Research on Production Optimization Based on Simulated Annealing Algorithm and Bayesian Sampling Optimization Model

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Abstract. Simulated annealing algorithm is an optimization algorithm in the field of machine learning that is good at processing high-dimensional complex data and seeking global optimal solutions. In this research, the simulated annealing algorithm is combined with mixed integer linear programming and applied to the production decision optimization scenario. Firstly, the normal distribution and hypothesis testing method and the Bayesian sampling optimization model are used to formulate the corresponding sample sampling scheme. This method provides the most comprehensive possible picture of the yield of a complete batch of parts, semi-finished products, or products with the minimum number of samples and the smallest batch size. Secondly, this paper takes the profit maximization of the enterprise as the objective function to model the production inspection and compensation process of the enterprise and uses the simulated annealing algorithm to solve the corresponding optimal production decision scheme. In general, this research not only expands the application scope of simulated annealing algorithm, but also provides a feasible optimization method and reference for similar multi-objective production decision-making problems.

Keywords: Hypothesis testing method, MATLAB, Bayesian sampling optimization model, Mixed integer linear programming, Simulated annealing.

1. Introduction

The simulated annealing algorithm is an efficient global optimization algorithm in the field of machine learning, which can effectively deal with high-dimensional complex data problems. The purpose of this paper is to integrate the simulated annealing algorithm with mixed integer linear programming to construct a new multi-objective production decision optimization model, to provide new solutions for related fields. Specifically, this paper first designs a sample sampling scheme based on normal distribution hypothesis testing and Bayesian sampling optimization, aiming to reflect the quality status of batch products as comprehensively as possible under the limited sampling resources. On this basis, this paper establishes a mixed integer programming model with the goal of profit maximization for the production inspection and compensation process of enterprises and uses the simulated annealing algorithm to solve and optimize and obtains the corresponding optimal production decision scheme. This not only expands the application scenarios of the simulated annealing algorithm, but also provides a useful reference for similar multi-objective production decision-making problems. In the future, we can further explore how to use this optimization model to make more accurate production decisions in a more complex production environment.

2. Establishment and Solution of Sampling Scheme Model

In the enterprise production process, the product is usually combined by a variety of spare parts processing and secondary. The product. In the assembled product, as long as one of the parts is unqualified, the finished product must be unqualified; if both parts are qualified, the assembled product may not be qualified. For unqualified products, enterprises can choose to scrap or dismantle them, which will not cause any damage to the parts, but requires dismantling costs. Enterprises

usually use sampling and testing to decide whether to accept batches of spare parts from suppliers, and the cost of testing is borne by the enterprise. The aim of this paper is to design a sampling method for companies that uses as few tests as possible. Firstly, this paper needs to identify a sampling method that ensures the highest possible accuracy for a given batch of spare parts using as few samples as possible.

Table 1. Acceptation standards

Numble	Scheme 1	Scheme 2	Scheme 3
1	456	456	123
2	789	213	644
3	213	654	649

As shown in Table 1, a certain value in the defective rate is defined as the nominal value, and the sampling detection method is combined with different reliability levels to decide whether to reject or accept a batch of products.

2.1. Sampling Detection Scheme Formulated by the Normal Distribution and Hypothesis Testing Methods

To develop a better sampling test plan, a hypothesis testing method is used here, which is a commonly used method in statistics. Hypothesis testing is a method of inference that uses sample data to explain the characteristics of a population. Here, this paper uses this method to better develop the corresponding detection protocol.

2.1.1 Selection and construction of indicators

Since the nominal value and reliability are not given, the sampling test scheme with a small number of tests needs to be formulated in this paper and no data is given, so the corresponding indicators need to be set as follows:

I_0 : Assuming that the defective rate of spare parts does not exceed the nominal value (10%).

I_1 : Assuming that the defective rate of the spare parts exceeds the nominal value (10%).

2.1.2 Establishment of the model

Here this paper uses the binomial distribution reference as the corresponding model [1-3], and here the main factor concerned in this paper is the defect rate. Suppose m tests are carried out and Y defective products are found, then the defective rate is:

$$Q = \frac{Y}{M} \tag{1}$$

As shown above, this article uses a symbolic formula to represent the defective rate, and then this article needs to identify the rejection domain to continue answering the question:

For the first case (rejection with 95% confidence), a cut-off value b_1 can be found in this paper, when $y > b_1$, I_0 is rejected.

For the second case (reception with 90% confidence), we can find a cut-off value b_2 that accepts I_1 when $y < b_2$.

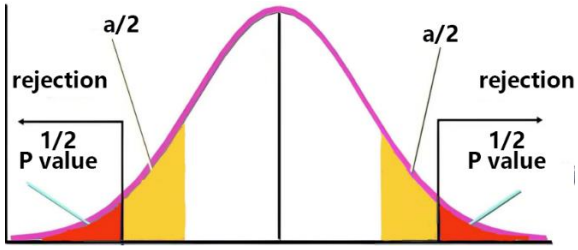


Figure 1. Reception or Rejection Schematic

Now that the assumptions and scenarios have been analyzed, the test statistic can be calculated, and the final decision can be made based on the actual test result y, compared with the critical values b_1 or b_2, whether to accept or reject the spare parts.

To this end, this paper uses MATLAB to draw the model, as shown in Fig. 1.

2.1.3 Calculation of sample size

Computation is a critical step after the model is established, and to ensure validity, this paper needs to distinguish whether to accept the minimum sample size required in the hypothesis under the condition of 90% and 95% confidence.

Calculation formula:

$$m = \left(\frac{V \frac{a}{2} \sqrt{q(1-q)}}{E} \right)^2 \tag{2}$$

Where: $\frac{a}{2}$ is the critical value of the distribution, determined with the confidence level, and q is the rate of inferiority (10%), E is the margin of error acceptable to the manufacturer.

When the conditional reliability is 95%, $V_{0.05} = 1.645, p = 0.10, E = 0.05$.

$$m = \left(\frac{1.96 \times \sqrt{0.10 \times 0.90}}{0.05} \right)^2 \approx 138 \tag{3}$$

When the conditional reliability is 90%, $V_{0.05} = 1.645, p = 0.10, E = 0.05$.

$$m = \left(\frac{1.645 \times \sqrt{0.10 \times 0.90}}{0.05} \right)^2 \approx 97 \tag{4}$$

The first visualization sample size can be obtained by complex arithmetic to be approximately equal to 97.

2.1.4 Determination of cut-off values

Cut-off value W95 under 95% reliability rejection: there is a cut-off value Y for the number of defective products in the sample, such that $q0 \leq 0.05$.

Cut-off value M90 under 90% reliability reception condition: there is a cut-off value Y for the number of defects in the sample, such that $q1 \geq 0.90$.

2.1.5 Analysis of the results of the calculations

Combined with the above algorithm results and chart analysis, it can be obtained:

With 95% confidence: a sample size of 138, which is greater than the cut-off value of W95, the batch of spare parts can be rejected in order not to affect the quality of production and the reputation of the company.

At 90% confidence: with a sample size of 97, less than or equal to the cut-off value of M90, parts can be received, preventing more costs for the business due to rejecting parts.

Better sampling and testing plan has been formulated for the enterprise, so that the production of the enterprise is more efficient and cost-saving.

2.2. Sampling Detection Scheme Formulated by the Bayesian Sampling Optimization Model

Bayesian sampling method is a special optimization scheme, to pass a small amount of detection sampling, and under the condition of specific reliability requirements, choose whether to accept the

accessories provided by the supplier, this paper is mainly divided into two kinds of reliability to discuss, using this method can effectively formulate a better sampling detection scheme [4, 5].

2.2.1 Assumptions of the model

Using the Bayesian sampling optimization scheme, this paper needs to make basic assumptions, if the defect rate (q) obeys the Beta distribution, and the analysis can only be better done by making full use of the assumptions, which is a common probability distribution case.

In this paper, the distribution of defective rate q is set as Beta (c_1, d_1), where c_1, d_1 can be a parameter comparison obtained from historical experience.

For example, if $c_1 = 9, d_1 = 1$, it can be considered as a defective rate of 10%.

$$E(q) = \frac{d_1}{C_1 + d_1} = 0.1 \tag{5}$$

Assuming that the sampling samples of each batch in this paper are m, and these m samples are to be tested, assuming the test results Y, conforms to the binomial distribution $\mathrm{Y}\sim\mathrm{Binomial}\ (m, q)$, there are k number of substitutions in m samples.

I_0: Assuming that the defective rate of spare parts does not exceed the nominal value (10%).

I_1: Assuming that the defective rate of the spare parts exceeds the nominal value (10%).

2.2.2 Establishment and solution of sampling testing program

This method is often used to describe the probability of success and failure, where the likelihood function is based on the number of defective products k and the number of parts m, and it has been assumed that the binomial distribution $\mathrm{Y}\sim\mathrm{Binomial}\ (m, q)$:

$$L(q|\ y) = \left(\frac{n}{x}\right) q^{y} (1-q)^{m-y} \tag{6}$$

The posterior distribution combines the likelihood function and is given by Bayes' theorem:

$$P(q|\ y) = L(q|\ y) \times p(q) \tag{7}$$

Since the prior distribution is a Beta distribution, the posterior remains unchanged, and its parameters are:

$$P(q|\ y) = \mathrm{Beta}(\alpha + y, \beta + m - y) \tag{8}$$

Where y is the number of defective products and $m - y$ is the number of conforming products.

We will use a right-tailed test to examine the relationship between the defective rate and the nominal value.

In a practical problem, we can first take a few batches of samples and fit the function using the algorithm as above to find the pattern to find a less sampling test solution.

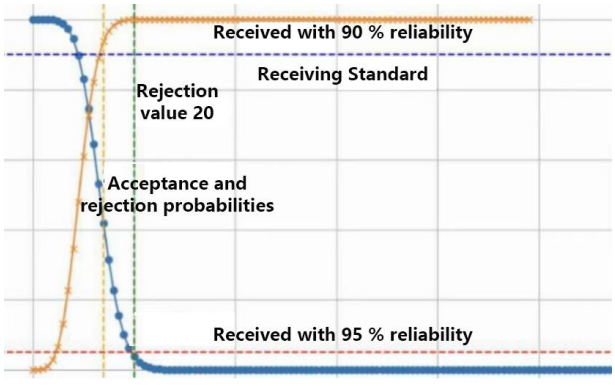


Figure 2. Reception or Rejection Schematic

As shown in Fig. 2, the right-tail test will be used to test the relationship between the defective rate and the nominal value. In the practical problem, this paper can first take several batches of samples, and use the above algorithm to fit the function to find out the rules, to find fewer sampling detection schemes.

2.2.3 Analysis of the results of the model

According to the algorithm code, it can be obtained:

At 95% confidence: the minimum required sample size is 317.

At 90% confidence: the minimum required sample size is 287.

The condition is that the nominal value is exceeded at 95% confidence: in this case, the 95% confidence interval for the posterior distribution needs to be calculated, and if the lower limit is greater than 5%, the batch of parts can be rejected. i.e. $I > I_a$, the batch of products should be rejected.

The condition is that the nominal value is not exceeded at 90% reliability: in this case, the 90% confidence interval of the posterior distribution need to be calculated, and if the lower bound is less than 10%, we can accept the parts.

2.3. Comparative and Comprehensive Analysis of Models

The results of the Bayesian sampling optimization model and graphical analysis were obtained. At 95% confidence level: the minimum required sample size is 317. At 90% confidence level: the minimum required sample size is 287.

The condition is the exceeding of the nominal value at 95% confidence level: in this case, we need to calculate the 95% confidence interval of the posterior distribution and reject the batch of parts if the lower bound is greater than 5%. Which means if $I > I_a$, the batch can be rejected.

The condition is the case where the nominal value is not exceeded at 90% confidence level: in this case, we need to calculate the 90% confidence interval of the posterior distribution, if the lower limit is less than 10%, the batch of parts can be accepted.

2.4. Synthesis of Analytical Models with the Median Approach

The specific formula for the integrated analysis method of taking the median is:

$$MID = \frac{a_1 + a_2}{2} \tag{9}$$

Substituting (138 and 317) in the 95 per cent of the above data:

$$MID_1 = \frac{138 + 317}{2} = 227.5 \tag{10}$$

Substituting (97 and 287) for 90 per cent of the above data:

$$MID_2 = \frac{97 + 287}{2} = 192 \tag{11}$$

In summary it can be found that: at 95 per cent confidence level, the minimum required sample size is 227.5; at 90 per cent confidence level, the minimum required sample size is 192.

2.5. Sensitivity Analysis

Using the modified data, we find the θ interval, which means the fluctuation range of sensitivity, and if the fluctuation of the coefficient is in this range, then it is considered a stable result. Change the upper bound of the constraints, the results obtained in this paper are not very different from the results that have not been changed before, and the ∂ interval is found by using the modified data, which means the fluctuation range of sensitivity, if the fluctuation of the coefficient is in this range, then it is considered a stable result.

3. Establishment and Solution of Corporate Profit Model

Factory production is closely related to the life of this paper, in which the assembly of multiple parts into semi-finished and finished products involves a variety of permutations. The basic idea is to combine the solution process of the optimization problem with the heat balance problem in statistical thermodynamics, so that the enterprise needs to take corresponding measures in the corresponding production stage to maximize the benefits. To model the production decision-making process of an enterprise, the production process of a product is selected for modelling and analysis in this paper. The production of the modified product contains a total of 8 parts, of which parts 1~3 can form semi-finished product 1, parts 4~6 can form semi-finished product 2, parts 7~8 can form semi-finished product 3, and then three semi-finished products to form finished products. The enterprise will give unconditional exchange for the non-conforming products purchased by the user and will incur exchange losses as a result. Therefore, the enterprise needs to decide the specific testing and dismantling scheme, including whether to test the spare parts, whether to test the semi-finished and finished products, and whether to dismantle the unqualified products, so as to maximise the profit of the enterprise after deducting the cost of testing and dismantling, and the logistics cost and reputation of the enterprise generated by the exchange.

Thus, the enterprise profit model needs to define the variables, parameters, and construct a mathematical expression to describe the problem. To build the mathematical model, this paper first uses Mixed Integer Linear Programming (MILP).

The Bayesian sampling optimization model combines Bayesian inference and optimization techniques, which can help firms respond better to decisions and save costs. Thus, this paper chooses Bayesian sampling optimization method in the sampling approach.

3.1. Objective Function Building

The goal of this paper is to maximize total profit, which is obtained by subtracting the cost from the sales revenue.

$$profit = sales\ revenue - cost \tag{12}$$

Sales revenue is the quantity of qualified finished products multiplied by the sales price:

$$sales\ revenue = y \times Q_{p1} \times Q_{p2} \times Q_{pf} \times Q_s \tag{13}$$

The total cost includes the cost of purchasing spare parts, the cost of assembly and the cost of dismantling:

$$total\ cost = m_1 \times D_1 + m_2 \times D_2 + y \times D_a + y \times (1 - Q_{p1} \times Q_{p2} \times Q_{pf}) \times Dd \tag{14}$$

Therefore, the objective function can be expressed as:

$$\begin{aligned} ptofit_{c1} &= y \times Q_{p1} \times Q_{p2} \times Q_{pf} \times Q_s \times Q_{p3} - \\ &\left(m_1 \times D_1 + m_2 \times D_2 + y \times D_a + m_3 \times D_3 + y \times \left(1 - O_{p1} \times O_{p2} \times O_{p2} \times O_{p3}\right) \times Dd\right) \end{aligned} \tag{15}$$

$$\begin{aligned} ptofit_{c2} &= y \times Q_{p4} \times Q_{p5} \times Q_{pe} \times Q_{p6} \times Q_s - \\ &\left(m_1 \times D_1 + m_2 \times D_2 + y \times D_a + m_3 \times D_3 + y \times \left(1 - O_{p1} \times O_{p2} \times O_{p2} \times O_{p3}\right) \times Dd\right) \end{aligned} \tag{16}$$

$$\begin{aligned} ptofit_{c3} &= y \times Q_{p7} \times Q_{p8} \times Q_{pg} \times Q_s - \\ &\left(m_7 \times D_7 + m_8 \times D_8 + y \times D_a + y \times \left(1 - O_{p7} \times O_{p8}\right) \times Dd\right) \end{aligned} \tag{17}$$

$$ptofit_{max} = ptofit_{c1} + ptofit_{c2} + ptofit_{c3} \tag{18}$$

3.2. Constraint Building

The number of finished products assembled cannot exceed the lesser of the two parts:

$$y \le min(m_1, m_2) \tag{19}$$

All variables should be non-negative integers:

$$y, m_1, m_2 \in N_o \tag{20}$$

3.3. Simulated Annealing

To give a decision-making scheme for the production process, this paper considers several factors, including the defective rate of semi-finished products composed of spare parts and parts, costs, sales prices, and dismantling costs. The following is a rough mathematical model and decision-making framework that can be used to guide production [6, 7].

In this article, the calculation steps are omitted, and Fig. 3 can be obtained using code MATLAB programming:

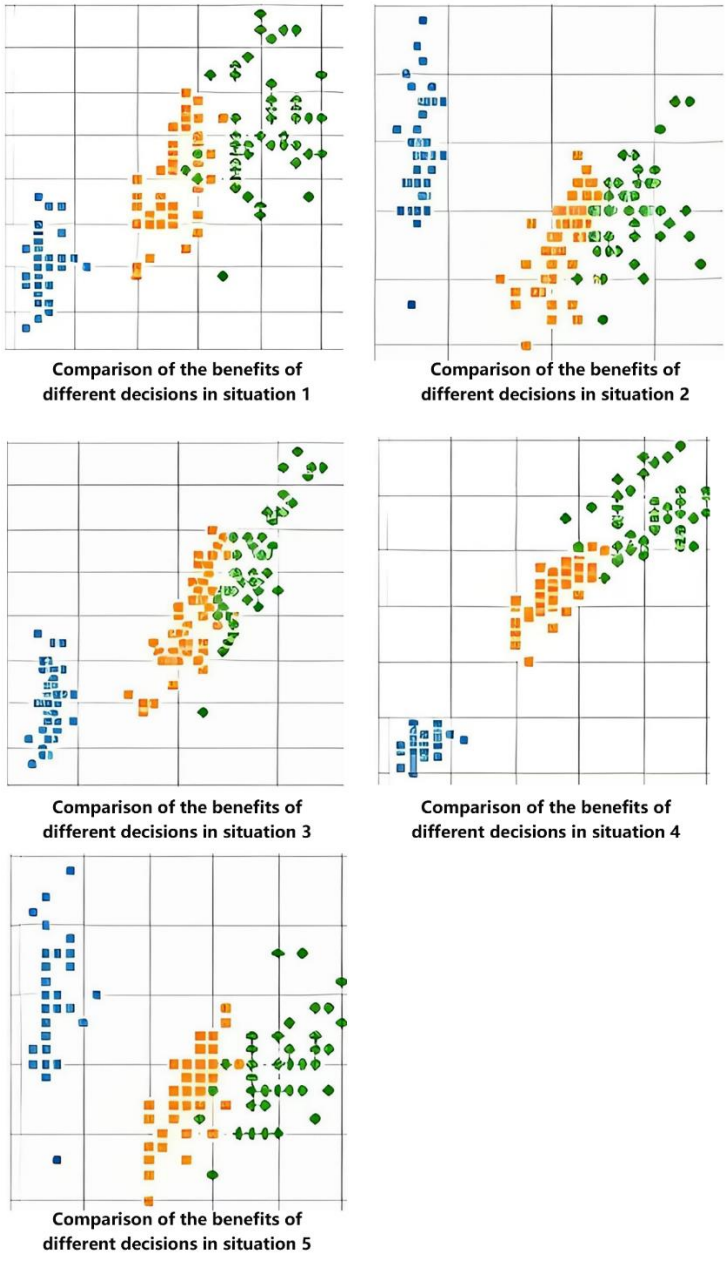


Figure 3. Comparison of the benefits of decision-making in different situations

3.4. Sensitivity Analysis

3.4.1 Mixed integer linear programming based on the decision scheme of finished product dismantling of parts

Using the modified data, we find the ω range, which means the fluctuation range of sensitivity, and if the fluctuation of the coefficient is in this range, then it is considered a stable result [8].

3.4.2 Simulated annealing algorithm based on the dismantling decision scheme of finished parts

Change the upper bound of the constraints, the results obtained in this paper are not very different from the previous unchanged results and use the modified data to find the interval, which means the fluctuation range of sensitivity, if the fluctuation of the coefficient is in this range, then it is considered a stable result.

4. Conclusion

In this research, the multi-objective production decision optimization model based on simulated annealing algorithm and mixed integer linear programming has made some innovations in theory and application. In terms of theory, the model integrates two classical optimization techniques, the simulated annealing algorithm and the mixed integer linear programming and gives full play to the advantages of each. On the one hand, as an effective global optimization algorithm, the simulated annealing algorithm can seek the optimal solution from the complex and multi-dimensional production decision space. On the other hand, mixed integer linear programming provides a suitable mathematical description for describing discrete decision variables in the production process. The organic combination of the two methods makes the optimization model better fit the complexity of actual production decisions. At the application level, the model cleverly adopts the normal distribution hypothesis test and Bayesian sampling optimization method under limited sampling resources, which effectively captures the quality of batch products and provides reliable data support for subsequent production decisions. On this basis, the model takes the maximization of corporate profit as the objective function and obtains a realistic optimal production decision scheme by solving the simulation annealing algorithm. This provides a concrete optimization path for enterprises to achieve a balance between quality control and economic benefits. In general, the optimization model established in this research not only expands the application of simulated annealing algorithm in multi-objective decision-making problems, but also provides new ideas and methods for researchers and practitioners in similar fields.

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