Study on Propeller Positioning and Trajectory Control of Underwater Robots Based on Bayesian Inference and Multi-Objective Optimization

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Abstract. In ocean exploration, the positioning and trajectory optimization of underwater vehicles are critical research areas that directly impact the success of ocean resource exploration, environmental monitoring, and search and rescue missions. However, the complex and uncertain marine environment poses significant challenges. This study first determines the three-dimensional spatial position of the underwater vehicle, calculates the vehicle's speed in the marine environment through mechanistic analysis, traversing the initial position, velocity, acceleration, angular velocity and angular acceleration of the submersible, fits the motion trajectory using a random walk, and uses Bayesian inference to estimate maximum probability positions of submersibles and select the optimal initial deployment point. To better cope with real-world scenarios involving ocean currents and the diverse motion modes of underwater vehicles, this study further used an improved genetic algorithm to divide the motion trajectory of the submersible into a series of discrete path points, and constructed a multi-objective optimization model with the shortest search and rescue time and the maximum probability of successful search and rescue as the objective function to realize the precise positioning of the lost underwater vehicle.

Keywords: Dynamic Model, Mechanistic Analysis, Random Walk Fitting Model, Bayesian Inference, Genetic Algorithm.

1. Introduction

In the vast field of marine science and engineering, the precise positioning and trajectory optimization of submersibles has always occupied a central position in research. These submersibles not only play a crucial role in the exploration of marine resources, environmental monitoring, and search and rescue missions, but their precise positioning and flexible trajectories are also the key to the success of the missions [1~3]. However, in the face of the unpredictable marine environment and various unforeseen factors, the positioning and trajectory optimization of submersibles still face unprecedented challenges.

To solve the problem of predicting the position of a submersible and determining the initial deployment point to minimize the time taken to locate a lost submersible, this study employs a Bayesian algorithm to analyze the real-time data to update the probability distribution of the submersible's position. The mathematical basis of the Bayesian Algorithm is the Bayes Theorem, which was proposed by the British mathematician Bayes. This algorithm is also widely used in the field of wireless positioning for optical communications, disaster chain prediction, traffic safety, etc [4~6]. In addition, to solve the problem of determining the search pattern to minimize the time to locate the lost submersible, a genetic algorithm is used in this study. The Genetic Algorithm was first proposed by John Holland in the United States in the 1970s and was mainly used to solve the

optimization search problem. This algorithm is also widely used in the field of logistics and transport, flexible job shop scheduling, and automated driving [7~9].

In this study, the information data received by the main ship is first analyzed and processed so that the main ship can analyze the environmental conditions around the submersible. Then, considering the case where the submersible is lost, a random walk is used to simulate the random motion of the submersible under the influence of currents, and a more realistic and accurate predicted trajectory of the submersible's motion is obtained by fitting the motion trajectory. Also considering the randomness of the data, this paper introduces Bayes Theorem to establish a prediction model, which more reliably determines the maximum probability of the diver's motion trajectory and position by updating the probability distribution of the diver's position under the influence of uncontrollable factors. To minimize the time to locate the lost submersible, this study also constructs a multi-objective optimization model based on Genetic Algorithm to determine the best initial deployment point and search pattern.

2. Submersible 3D Space Positioning and Trajectory Analysis

This study's data sources are from www.comap.com/undergraduate/contests, to achieve accurate positioning of the lost vehicle, this study constructs a three-dimensional random walk simulation reasoning model, comprehensively considering multiple factors to improve the accuracy and reliability of vehicle position prediction.

First, this paper uses the information received by the main ship to establish the three-dimensional spatial position by deeply analyzing the inertial measurement unit information of the vehicle. The vehicle regularly sends environmental data to the main ship, helping the main ship analyze the surrounding environmental conditions.

Then, in the case of vehicle loss, since the vehicle's pitch state, initial acceleration, and initial position are unknown, this paper uses a random walk to simulate the random movement of the vehicle under the influence of ocean currents, fitting the motion trajectory to obtain a more realistic and credible prediction of the vehicle's movement trajectory.

Finally, considering data randomness, Bayesian inference is introduced to establish a prediction model, updating the probability distribution of the vehicle's position under the influence of uncontrollable factors, and more reliably determining the vehicle's most probable movement trajectory and position. To improve the accuracy and reliability of vehicle position prediction, this paper suggests equipping the vehicle with the following devices:

(1) Inertial Measurement Unit: Regularly sends the vehicle's operation information to the main ship, including speed, acceleration, angular velocity, and angular acceleration, used to establish a three-dimensional spatial position.

(2) Acoustic Positioning System: Such as Long Baseline (LBL), Short Baseline (SBL), and Ultra-Short Baseline (USBL) positioning systems, calculate the vehicle's actual position through the signal of acoustic beacons, assisting in determining X and Y coordinates.

(3) Depth Sensor: Uses optical principles to measure the vehicle's depth, assisting in determining the Z coordinate.

2.1. D Space Acquisition of Vehicle Position

Based on the information regularly sent by the vehicle to the main ship, determine the vehicle's movement state in three-dimensional space, and then calculate the vehicle's position information. Suppose the initial position of the vehicle is:

$$P = \left[P_1,\ P_2,\ P_3 \right] \tag{1}$$

Where, P_1, P_2, *and* P_3 are the initial positions of the vehicle in three-dimensional space.

The initial speeds in the three directions are V_1, V_2, *and* V_3, and the accelerations in the three directions areα_1, α_2, α_3 After time T, the vehicle's speed and position have the following relationship:

$$\begin{cases} V_x' = V_1 + a_1 \times T \\ V_y' = V_2 + a_2 \times T \\ V_z' = V_3 + a_3 \times T \end{cases} \tag{2}$$

$$\begin{cases} x' = P_1 + V_1 \times T + \frac{1}{2} \times a_1 \times T^2 \\ y' = P_2 + V_2 \times T + \frac{1}{2} \times a_2 \times T^2 \\ z' = P_3 + V_3 \times T + \frac{1}{2} \times a_3 \times T^2 \end{cases} \tag{3}$$

At this time, the final position of the vehicle is:

$$P' = \left[x',\ y',\ z' \right] \tag{4}$$

Next, this paper establishes the dynamic model and the mathematical model of ocean currents, according to the different coordinate system transformations shown in Figure 1, as follows[10].

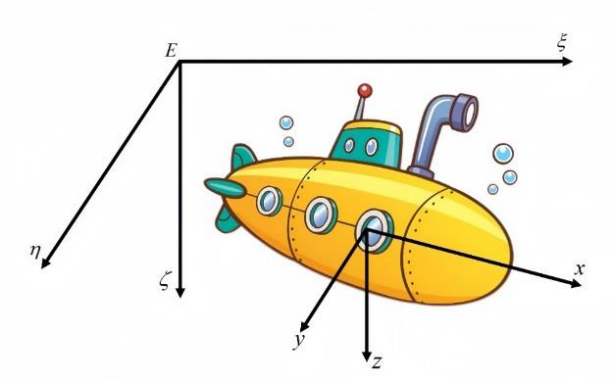


Figure 1. Vehicle Ground Coordinate System and Carrier Coordinate System.

$$\begin{cases} Mv + C(v)v + D(v)v + g(\eta) = f \\ \eta = J(\eta)v \end{cases} \tag{5}$$

$$\begin{cases} v_{w0} = v_{wxo} i_0 + v_{wyo} j_0 + v_{wzo} k_0 \\ v_{T0} = v_{Tw0} + v_{w0} \\ \begin{cases} v_{Twx0} = v_{Tx0} - v_{wx0} \\ v_{Twy0} = v_{Ty0} - v_{wy0} \\ v_{Twz0} = v_{Tz0} - v_{wz0} \end{cases} \\ \begin{bmatrix} v_{Twx}^0 \\ v_{Twy}^0 \\ v_{Twz}^0 \end{bmatrix} = T_1 \begin{bmatrix} v_{Tx0}^0 - v_{wx0} \\ v_{Ty0}^0 - v_{wy0} \\ v_{Tz0}^0 - v_{wz0} \end{bmatrix} \\ v_{Tw0} = 0 \end{cases} \tag{6}$$

Where, M, C(v), D(v), and g(η) respectively represent the mass matrix, Coriolis matrix, damping matrix, and restoring force moment vector, and f represents the force on the vehicle. In the formula, vw0 represents the ocean current speed in the ground coordinate system, vwx0,vwy0, and vwz0 are its three velocity components, i0,j0, and k0 represent unit vectors along the three coordinate axes in the coordinate system, vT0represents the vehicle's speed in the ground coordinate system, and vTw0 represents the vehicle's speed relative to the ocean current. The superscript 0 indicates the parameter value at zero time, and the subscript 0 indicates the parameter value in the ground coordinate system. T1 is the transformation matrix.

The initial speed of the vehicle is solved as:

$$V_0 = 0.23\ m/s \tag{7}$$

Through such mathematical modeling work, we can more comprehensively understand the vehicle's movement state and its interaction with ocean currents.

2.2. Fitting the Vehicle's Motion Path Under the Influence of Ocean Currents Using the Random Walk Model

To simplify the problem, this study sets the time step to 1 minute, and the vehicle can move in any direction within each time step. By traversing the initial position, speed, acceleration, angular velocity, and angular acceleration of the vehicle, and using Matlab to traverse and solve the inertial measurement unit information, the complete motion trajectory within 1 hour is simulated.

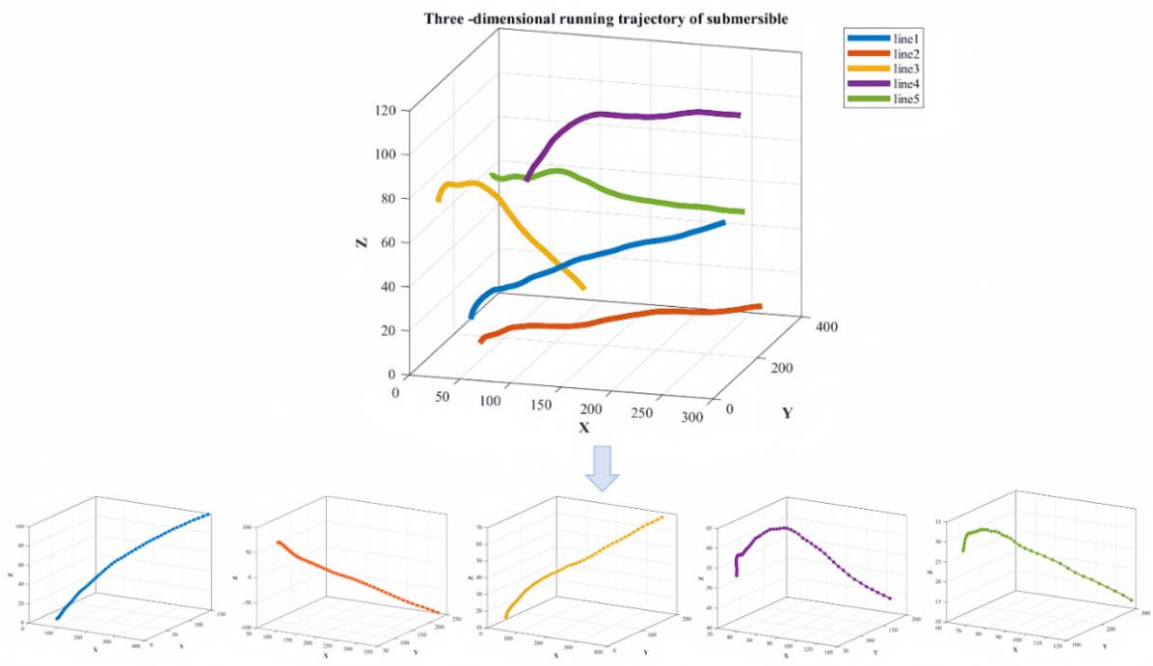


Figure 2. Random Walk Fitting Vehicle Motion Trajectory.

The visualization diagram is shown in Figure 2, listing five possible complete motion trajectory curves of the vehicle within 1 hour. Lines 1-5 respectively represent the trajectory curves of the vehicle under different initial positions, speeds, accelerations, angular velocities, and angular accelerations.

2.3. Estimating the Maximum Probability Position of the Vehicle Using Bayesian Inference

Using Bayesian inference, analyze real-time data to update the probability distribution of the vehicle's position, considering factors such as ocean current disturbance to predict the vehicle's most probable movement trajectory and position. This paper uses the following formula to calculate the posterior probability:

$$P(A \mid B) = \frac{P(B \mid A) \cdot P(A)}{P(B)} \tag{8}$$

Where, *P (A|B)* is the conditional probability of event A occurring given event B. *P (B|A)* is the conditional probability of event B occurring given event A. *P (A)* is the prior probability of event A, which is the probability of event A occurring before obtaining information about event B. *P (B)* is the total probability of event B, which can be considered a normalization constant to ensure the total probability sums to 1.

Using Matlab to calculate the probability distribution of the vehicle's position, the most probable movement trajectory and position are selected. The motion trajectory diagram is shown in Figure 3, where the left diagram shows the probability distribution of the vehicle's position in the ocean, with the darker color representing a higher probability. The right diagram shows the most probable movement trajectory and position of the vehicle predicted by the Bayesian model.

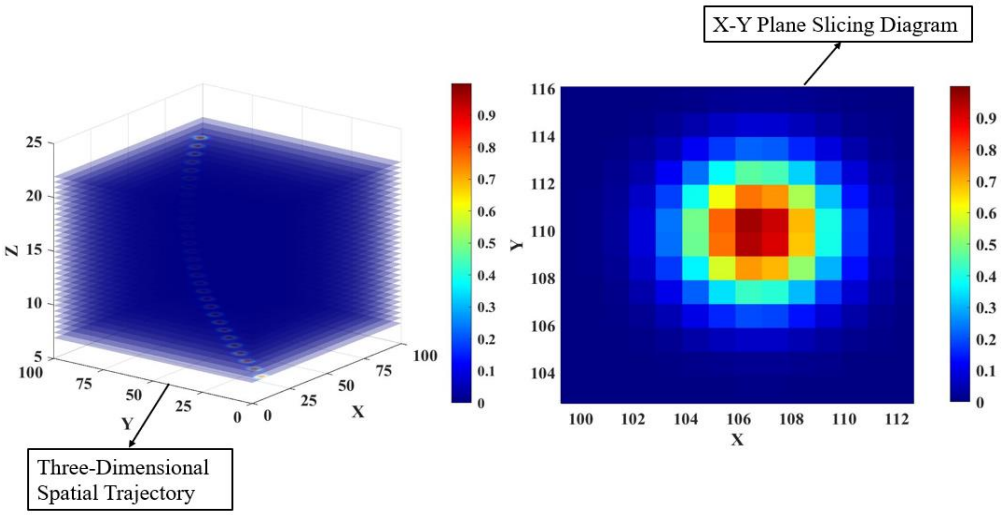


Figure 3. Probability Distribution of Vehicle Position.

3. Initial Deployment and Search Strategy Based on Multi-Objective Optimization

In the previous chapter, the three-dimensional spatial positioning and trajectory analysis of the submersible through Bayesian inference roughly determined the location of the lost submersible. To select the optimal initial deployment point and search mode, minimize the time to locate the lost submersible, improve the success rate of the positioning, and reduce the waste of time and resources for search and rescue, a highly efficient search model is first established preliminarily.

Considering the influence of marine environmental factors, the motion modes of unpowered submersibles in the ocean are diverse and can be roughly classified into three motion modes: fixed-depth motion, non-fixed-depth linear motion, and non-fixed-depth curved motion. The motion trajectory of the submersible is divided into a series of discrete path points, and the optimal search pattern is obtained by constructing an efficient search model based on a genetic algorithm, as shown in the figure 4:

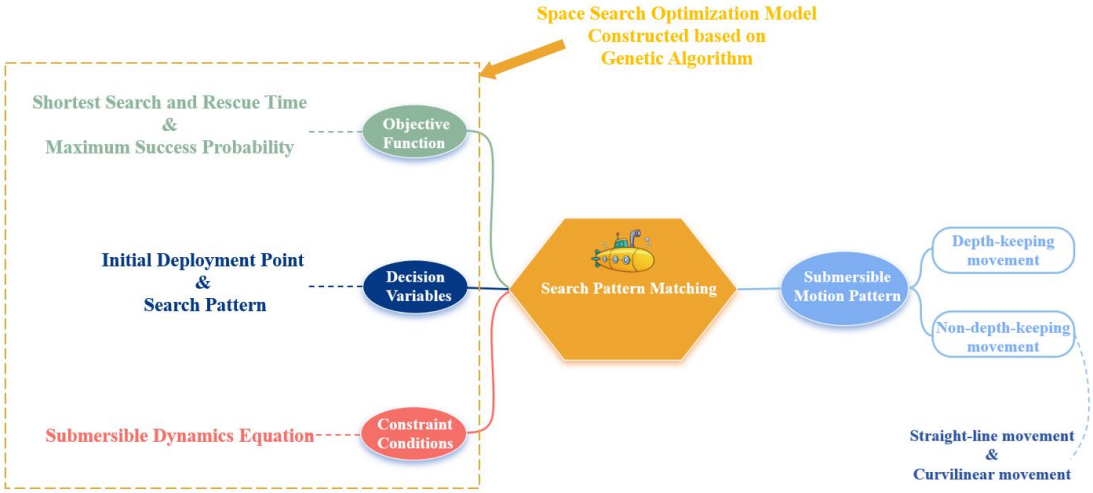


Figure 4. Submersible search pattern solving.

Based on the submarine motion trajectory map derived from Bayesian inference in the previous chapter, the darkest-colored region in the map is selected as the optimal initial deployment point as in Figure 5:

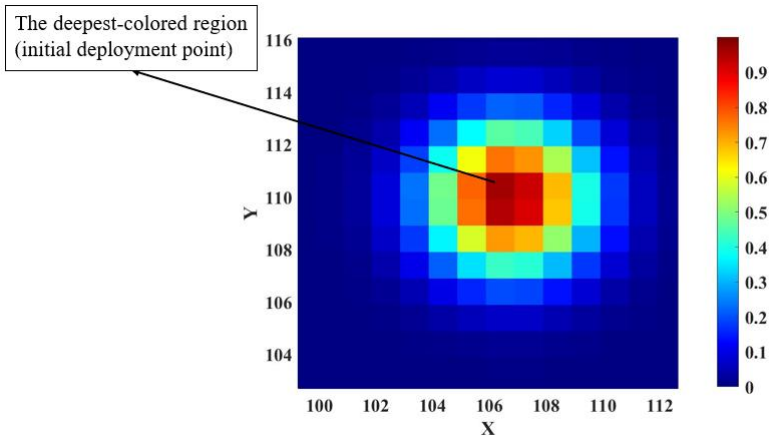


Figure 5. Methodology for selecting the optimal initial deployment point.

Divide the submersible's trajectory into a series of discrete waypoints, with the objectives of minimizing search and rescue time and maximizing the probability of successful search and rescue as the target functions:

$$\begin{cases} T_m = (t_1 + t_2 + \ldots + t_n)_{\min} \\ P_m = P_{\max} \end{cases} \tag{9}$$

Where, $t_1, t_2, \ldots, t_n$ represent the time it takes to reach each consecutive waypoint during the search and rescue, T_m is the minimum search and rescue time, and P_m is the maximum probability of successful search and rescue.

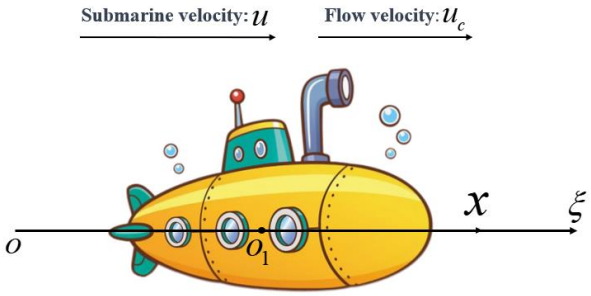


Figure 6. Submersible motion platform coordinate model.

Modeled from the coordinates of the submersible shown in Figure 6, using the initial deployment point of the submersible and the choice of search pattern as decision variables, with a series of dynamic equations of the submersible as constraint conditions:

$$st\begin{cases} m\cdot\dot{u} = F(n,\ u,\ u_c) + X(\dot{u},\ \dot{u}_c,\ u,\ u_c) \\ X(\dot{u},\ \dot{u}_c,\ u,\ u_c) = -m_{11}(\dot{u}-\dot{u}_c) + X_1(u-u_c) \\ (m+m_{11})\dot{u} = X_1(u-u_c) - m_{11}\dot{u}_c \\ X_1(u) = X_u u + X_{uu}u^2 + X_{uuu}u^3 \\ m = 3\times10^5 kg,\ \mathrm{m}_{11} = 1.178\times10^5 kg \\ X_u = 9.317\times10^3,\ X_{uu} = 9.317\times10^4,\ X_{uuu} = -1.557\times10^4 \\ u_c = 1.0kn \end{cases} \tag{10}$$

Where m represents the submersible mass, F represents the propeller thrust, $X(\mathring{u}, \mathring{u}_c, u, u_c)$ represents the submersible's longitudinal dynamics, m_{11} represents the additional longitudinal mass of the submersible, $X_1(u-u_c)$ represents the non-inertial hydrodynamic forces in the longitudinal direction.

When $(u-u_c) > 0$, fluid flows from the submersible's head to tail, equivalent to forward motion; when $(u-u_c) < 0$, fluid flows from the submersible's tail to head, equivalent to reverse motion. $X_1(u)$ represents the longitudinal resistance of the submersible, and X_u, X_{uu}, *and* X_{uuu} represent the corresponding resistance regression coefficients.

Setting parameters: $M=50$, maximum generations $G=1000$, crossover rate $P_c=1$, mutation rate $P_m=0.001$, perform initial population calculation. The submersible's trajectory obtained from the previous chapter can be used as the initial population.

$$C = Q_1,\ Q_2,\ ...,\ Q_n \tag{11}$$

Where, Q1, Q2, …, and Qn represent the n waypoints dividing the motion trajectory.

Perform the following operations on the submersible's motion trajectory:

(1) Swap the positions of any two waypoints to obtain a new trajectory:

$$Q_1...Q_u Q_{u-1}...Q_v Q_{v+1}...Q_n \quad (1 < u < v < n) \tag{12}$$

$$\Delta f = (d_{Q_{u-1}Q_v} + d_{Q_u Q_{v+1}}) - (d_{Q_{u-1}Q_u} + d_{Q_u Q_{v+1}}) \tag{13}$$

If $\Delta f < 0$, the new trajectory iterates the original trajectory, continuously iterating and modifying until no further optimization is possible, generating multiple feasible solutions.

(2) Crossover Operation: This paper employs an improved crossover approach. Firstly, based on the principle of 'like with like,' the original trajectory is paired, meaning waypoints with smaller objective function values are paired with each other, and waypoints with larger objective function values are paired. Then, using a chaotic sequence, the positions of crossover points are determined. Finally, the identified crossover items are subjected to crossover. The specific steps are as follows:

Take a random number in the (0, 1) interval as the initial value, and then use the formula:

$$X(n+1) = 4X(n)\left[1 - X(n)\right] \tag{14}$$

Iterate to generate a chaotic value in the (0, 1) interval, perform chaotic iteration, then multiply this value by 100 add 2, and finally round it to the nearest integer. This is done to ensure that, during the selection of crossover points, the algorithm prioritizes maximizing the probability of successful search and rescue of the submersible, thereby mitigating the optimization oscillation issue in the genetic algorithm's combinatorial optimization process.

(3) Mutation Operation: To avoid falling into local optimum solutions, this paper, based on the given mutation rate, selects individuals for mutation. Three random integers μ_1, μ_2, μ_3, are chosen suchthat $1<u<v<\mu_1, \mu_2, \mu_3<n$, ensuring mutation occurs within the global range.

(4) Deterministic Strategy: In the original and new trajectories, select the path with the optimal objective function value to obtain a search pattern with the shortest search and rescue time and the highest probability of successful search and rescue.

The optimal search paths for different motion modes are obtained by the above efficient search model as shown in Figure 7.

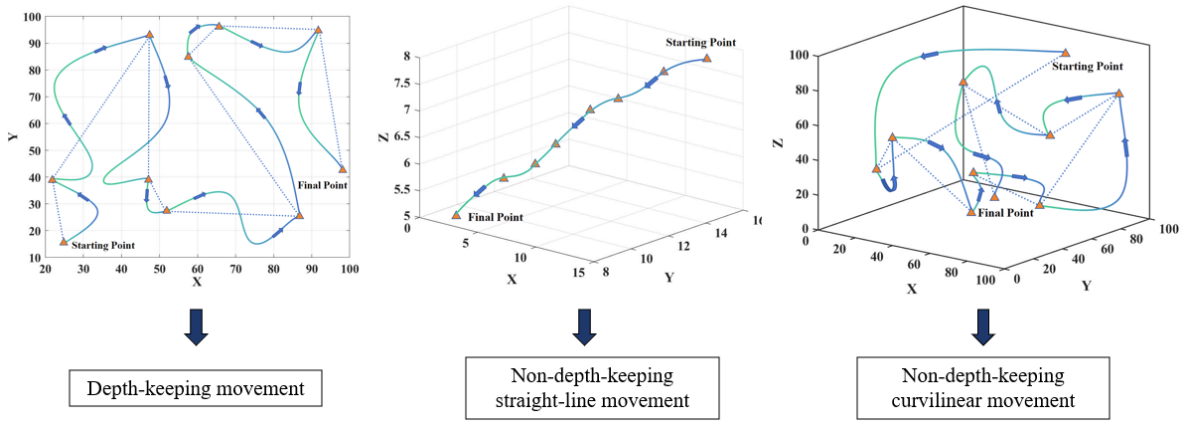


Figure 7. Optimal search paths for different motion patterns.

Table 1. Matching optimal search patterns for different motion patterns.

Submersible Motion Pattern	Optimal Search Pattern
Fixed-Depth Movement	Grid Search
Non-Fixed-Depth Linear Movement	Line Search
Non-Fixed-Depth Curved Movement	Jackstay Search

The results shown in Table 1 above are obtained: when the motion mode of the submersible is fixed depth motion, the optimal search mode is grid search; when the motion mode of the submersible is non-fixed depth curved motion, the optimal search mode is line search; and when the motion mode of the submersible is non-fixed depth curved motion, the optimal search mode is Jackstar search.

4. Conclusions

In this study, the position of the submersible in the sea area is obtained by constructing the basic 3D space, and then the velocity of the submersible under the influence of the marine environmental factors is obtained through the mechanism analysis, and the random walk is fitted to the trajectory map of the submersible. Bayes Theorem is used to improve the accuracy of the model and further precise the position of the submersible, and then the prediction model for the positioning of the submersible is obtained. To minimize the shortening of locating the position of the lost submersible, the optimal search model is investigated in this study. Based on the optimal initial deployment point, i.e. the point with the darkest color in the Bayesian inference map, due to the large influence of the diver by the sea current and the various motion patterns, to accurately match the search and rescue mode, this study adopts a multi-objective optimization model constructed based on the improved Genetic Algorithm to take the shortest search time and the maximum probability of successful search and rescue as the objective function, take the diver's initial deployment point and the search mode as the decision variables, and take the dynamics equation The initial deployment point and search mode of the submersible are taken as the decision variables, and the dynamics equation is taken as the constraints, which results in grid search for fixed-depth motion, line search for non-fixed-depth linear motion, and Jackstar search for non-fixed-depth curved motion.

This paper provides a research idea and framework applied to marine search and rescue-related fields, based on Bayes Theorem and multi-objective optimization, and proves the feasibility of the method through 3D positioning and trajectory prediction simulation experiments.

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