Multi-beam Depth Coverage Modeling and Solution Based on Geometric Relationships

Yipeng Liu [#], Xun Wang [#], Tingting Shan [*, #], Jing Shi [#]

Tianjin University of Technology Tianjin, China

* Corresponding Author Email: shantingting@tjut.edu.cn

#These authors contributed equally.

Abstract. In recent years, precise measurements have become increasingly important in various fields. Multibeam bathymetry, known for its wide coverage, high data accuracy, and efficiency, has been widely adopted in scientific research and engineering projects. However, its precision and efficiency are influenced by depth, slope, and survey line direction. Thus, optimizing survey line coverage and overlap is a pressing issue. This study establishes a mathematical model to explore the coverage width and overlap rate of multibeam bathymetry under different depth conditions. Using geometric relationships and trigonometric functions, the model relates measurement range to depth and slope, and its effectiveness is validated with actual survey data. The study proposes optimized survey routes, providing theoretical support and technical guidance for practical measurement work.

Keywords: Precise measurement, multibeam bathymetry, coverage optimization, mathematical model.

1. Introduction

In recent years, with the rapid development of science and technology, precise measurements have become increasingly important in various fields [1] [2] [3]. High-precision measurement and data analysis techniques are crucial making informed decisions. Traditional measurement methods often suffer from low accuracy and inefficiency, but modern advancements, such as multibeam bathymetry, offer new solutions to these issues [4, 5].

Multibeam bathymetry utilizes the propagation characteristics of sound waves in a medium by emitting and receiving multiple beams of sound to obtain detailed data on the survey area [6, 7]. This technology, known for its wide coverage, high data accuracy, and high efficiency, has been widely adopted in various scientific research and engineering projects. However, the accuracy and efficiency of multibeam bathymetry are influenced by numerous factors in practical applications, such as water depth, seabed slope, and the direction of the survey lines [8, 9]. Therefore, optimizing the coverage and overlap rate of survey lines to enhance measurement accuracy and efficiency is a pressing research problem [10].

This study aims to explore the coverage width and overlap rate of multibeam bathymetry under different water depth conditions by establishing a mathematical model. It analyzes the relationship between the direction of the survey lines and the coverage area and proposes optimized survey routes. Through the application of geometric relationships and trigonometric functions, the study establishes a model relating the measurement range to water depth and slope. It validates the model's effectiveness using actual survey data and provides specific optimization schemes for survey routes based on the optimized model, offering theoretical support and technical guidance for practical measurement work.

2. The Influence of Sea Depth on Multibeam Survey Line Coverage

The data for this study is sourced from http://www.mcm.edu.cn/. To simplify the scope of the research, a specific marine area is chosen for multibeam survey line coverage modeling. This study focuses on modeling a single survey process, aiming to address the width covered by light waves at

different seawater depths and the overlap rate with the previous survey line. The steps for establishing the model are as follows:

This study defines the parameters as follows: the slope gradient is 1.5°, the opening angle of the multibeam transducer is 120°, and the central seawater depth is 70 meters. The swath coverage width needs to be determined. First, using geometric relationships, we can derive an expression for the relationship between various seawater depths and the distance from the center point of the survey line.

$$H = \tan(h - \tan\alpha * d) \tag{1}$$

Next, by calculating the seawater depth at each distance, and then applying the law of sines, we can derive the expression for the horizontal coverage width about the distance from the center point of the survey line.

$$W = \frac{H * \sin\theta}{\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} * \tan^2\alpha} \tag{2}$$

Finally, calculate the coverage rate between adjacent survey lines:

$$\eta = 1 - \frac{d}{W} \tag{3}$$

Using the above model, this study can determine the coverage width at different seawater depths and establish a mathematical model for the overlap between adjacent swaths. As shown in Figure 1:

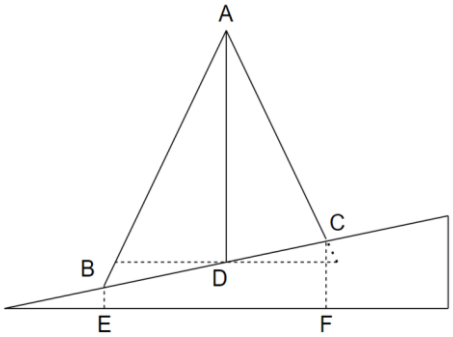


Figure 1. Schematic diagram of the model (where EF represents the coverage area).

Furthermore, by using geometric relationships, a simulation can be drawn showing the coverage width EC of light waves emitted by the vessel at a specific point on the slope. By drawing a perpendicular line AD from point D, sequential calculations can be carried out, with the results presented in Table 1.

Table 1. Survey Line Coverage Data.

Distance from Center Point of Survey Line (m)	-800	-600	-400	-200	0	200	400	600	800
Seawater Depth (m)	91.0	85.7	80.5	75.2	70.0	64.8	59.5	54.3	49.1
Coverage Width (m)	315.7	297.5	279.4	261.2	243.0	224.8	206.6	188.5	170.3
Overlap Rate with Previous Survey Line (%)	0.37	0.33	0.28	0.23	0.17	0.11	0.03	-0.06	-0.17

3. Relationship Between Survey Line Direction and Coverage Area in Multibeam Bathymetry

Building on the previous section, this paper further considers the rectangular survey area the angle between the survey line direction, and the projection of the seabed slope's normal vector on the horizontal plane. Therefore, additional constraints are added to the mathematical model established in the previous section to develop a mathematical model for the coverage width of multibeam echo sounding. The steps are as follows:

Define the parameters: the seawater depth at the center point of the marine area is 120 meters, and the angle between the survey line direction and the projection of the seabed slope's normal vector on the horizontal plane is specified.

Using the default direction in the problem as the positive direction, the center of the survey rectangle is set as the origin. The angle between the projection of the survey line direction onto the slope and its projection onto the horizontal plane is specified. This scenario is equivalent to the one described in the previous section. As shown in Figure 2, using geometric principles, the following expression is derived:

$$\tan\gamma = \sin\beta * \tan\alpha \tag{4}$$

$$H = l * \cos\beta + 120 \tag{5}$$

$$W = \frac{H * \sin\theta}{\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} * \sin^2\beta * \tan^2\alpha} \tag{6}$$

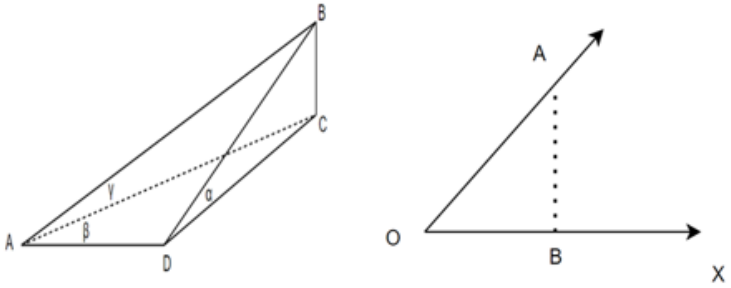


Figure 2. The left panel and right panel represent visualizations for the two respective directions.

Now calculate the seabed depth corresponding to each point. Assuming the distance from the survey vessel to the center point of the marine area is l and the survey line direction angle is β as shown in Figure 2, $\angle AOB$ is β, O is the center point, A is the location of the vessel, and OX is the projection of the sloped seabed direction onto the horizontal plane. The depth at the vessel's position is:

$$H(A) = 120 + l\cos\beta \tag{7}$$

Further, write code to input distance and angle to obtain the coverage width. The final results are shown in Table 2.

Table 2. Coverage Widths at Different Distances and Survey Line Angles.

Coverage width /m		Distance between the measuring ship and the center of the sea area (nautical miles)							
		0	0.3	0.6	0.9	1.2	1.5	1.8	2.1
Survey line direction angle /°	0	415.69	466.09	516.49	566.89	617.29	667.69	718.09	768.48
	45	416.31	452.00	487.69	523.38	559.07	594.76	630.45	666.15
	90	416.38	416.38	416.38	416.38	416.38	416.38	416.38	416.38
	135	415.70	380.06	344.72	308.79	243.15	237.51	201.87	166.23
	180	416.24	365.78	315.31	264.84	214.39	163.91	113.45	62.98
	225	416.43	380.73	345.03	309.33	273.63	237.93	202.23	166.53
	270	415.72	415.72	415.72	415.72	415.72	415.72	415.72	415.72
	315	416.17	451.85	487.52	523.20	558.89	594.56	630.23	665.91

4. Line Optimization Model and Coverage Area Analysis

Using the northeast corner as the origin, the positive direction of the X-axis as west, and the positive direction of the Z-axis as up, establish a right-handed Cartesian coordinate system. The total length of the survey line is influenced by two factors: direction and spacing, both of which are constrained by the overlap rate. Since the other scenario is similar, this model only considers cases

where the survey line slope is greater than zero. In this context, the scenarios can be roughly divided into four cases:

1. The survey line is parallel to the X-axis.
2. The survey line passing through the origin intersects with the boundary of the deep-water area.
3. The survey line passing through the origin does not intersect with the boundary of the deep-water area.
4. The survey line is perpendicular to the X-axis.

For any given survey line, there are at least two intersection points with the boundary of the water area. Let the intersection point closer to the Y-axis be A and the other be B Let the slope of the line be m. The coverage area of this survey line is analyzed as follows: Each point's coverage range is linearly related to the depth and the angle between the survey line direction and the horizontal projection of the slope's normal vector. In this problem, each point's depth is also linearly related to its X-coordinate. Therefore, the coverage range at every point on the survey line can be determined. Since the overlap rate needs to be between 10% and 20%, the following analysis applies:

$$10\% \le 1-\frac{d}{W} \le 20\% \tag{8}$$

Furthermore, the following conclusions can be derived:

$$0.8W \le d \le 0.9W \tag{9}$$

Since any point on this survey line must satisfy the above equation, it is only necessary to ensure that:

$$0.8W_{\max} \le d \le 0.9W_{\min} \tag{10}$$

The relationship between the coverage width at each point and its x-coordinate is given as follows:

$$h = 110 + (x - 3704)\tan\alpha \tag{11}$$

$$W = h\frac{\sin\theta}{\cos^2(0.5\theta) - \sin^2(0.5\theta)\tan^2\alpha\sin^2\beta} \tag{12}$$

Now, let's discuss each of the four scenarios separately.

When the survey line is parallel to the X-axis, it intersects the left and right boundaries of the water area. At this time, x_A is 0 and x_B is 7408. When substituted into the formula, the variable d does not exist, so this scenario is excluded.

For the scenario where the survey line passing through the origin intersects the boundary of the deep-water area: At this time, x_A is 0 and x_B is 7408, which is the same as the first case, and therefore this scenario is also excluded.

When the survey line passing through the origin intersects the boundary of the deep-water area, x_A is 0 and x_B is 7408, which is the same as the previous case, and therefore this scenario is also excluded.

For the remaining cases, when the survey line passing through the origin intersects the boundary of the deep-water area, as shown in Figure 3, the coverage width of the survey line reaches its minimum value at point A and its maximum value at point B. Therefore, it must satisfy the following conditions:

$$0.8W_B \le d \le 0.9W_A \tag{13}$$

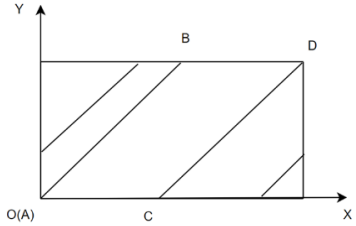


Figure 3. Schematic diagram of the scenario A.

$$W_B = (110 + (\frac{3704}{\tan\beta} - 3704)\tan\alpha)\frac{\sin\theta}{\cos^2(0.5\theta) - \sin^2(0.5\theta)\tan^2\alpha\sin^2\beta} \tag{14}$$

$$W_A = (110 + (0 - 3704)\tan\alpha)\frac{\sin\theta}{\cos^2(0.5\theta) - \sin^2(0.5\theta)\tan^2\alpha\sin^2\beta} \tag{15}$$

By solving the simultaneous equations, the range of values β can be determined.

$$\tan(\beta) \geq \frac{8*3704*\tan\alpha}{110 - 3704\tan\alpha} \tag{16}$$

Considering the survey line on the left side of AB, compared to AB, the minimum overlap rate remains unchanged while the maximum overlap rate decreases. This is because the angle β\betaβ makes the overlap rate decrease, so this is valid.

Similarly, considering the survey line intersecting with the upper and lower boundaries, compared to the survey line CD, the minimum overlap rate increases while the maximum overlap rate remains unchanged, which is also valid.

The value of β is related to α. The final width of the sea area depends on the depth and length of the sea area and is independent of the coordinates.

Next, this study establish a model to find the optimal survey route:

It is not difficult to see that the total length of the survey line L is determined by the survey direction β\betaβ, the survey line spacing d, and the initial starting position. Therefore, a simplification is made here by setting a single starting point, meaning that survey lines in all directions originate from the coordinate origin, as shown in Figure 4.

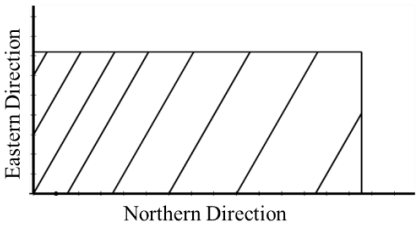


Figure 4. Schematic diagram of the scenario B.

Assume the inclination angle of this set of survey lines is β, the slope $k = \tan\beta$, the equation of the survey line l0 passing through the origin is:

$$y = x*\tan\beta \tag{17}$$

To minimize the total survey route, it is evident that, for a given β the larger the d the shorter the total route. Based on previous analysis, the maximum value of d ensures that each adjacent survey line overlaps by 90%. First, consider the upper left region. Define the distance $l1$ as the nearest survey line, with its equation set as follows:

$$l_1 = x*\tan\beta + b_1 \tag{18}$$

$$d_1 = 0.9w_0 = \frac{b_1}{\sqrt{\tan^2+1}} \tag{19}$$

From this, we can solve for l1. Similarly, through l1, we can find l2, and so on. It is observed that due to the minimum coverage constraint in the right region, the survey line spacing is fixed, and the length of each survey line varies linearly.

Next, consider the lower right region. Now define the distance l-1 as the nearest survey line, denoted as l-1, with its equation set as follows:

$$l_{-1} = x*\tan\beta + b_{-1} \tag{20}$$

$$d_1 = 0.9w_0 = \frac{b_{-1}}{\sqrt{\tan^2+1}} \tag{21}$$

By solving for the equation of l-1, we can then determine its intersection with the X-axis. Using these intersections, we solve for l-2. By continuing this process, it is found that the value of d gradually increases.

Combining the above considerations, the survey line spacing initially decreases and then gradually increases, as shown in Figure 4. From this, we derive the optimal survey route.

Next, we calculate the total length of the survey route. We determine the intersection points of each survey line with the boundaries of the water area. The difference between the two horizontal coordinates, multiplied by $\sqrt{1+\tan^2\beta}$, gives the length of each survey line. Thus, the total length L of the survey lines is expressed as:

$$\mathrm{L}_{总} = \sum_i (x_{ir} - x_{il})\sqrt{1+\tan^2\beta} \tag{22}$$

Where xir and xil are the horizontal coordinates of the two intersection points of the survey line with the water area boundary, and xir≥xil.

According to the results of problem two, when the measurement line direction is perpendicular to the projection direction of the slope normal vector on the horizontal plane, the coverage range of each measurement line remains unchanged and is divided into two sides. At this time, if l0 continues to start from the origin, it will completely waste half of l0's coverage range. Therefore, it cannot start from the origin again. Now, it is necessary to find an accurate x0 such that l0 passes through it and perfectly covers the shallow water area.

It only needs to satisfy that half of the coverage range at x0 is exactly equal to x0, which needs to satisfy the following formula:

$$x_0 = 0.5(110 + (x_0 - 3704)\tan\alpha)\frac{\sin\theta}{\cos^2(0.5\theta) - \sin^2(0.5\theta)\tan^2\alpha} \tag{23}$$

Let:

$$x_1 = x_0 + 0.9(110 + (x_0 - 3704)\tan\alpha)\frac{\sin\theta}{\cos^2(0.5\theta) - \sin^2(0.5\theta)\tan^2\alpha} \tag{24}$$

As the x-coordinate of l−1, sequentially obtain all the $x_0\ x_1\ x_2\ .....\ x_n$ The expression for the total length of the measurement line $\mathrm{L}_{\mathrm{total}}$ is:

$$\mathrm{L}_{总} = 3704*(n+1) \tag{25}$$

Finally, summarize the four situations, select the optimal solution in each case, and compare them with each other to find the shortest measurement line length and position.

In summary, the first two cases do not hold; for the third case, it follows from Figure 5 that there are three types of survey lines, called here left, center, and right.

According to the previous derivation, the measured lines in this section are equally spaced, so the total length can be easily calculated by knowing the spacing and slope, and the calculation procedure is given here:

$$n = \left\lfloor \frac{3704\cos\beta}{d_1} \right\rfloor \tag{26}$$

n Indicates that there are n measured lines in the upper left.

$$d_1 = (110 + (0 - 3704)\tan\alpha) * 0.9 * \frac{\sin\theta}{\cos^2(0.5\theta) - \sin^2(0.5\theta)\tan^2\alpha\sin^2\beta} \tag{27}$$

d_1 Denotes the distance between a line passing through the origin and a neighboring line.

There are similar triangles to get the total distance traveled by the upper left measuring line:

$$L_{左} = \frac{7408\cos\beta - n(n+1)d_1}{\sin 2\beta} \tag{28}$$

The distance between the first line on the right l_{-1} and l_0 is d_1, then the coordinates of its intersection with the x-axis are.

$$x_1 = \frac{d_1}{\sin\beta} = 0.9\frac{w(0)}{\sin\beta} \tag{29}$$

Thus its distance from the second line is.

$$d_2 = 0.9W(x_1) \tag{30}$$

The coordinates of the intersection of the second survey line with the X-axis are.

$$x_2 = x_1 + \frac{d_2}{\sin\beta} = x_1 + \frac{0.9W(x_1)}{\sin\beta} \tag{31}$$

And so on, until.

$$x_m \le 7408 \le x_{m+1} \tag{32}$$

Explain that there are a total of $n+1$ survey lines (including the one past the origin) in the middle and lower right sections.

Running it through your computer can find all the x_i, which exists:

$$x_{k-1} \le 7408 - \frac{3704}{\tan\beta} < x_k \tag{33}$$

It shows that there are k entries in the middle region and $n-k+1$ entries in the right region, with respective lengths:

$$L_{total} = \frac{3704k}{\sin\beta} \tag{34}$$

$$L_r = \frac{(7408 - x_k) + (7408 - x_{k+1}) + \ldots + (7408 - x_m)}{\cos\beta} \tag{35}$$

Combined, the total length can be calculated, and ultimately, the computer calculations are as follows:

The total length is minimized when β takes 89.371, which is 137,000 meters.

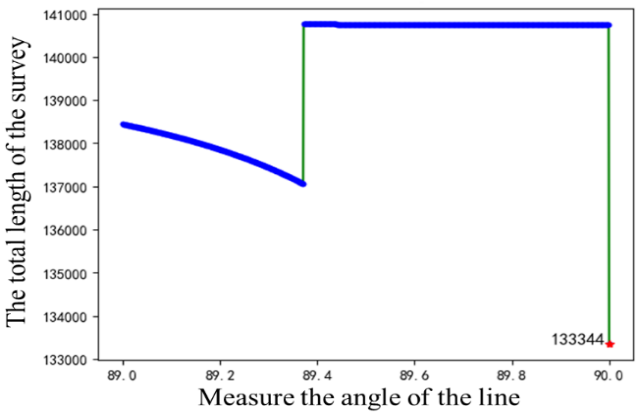


Figure 5. Measurement Line Angle Relationship Diagram.

For the fourth case, the computer run results in a total length of 133,000 meters.

Final summary: The optimized best route is: direction due south, with 36 survey lines, totaling 133,344 m. The distance of each line from the eastern boundary is shown in Table 3 below.

Table 3. Coverage Widths at Different Distances and Survey Line Angles.

serial number	1.0	2.0	3.0	4.0	5.0	6.0
gap	45.2	89.5	137.4	189.3	245.4	306.2
serial number	7.0	8.0	9.0	10.0	11.0	12.0
gap	371.8	442.9	519.8	602.9	692.9	790.2
serial number	13.0	14.0	15.0	16.0	17.0	18.0
gap	895.5	1,009.4	1,132.6	1,265.9	1,410.1	1,566.1
serial number	19.0	20.0	21.0	22.0	23.0	24.0
gap	1,734.8	1,917.4	2,114.9	2,328.5	2,559.7	2,809.7
serial number	25.0	26.0	27.0	28.0	29.0	30.0
gap	3,080.2	3,372.8	3,689.4	4,031.8	4,402.3	4,803.1
serial number	31.0	32.0	33.0	34.0	35.0	36.0
	5,236.6	5,705.7	6,213.1	6,762.0	7,355.8	7,998.2

5. Optimization of Survey Line Direction and Spacing for Irregular Seafloor Topography

Further, based on the previous research, this study changed the terrain in the water, so that it changed from a flat slope to an irregular shape, then we need to use the calculus method, combined with regression analysis, to find the optimal solution for the direction of the measurement line, and then according to the requirements of the overlap rate of the distance between the measurement line, to meet the conditions under the premise of the shortest distance to get the measurement of the route.

First, the model to find the normal vector of the tangent plane at each point of the seafloor is established:

This study sets the sea area as 5 nautical miles long from north to south and 4 nautical miles long from east to west, and there is a record for every 0.02 nautical miles, with a total of 200*250=50,000 positions of depth data.

A right-handed right-angle coordinate system is now established with the origin at the water surface in the southwest corner of the watershed, due east as the positive X-axis direction, and vertically down as the positive Z-axis direction, with one meter as the unit length.

Definition:

(1) $\mathrm{C}(m,n)$ denotes the depth of the m th from east to west and the n th from south to north (i.e., the point with positional coordinates of (0.02* * p m, 0.02* * p n), where it is considered that (0, 0) corresponds to the 0th point) point.

(2) D (m,n) Indicates the point with position coordinates (0.02* * p m, 0.02* * p n, $\mathrm{C}(m,n)$).

(3) x_m Denotes the horizontal coordinate of D (m,n) and y_n denotes the vertical coordinate of D (m,n).

(4) d (m,n) Denotes the tangent normal vector of the point with position coordinates (0.02* * p m, 0.02* * p n, $\mathrm{C}(m,n)$).

(5) p Indicates the conversion ratio between nautical miles and meters, i.e., 1852.

(6) $\alpha(m,n)$ Indicates the angle between the tangent plane where D (m,n) is located and the horizontal plane, i.e., the slope.

(7) $\beta(m,n)$ Denotes the angle between the direction perpendicular to the direction of projection d (m,n) on the horizontal plane and the X-axis, and its magnitude is between -π/2 and π/2.

(8) $\overline{\beta}$ Indicates the angle between the direction of the final optimized survey line and the X-axis.

(9) γ Indicates the angle between the actual direction of travel of the ship and the direction of the horizontal projection of the normal vector of the slope of the seabed directly below the ship.

(10) $W(m,n)$ Indicates the width of coverage when the ship located at coordinates (m,n, 0) is measured.

Using the differential method, we consider the plane defined by the points $D(m,n)$ $D\ m+1,(,n$), D ($m,n+1$) as a tangent to the point $D(m,n)$, then we can obtain:

$$d(m,n)=D(m,n)D(m,n+1)*D(m,n)D(m+1,n) \tag{36}$$

Substituting the data gives:

$$d(m,n)=(0.02(c(m+1,n)-c(m,n))p,0.02(c(m,n+1)-c(m,n))p),-0.04p) \tag{37}$$

And then can get:

$$\tan(\alpha(m,n))=\left|\frac{\sqrt{(c(m,n+1)-c(m,n))^2+(c(m+1,n)-c(m,n))^2}}{0.02p}\right| \tag{38}$$

According to the previous section, it can be concluded that the total length of the line is shortest when the direction of the line is perpendicular to the normal vector of the slope, so β is the optimal angle of the line for each point, and the expression β is calculated:

$$\tan(\beta(m,n))=-\frac{c(m+1,n)-c(m,n)}{c(m,n+1)-c(m,n)} \tag{39}$$

A total of 50,000 data are given in the attachment, so it is possible to come up with 50,000 ($tan\ \beta$), which nowadays has two directions:

One takes its average as the value of ($tan\ \bar{\beta}$);

One is to first calculate the value of β and then $\bar{\beta}$, the

Comparing the two algorithms, the first one is not valid because according to the simulated underwater contour map and the conclusion of the third question, the optimal direction of the survey line at each point should be the same as the tangent direction of the contour line, so there are β-values that are less than 90° and greater than 90°, and if the first algorithm is chosen, 89° and 91° will cancel each other and affect the results, so the second algorithm is chosen.

Brought into the program can be derived:

$$\bar{\beta}=58.40258336571707 \tag{40}$$

$$\tan(\bar{\beta})=1.625641032161666415441175691881 9 \tag{41}$$

Based on the formula in the previous section:

$$W=H\frac{\sin\theta}{\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}\sin^2\beta\tan^2\alpha} \tag{42}$$

Where θ for the multibeam transducer opening angle of 120 °, α for the slope, β for the direction of the measurement line and the slope where the plane of the normal vector in the horizontal plane of the projection direction of the angle, in this problem α and the location of the ship, is related to the above formula β in this problem should be the actual direction of the ship traveling with the ship is directly below the bottom of the sea surface of the normal vector of the slope of the horizontal projection direction of the angle (i.e., γ), this time the above formula changed to:

$$W(m,n)=\mathrm{C}(m,n)\frac{\sin\theta}{\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}\sin^2\gamma\tan^2\alpha(m,n)} \tag{43}$$

As shown in Figure 6, $\angle COX$ is $\overline{\beta}$, OA is the direction of the horizontal projection of the normal vector of the slope of the seabed directly below the ship, $\angle$BOX is $\beta(m,n)$, $\angle COA$ is γ, then the direct mathematical relationship between the three is:

$$\sin(\gamma(m,n)) = \sin(\overline{\beta} - \beta(m,n) + \frac{\pi}{2}) = \cos(\overline{\beta} - \beta(m,n)) \tag{44}$$

After mathematical derivation, the equation holds when $\beta(m,n)$ satisfies a size between -π/2 and π/2.

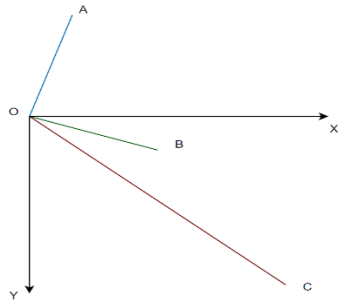


Figure 6. Measurement Line Angle Diagram.

Thus we can get the coverage width of the survey line along the $\overline{\beta}$ direction at any point on the water surface.

Now let's make a line that lies in the XOY plane (see below), i.e. l_0, such that its angle with the X-axis is $\overline{\beta}$, then the equation of l_0 is

$$y = \tan\overline{\beta}x \tag{45}$$

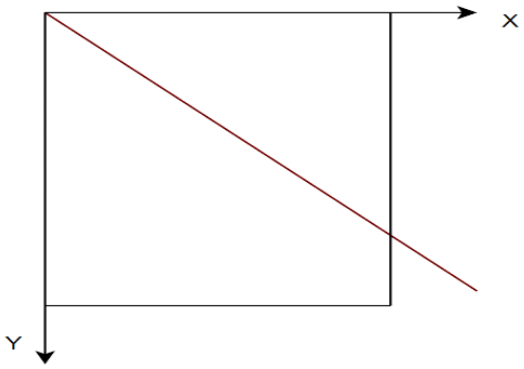


Figure 7. Top View.

As shown in Figure 7, when $x_{0,0} = 0$ is $y = 0 * \tan\overline{\beta}$, since we don't know the depth of each of the unknowns, we need to determine n, so that:

$$x = x_{0,0}\ y_n \le 0 * tan\,\overline{\beta} < y_{n+1} \tag{46}$$

If $2*0.02*p*\tan\overline{\beta} \le y_n + y_{n+1}$ so, then $h_{0,0} = C(0,n)$; otherwise $h_{0,0} = C(0,n+1)$.

When $x_{0,1} = 0.02*p$ is $y = 0.02*p*\tan\overline{\beta}$, you also need to find n in "Attachment.xlsx", so that:

$$x = x_{0,1}\ y_n \le 0.02 * p * tan\,\overline{\beta} \le y_{n+1} \tag{47}$$

If $2*0.02*p*\tan\overline{\beta} \le y_n + y_{n+1}$ so, then $h_{0,1} = C(1,n)$; otherwise $h_{0,1} = C(1,n+1)$.

When $x_{0,2} = 2*0.02*p$ $y = 2*0.02*p*\tan\overline{\beta}$, similarly you need to find n in "Attachment.xlsx", so that:

$$x = x_{0,2} \ \ y_n \le 2 * 0.02 * p * tan\,\overline{\beta} \le y_{n+1} \tag{48}$$

If $2*2*0.02*p*\tan\overline{\beta} \le y_n + y_{n+1}$ so, then $h_{0,2} = C(2,n)$; otherwise $h_{0,2} = C(2,n+1)$.

And so on, until the appearance of $x_{0,k} \ge 4*p$ or $k*0.02*p*\tan\overline{\beta} \ge 5*p$ stop, we can get the measurement line l_0 on the k group of data, this group of data shows the relationship between the X coordinate of each point l_0 and the depth of h_0, then $h_0 = h_0(x_0)$, combined with the above formula, we can derive the relationship between the X coordinates of each point on the l_0 and the width of the coverage, according to all the coverage of the width of $W_0 = W_0(x_0)$, we can calculate the average value of $\overline{W_0}$ as the coverage width of the entire line, to meet the overlap rate of 10% ~ 20% requirements, we might as well let the average overlap rate of 10%, i.e., at this time and are adjacent to the two lines and are at a distance of, respectively, to and. To meet the overlap rate to meet the requirements of 10% ~ 20%, we might as well let the average overlap rate of 10%, that is $d_0 = 0.9\overline{W_0}$, at this time and l_0 adjacent to the two lines of measurement l_1 and l_{-1} within the distance of l_0 d_0, respectively, l_1 and l_{-1} repeat the above steps, and then recursive down to find all the equations of the line, but also to get the optimal choice of the line of measurement, the total length of the line.

The next step is to solve for the percentage of area that is missed and the total length where the overlap is higher than 20%:

Taking l_0 as an example, a point scatter plot W_0, x_0 is made, the relationship equation $W_0 = W_0(x_0)$ is fitted using regression analysis, and the line $W_0 = 0.9*\overline{W_0}$ is drawn as shown below:

The requirement to satisfy the overlap between 0 and 20% is obtained:

If $W_0 < 0.9\overline{W_0}$, then the test is missed;

If $W_0 > 1.125\overline{W_0}$, then the overlap is greater than 20%.

l_0 Lengths with upper overlap greater than 20%.

$$g_0 = \frac{\Delta x}{\cos\overline{\beta}} \tag{49}$$

Total length of overlap greater than 20%. $g = \sum_i g_i$

l_0 The length of the missed measurements on the $lc_0 = \frac{\Delta x'}{\cos\overline{\beta}}$

l_0 The area of missed measurements on the $Lc_0 = \int_{x_1}^{x_1+\Delta x} (0.9\overline{w_0} - w_0(x))dx$

Total area of missed measurements: $Lc = \sum_i Lc_i$

Bringing the above formulas into PyTorch, MLPRegressor, SVR, and Sequential for calculations, the results are shown in Table 4:

Table 4. Model Performance Comparison.

Model	omit	over	X Distance (m)	y Distance (m)	Total Distance (m)
PyTorch Deep learning	0.0415	69099.3838	42642.2953	1052663.6690	1095305.9643
MLPRegressor	0.1262	140518.1144	36458.7085	860042.662549	896501.3710
SVR	0.0706	149397.5080	39204.7786	846402.790150	885607.5688
Sequential	0.0732	90140.8664	36311.7026	860986.165986	897297.8686

6. Conclusions

This study explores the influence of depth on multibeam survey line coverage and the relationship between survey line direction and coverage area. By establishing a mathematical model, it determines the swath coverage width and overlap rate at various depths, demonstrating that greater depth increases coverage width but decreases overlap rate. The study further refines the model to account for the angle between the survey line direction and the slope, optimizing coverage for different topographies. An optimization model is developed to minimize the total survey route length while maintaining an overlap rate of 10-20%, identifying the best route as having 36 survey lines oriented due south with a total length of 133,344 meters. Finally, the study adapts the model for irregular topography, using calculus and regression analysis to optimize survey line direction and spacing, resulting in efficient and accurate mapping. The research highlights the importance of considering various factors in survey planning to enhance the efficiency and accuracy of multibeam echo sounding projects.

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