{1}$$

The above formula ε_i called the random error term, has a mean of 0. If the event in i observation (i=1.... n). The probability of birth and the probability of an event not occurring are respectively

$$P(y_i = 1|x_i) = \pi_i = \frac{e^{x_i^T \beta}}{1+e^{x_i^T \beta}}, P(y_i = 0|x_i) = 1-\pi_i = \frac{1}{1+e^{x_i^T \beta}} \tag{2}$$

According to the definition of the expected value of the discrete random variable, the conditional mean of the dependent variable y_i can be defined as

$$E(y_i|x_i) = \pi_i = \frac{e^{x_i^T \beta}}{1+e^{x_i^T \beta}} \tag{3}$$

Since y is the qualitative variable taking the values 0 and 1, for a more accurate description of the Logistic regression model. The Logit function is introduced first, and the variable z is [0,1], then

$$\log it(z) = \ln(\frac{z}{1-z}) \tag{4}$$

The function (4) expands the value of the variable z to $(-\infty, +\infty)$, and this transformation is called the Logit transformation, so the Logistic regression model can be expressed as

$$\log it(\pi_i) = \ln(\frac{\pi_i}{1-\pi_i}) = x_i^T \beta, i = 1,..., n \tag{5}$$

The model, $\beta = (\beta_0, ..., \beta_p)^T$ indicates the regression coefficient, $x_i = (1, x_{1i}, x_{2i}, ..., x_{pi})^T$ is in non-occurrence $\frac{\pi_i}{1-\pi_i}$ and indicates the occurrence ratio of an event. The model above is also known as the Logistic fixed-intercept regression model.

In random sampling, the data source may have a hierarchical structure and the levels remain independent of each other, but the observations within the hierarchy are clustered, so the intercept term β_0 in the model (5). Value adds random effect, that is, constitute a new random intercept $\beta_{0j} = \beta_0 + b_j$, random intercept model layer j (j=1,2,..., m) and i th observation (i=1,2,...,n). The result is expressed as y_{ij}, and, for layer i, the conditional expectation of the dependent variable y_{ij} is

$$E(y_{ij}|x_{ij}, b_j) = \pi_{ij} \tag{6}$$

At this time, the Logistic random intercept regression model can be expressed as

$$logit(\pi_{ij}) = \ln(\frac{\pi_{ij}}{1-\pi_{ij}}) = f_{ij}^T \beta + b_j, i = 1,..., n, j = 1,..., m \tag{7}$$

where b_j representing the random part of different layers and is independent of each other, b_j following a certain distribution, The random intercept of the model (7)is b_j , $f_{ij} = (1, x_{ij1},..., x_{ijp})^T, \beta = (\beta_0,..., \beta_p)^T, x_j = x_{ij} = (x_{1j}, x_{2j},..., x_{nj})^T, X_j = (x_{ij1}, x_{ij2},..., x_{ijp})$

Note $F_j = (1_n \vdots X_j) = (f_{1j},..., f_{nj})^T$, F_j is the matrix of $n \times (p+1)$ dimension, $Y_j^* = (y_{1j}^*, y_{2j}^*,..., y_{nj}^*)^T, \pi_j = (\pi_{1j},..., \pi_{nj})^T$ the linear regression model corresponding to the j th layer of the random intercept model for convenient description. The meaning is

$$Y_j^* \triangleq logit(\pi_j) = F_j \beta + 1_n b_j \tag{8}$$

The whole body of the linear regression model corresponding to all layers can be expressed as

$$Y^* \triangleq \begin{pmatrix} F_1 \\ \vdots \\ F_m \end{pmatrix} \beta + \begin{pmatrix} 1_n & & 0 \\ & \ddots & \\ 0 & & 1_n \end{pmatrix} \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix} = F\beta + I_{nm} b \tag{9}$$

2.2. Model of sex-ratio change

The sex ratio changes in lamprey populations may be influenced by environmental factors (such as temperature, and food resources) and individual behaviour (such as competition, and reproductive strategies), and these variables are introduced to study the sex ratio changes in lamprey populations. Figure 1 shows the impact of the model machine required to study the sex ratio of lampreys.

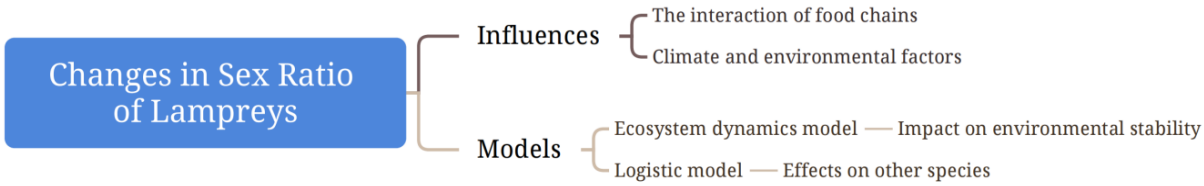


Figure 1. Changes in the Sex Ratio of Lampreys

An elementary differential equation model was employed to depict alterations in the size of lamprey populations and their sex ratio. Herein, N represents the aggregate number of lampreys, F denotes the female lamprey count, and M signifies the male lamprey count. On this basis, the subsequent difference equation was formulated.

$$\frac{dN}{dt} = \text{Birth rate - Mortality rate} \tag{10}$$

$$\frac{dF}{dt} = \text{Female birth rate - Female mortality rate} \tag{11}$$

$$\frac{dM}{dt} = \text{Male birth rate - Male mortality rate} \tag{12}$$

The rates of natality and mortality are influenced by numerous factors, including the availability of resources and predation pressure. To formulate a predictive model for alterations in sex ratios, this passage postulated that the sex ratios of lampreys are influenced by extrinsic environmental factors, particularly the availability of food. Such factors could potentially induce variations in the rates of natality and mortality among both male and female lampreys.

The male ratio (p_m) denotes the proportion of the number of males (M) to the overall population (P). It is calculated by dividing the number of males by the total population.

$$p_m = \frac{M}{P} \tag{13}$$

The female ratio (p_f) denotes the proportion of females (M) relative to the overall population (P). It is calculated by dividing the number of females by the total population to derive the ratio.

$$p_f = \frac{F}{P} \tag{14}$$

To investigate the influence of external environmental factors, such as food availability, on the sex ratio dynamics of lampreys, this passage have enhanced the previously mentioned differential equation model to incorporate these effects. This passage assume that food availability has a direct bearing on lamprey growth rates, ultimately shaping the birth rates of both female and male lampreys. Denoting the level of food availability as R, the expression for birth rate has been revised to:

$$\text{Female birth rate} = \alpha \cdot R \tag{15}$$

$$\text{Male birth rate} = \beta \cdot R \tag{16}$$

The parameters denoted by α and β pertain to food availability, signifying the proportional relationship between the birth rates of female and male lampreys and the level of food availability. In this model, it is postulated that the mortality rates of both female and male lampreys are independent of food availability. Consequently, the comprehensive model can be represented by the following system of differential equations.

$$\frac{dN}{dt} = \text{Birth rate - Mortality rate} \tag{17}$$

$$\frac{dF}{dt} = \alpha \cdot R \text{ - Female mortality rate} \tag{18}$$

$$\frac{dM}{dt} = \beta \cdot R \text{ - Male mortality rate} \tag{19}$$

3. Results

This passage established a logistic regression model to obtain the relationship between lampreys population changes and food, human activities, and food chains. In addition, taking into account parasite control and niche competition, this passage used a species interaction model to analyze the effects of changes in lampreys' sex ratio on parasite abundance and distribution and the effects on other species' niches. The analysis found that if one sex significantly outpaces the other, competition will increase and reproductive rates may decline [5]. Resources are overused, affecting the entire population. If the proportion of male lampreys increases, the number of prey species may decrease, and the increase in the number of lampreys population may provide more food sources for predators on the food chain, but it may also lead to over-utilization of resources, which in turn affects the survival and reproduction of other species, thus affecting the balance of the food chain [6]. If the number of a given sex is significantly too high or too low, it may lead to a reduction in genetic diversity, which may increase the genetic risk of the population. The result is shown in Figure 2.

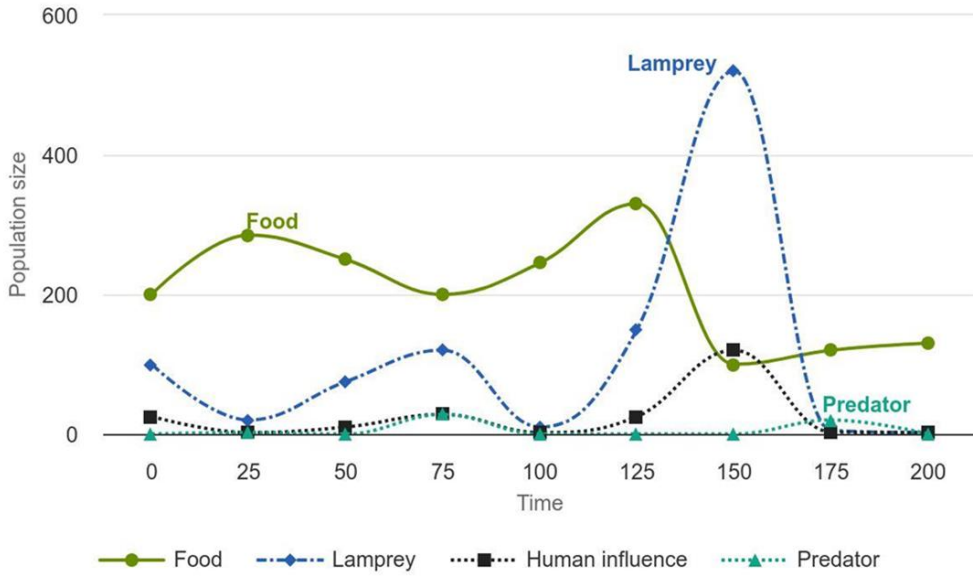


Figure 2. Ecosystem Simulation of Lampreys

Advantages in these cases may include increased reproductive potential and rapid adaptation to environmental changes [7]. If the sex ratio adjustment leads to an increase in the number of females, it may increase the reproductive potential of the population in the short term. Enables populations to adapt more quickly to environmental changes, such as fluctuations in food supply. Disadvantages may include reduced reproductive rate and increased population fluctuation. If the sex ratio results in fewer effective breeding individuals, it may reduce the reproductive rate of the entire population [8]. Changes in the sex ratio can lead to fluctuations in population size and structure, affecting its stability and predictability. The result is shown in Figure 3.

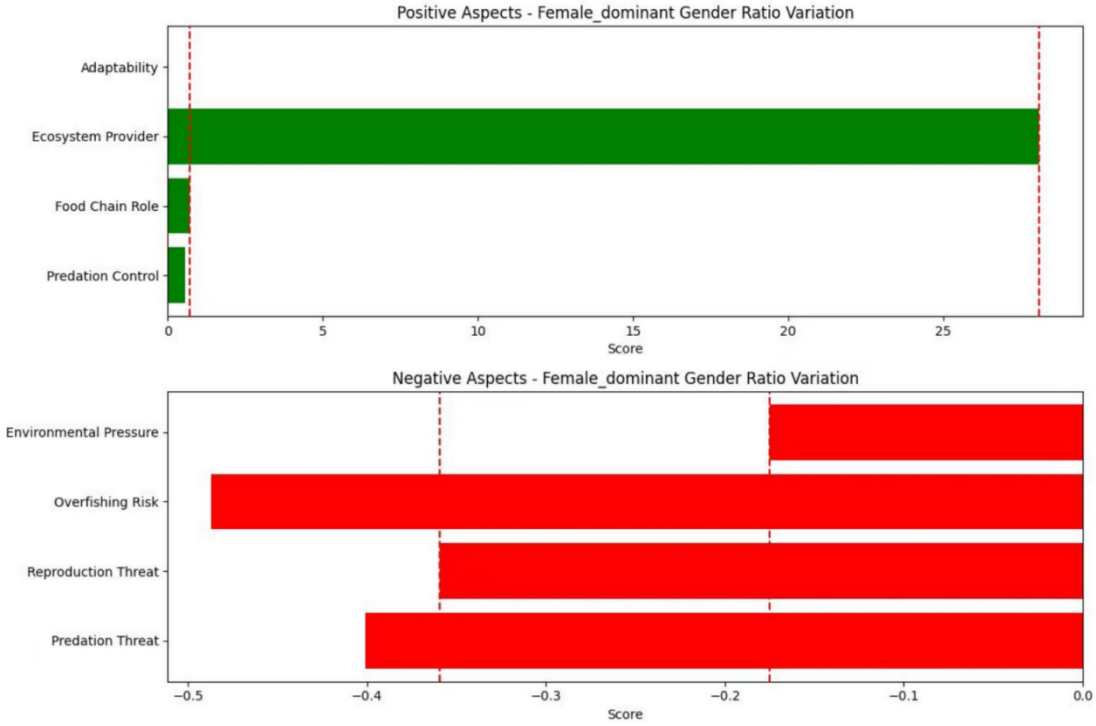


Figure 3. Advantages and Disadvantages of Female Dominance of Gender Ratio

Considering environmental factors, this passage study the effects of changes in the lamprey sex ratio on the ecosystem. Based on the stability of the ecosystem and the reproduction strategy of lampreys, mathematical models were used to study the effects of changes in the sex ratio of lamprey populations on the stability of the ecosystem [9].

It is hypothesized that changes in sex ratios are a strategy for lampreys to adapt to the current availability of environmental resources and that females may adjust their reproductive strategies under different sex ratios to maximize population survival and reproductive success. This passage speculate that changes in sex ratios may have an impact on overall ecosystem stability, while interdependence between lampreys and other species may contribute to or threaten overall ecosystem stability.

This passage established ecosystem stability models and Logistic models to study the effects of changes in lampreys' sex ratio on ecosystem stability, to explore the chain reaction and the risk of ecosystem collapse that may be triggered by changes in lampreys' sex ratio. The result is shown in Figure 4.

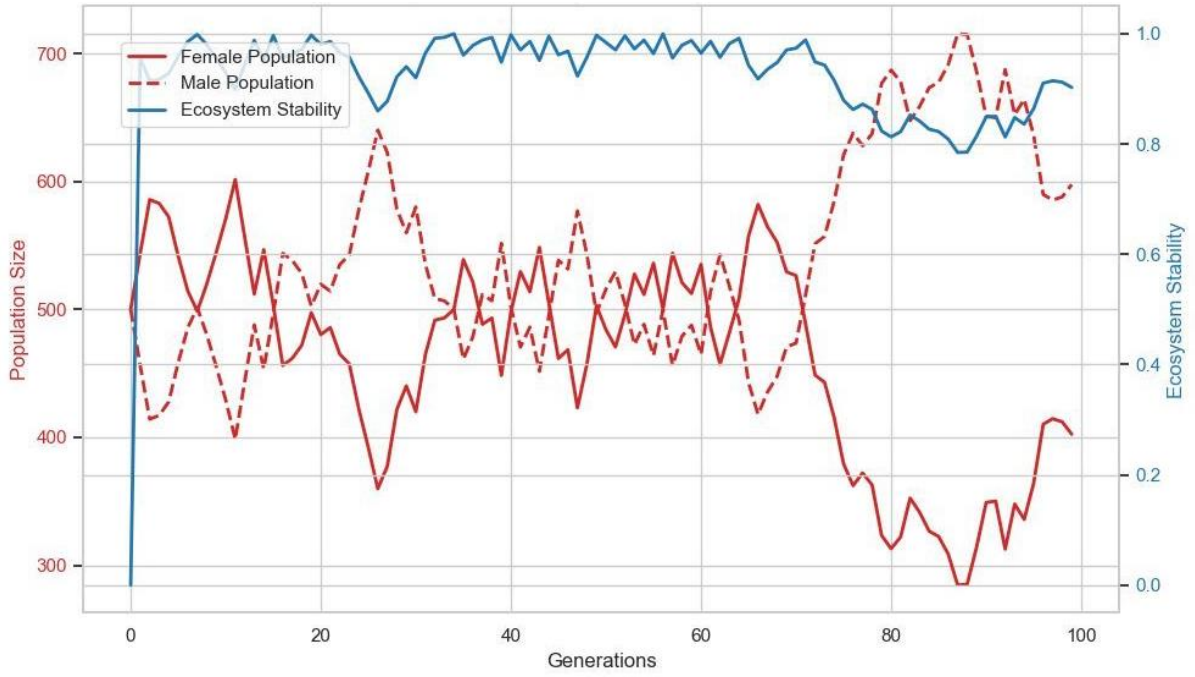


Figure 4. Population Dynamics and Ecosystem Stability of Lampreys

The mechanism of sex ratio change is introduced into the established ecosystem model, and the ecosystem dynamics under different sex ratio change schemes are simulated by setting the rules of sex ratio change, the influence of external factors or the behaviour of individuals [10]. It is concluded that changes in population density caused by changes in sex ratio will affect the food chain structure of the entire ecosystem, and may also affect the role and function of lampreys in the ecosystem. If a change in the sex ratio leads to an increase or decrease in population density, the food chain structure of the entire ecosystem may be affected.

4. Conclusions

This passage established the Logistic model, sex ratio change model and ecosystem dynamics model to examine the position of lampreys in the food chain and analyze the effects of sex ratio change on other species and ecosystem stability. Taking into account parasite control and niche competition, this passage also used species interaction models to analyze the effects of changing sex ratios in lampreys on parasite abundance and distribution and their effects on other species niches. To understand the chain reaction that changes in the sex ratio of lampreys can trigger and the risk of ecosystem collapse.

The results show that fluctuations in the sex ratio of lampreys have profound effects on their reproductive rate, population density, genetic diversity and the overall stability of the ecosystem. If there is significantly more of one sex than the other, competition will increase and reproduction rates may decline; Resources can be overused, affecting entire populations. If the proportion of male

lampreys increases, the number of species they prey on May decrease, affecting the balance of the food chain and affecting the stability of the ecosystem. If the number of a particular sex is significantly too high or too low, it can lead to a reduction in genetic diversity, thereby increasing the genetic risk of the population.

The advantage of these models is that they provide a systematic analytical framework for better-predicting ecosystem dynamics. These models can be applied not only in studies related to lampreys but also to other species and ecosystems. Through these studies, this passage can better maintain the stability of the ecosystem and improve the sustainable management and protection of the natural environment. The modelling process considered dynamic changes, especially the dynamic changes of various factors in the ecosystem were modelled, which helped to better understand the process of lamprey sex ratio changes. The modelling fully considers the resource dependence of lamprey sex ratio changes, which provides insights into understanding the ecology of this phenomenon.

References

[1] York J R, Thresher R E, McCauley D W. Applying functional genomics to the study of lamprey development and sea lamprey population control[J]. Journal of Great Lakes Research, 2021, 47: S639-S649.

[2] Hagen R, Ortmann S, Elliger A, et al. Evidence for a male-biased sex ratio in the offspring of a large herbivore: The role of environmental conditions in the sex ratio variation [J]. Ecology and Evolution, 2022.

[3] Lewandoski S A, Bravener G A, Hrodey P J, et al. Environmental and Biological Factors Influence Migratory Sea Lamprey Catchability: Implications for Tracking Abundance in the Laurentian Great Lakes [J]. Journal of Fish and Wildlife Management, 2020, 11(1):68-79.

[4] Hume J B, Bravener G A, Flinn S, et al. What can commercial fishery data in the Great Lakes reveal about juvenile sea lamprey (Petromyzon marinus) ecology and management? [J]. Journal of Great Lakes Research, 2021(1).

[5] Lakshmi S, Karthik R. Mathematical Modelling on Lamprey GnRH-III using Autoregressive Logistic Processes [J]. International Journal of Computer Applications, 2014.

[6] Nhu V H, Shirzadi A, Shahabi H, et al. Shallow landslide susceptibility mapping: A comparison between logistic model tree, logistic regression, naïve Bayes tree, artificial neural network, and support vector machine algorithms [J]. International journal of environmental research and public health, 2020, 17(8): 2749.

[7] Geary W L, Bode M, Doherty T S, et al. A guide to ecosystem models and their environmental applications [J]. Nature Ecology & Evolution, 2020, 4(11): 1459-1471.

[8] Wang C J, Hudson J M, Géraldine Lassalle, et al. Impacts of a changing climate on native lamprey species: From physiology to ecosystem services [J]. Journal of Great Lakes Research, 2021(4).

[9] Lucas M C, Hume J B, Almeida P R, et al. Emerging conservation initiatives for lampreys: Research challenges and opportunities [J]. Journal of Great Lakes Research, 2020.

[10] Johnson N S, Swink W D, Brenden T O. Field study suggests that sex determination in sea lamprey is directly influenced by larval growth rate [J]. Proceedings of the Royal Society B: Biological Sciences, 2017, 284(1851): 20170262.