Mathematical modeling and numerical simulation analysis of bench dragon motion

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Abstract. When performing, bench dragons often face challenges such as lack of synergy and poor articulation of movements.Currently, the research on Bench Dragon mostly relies on qualitative descriptions and lacks a systematic scientific method to quantify and optimize the performance. The aim of this paper is to enhance the spectacle of its performance and optimize its circling movements through mathematical modeling. The solenoidal equation and the microelement method are used to establishhe dragon head motion model, and the cosine theorem is used to recursively calculate the positions of the dragon body and the dragon tail, and then the velocity equation is constructed based on the rate of change of the position. Through numerical simulation and ode45 function, the model was solved and the position and velocity data per second in 300 seconds were derived. The results show that the model can significantly improve the performance effect of the bench dragon and provide a scientific quantitative analysis tool for the inheritance and innovation of traditional folk art.

Keywords: Bench Dragons, Microelement Analysis, Numerical Simulation, Mathematical Modeling.

1. Introduction

The charm of the Bench Dragon [1] performance lies in the display of inter-segmental coordination and dynamic interaction. These variations are both unique and interrelated. It is this coordination and interaction that brings fluidity and strong visual impact to the entire performance. Li et al [2]. Proposed a new algorithm which is based on the multipoint correction of quantized state systems (MC-QSS) explicit algorithm. The algorithm is specifically designed to solve the rigidity equations in multi-body system dynamics. By discretizing the hand state variables and introducing the multipoint correction technique, the algorithm can effectively improve the accuracy and efficiency of solving rigid systems, Li Can et al. proposed a method based on the principle of optimization for solving the coexisting state of suspended and recumbent chains in the multi-body system of anchor chains, Li Can et al [3]. Proposed a method based on the optimization principle for solving the coexistence state problem of hanging and lying chains in an anchor chain multibody system. This method simplifies the computational process and effectively handles the equilibrium problem of the multibody system in a unified framework, Du Wenjing et al [4]. Proposed a numerical solution of the dynamically constrained Hamilton equation of multi-body system based on the multi-step block method and solved the nonlinear algebraic equations by the Newton iteration method. They used the Newton iteration method to solve nonlinear equations and conducted simulations with a two-link robotic arm. The results show that this method can effectively control energy and constraint errors, making it suitable for long-term simulations. Bowen Li et al [5]. Proposed an L-stabilization method to solve differential-algebraic equations in the dynamics of multibody systems. They constructed a solution format based on different nodes and applied Ehle's theorem with Padé approximation technique to obtain solution formulas, which is suitable for long-term simulations, and Hao Yongjiang et al [6]. Proposed a kinetic modeling method based on the Euler-Poincare equation. They introduced pseudo-velocity and shift operators for global dynamics description, which improved the modeling efficiency and reduced the repetitive and crossover operations.

However, none of the multi-body system dynamics studied in the above literature has applied the problem to traditional folk activities; therefore, this paper is the first to apply the multi-connector

motion model to the bench dragon, a traditional folk activity, through multi-connector motion modeling to provide a new perspective and method for the modernization and inheritance of this traditional art. The introductory part of this paper describes the problems in the bench dragon performance, and puts forward the goal of optimizing the performance through mathematical modeling. Then, the relevant theoretical part describes in detail the Archimedean spiral, and the relationship between the derived linear velocity and angular velocity, which lays the mathematical foundation for the establishment of the bench dragon motion model. The experimental part describes how to construct the motion model of the dragon head and solve the motion of the whole bench dragon using numerical simulation. The results section shows the trajectory and velocity data obtained from the simulation to verify the accuracy of the model. Finally, the summary section emphasizes the significance of these research results in enhancing the spectacle and professionalism of the bench dragon performance, and points out the rationality of the numerical simulation method and its value in practical application.

2. Relevant theory

2.1. Archimedes spiral

The isometric solenoid, also known as the Archimedean solenoid [7], is parametrically equated in polar coordinates as

$$\rho = \rho_0 + a \cdot \theta \tag{1}$$

Among them, ρ_0 is the starting radius of the solenoid, θ is the current polar angle, a is the growth factor, and the relationship with p can be expressed as

$$a = \frac{p}{2\pi} \tag{2}$$

Thus equation (1) can be written as

$$\rho = \rho_0 + \frac{p}{2\pi}\theta \tag{3}$$

2.2. Linear and Angular Velocity

Take a small segment of microelements on the spiral [8] and express the linear velocity v in terms of the rate of change ds of a small segment of arc length elements as

$$v = \frac{ds}{dt} \tag{4}$$

Take radial minima $d\rho$ and angular minima $\rho d\theta$ on the solenoid as shown in Figure 1

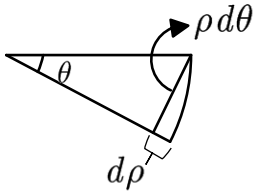


Fig 1. Schematic diagram of arc length calculation.

Then the arc length element ds can be expressed as

$$ds = \sqrt{d\rho^2 + (\rho d\theta)^2} \tag{5}$$

According to the parametric equation of the Archimedean spiral, there are

$$\begin{cases} d\rho = ad\theta \\ \rho = \rho_0 + a\theta \end{cases} \tag{6}$$

Among them $a = \frac{p}{2\pi}$ .Bringing the above equation into equation (4), there are

$$ds = \sqrt{(ad\theta)^2 + (\rho_0 + a\theta)^2 d\theta^2} = \sqrt{a^2 + (\rho_0 + a\theta)^2}\, d\theta \tag{7}$$

Therefore, the linear velocity v is

$$v = \frac{ds}{dt} = \sqrt{a^2 + (\rho_0 + a\theta)^2} \cdot \frac{d\theta}{dt} \tag{8}$$

According to the defining equation for angular velocity $\omega = \frac{d\theta}{dt}$ then the relationship between linear velocity and angular velocity can be derived as

$$v = \sqrt{a^2 + (\rho_0 + a\theta)^2} \cdot \omega \tag{9}$$

Let $h = \sqrt{a^2 + (\rho_0 + a\theta)^2}$ then the expression for the angular velocity ω is

$$\omega = \frac{v}{h} \tag{10}$$

According to $\theta = \omega t$ equation (11) can be rewritten as

$$\rho = \rho_0 - \frac{p}{2\pi}\omega t \tag{11}$$

3. Experiment

3.1. Faucet front handle movement position model

3.1.1. Equation of the bench dragon-screw line

The diagram of the dragon dance team reeling in clockwise is shown in Figure 2,

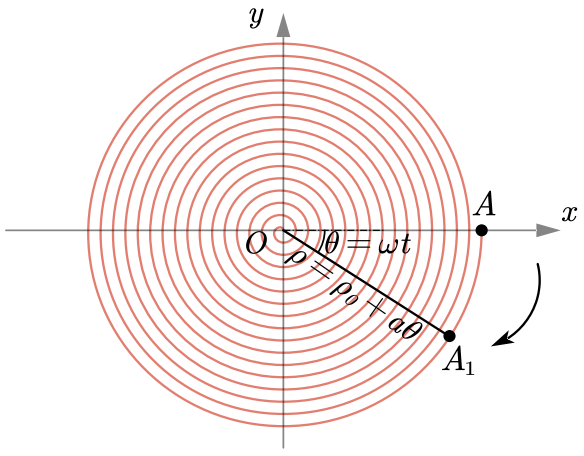


Fig 2. Schematic diagram of the movement.

θ In Fig. 2 represents the angle between the line between the point and the center of the helix and the x-axis, i.e., the radial angle in the polar coordinate system, and ρ is the distance between the point and the center of the circle, which is expressed as the polar diameter in the polar coordinate system.

Figure 3 shows the flowchart of the bench dragon motion modeling and solving.

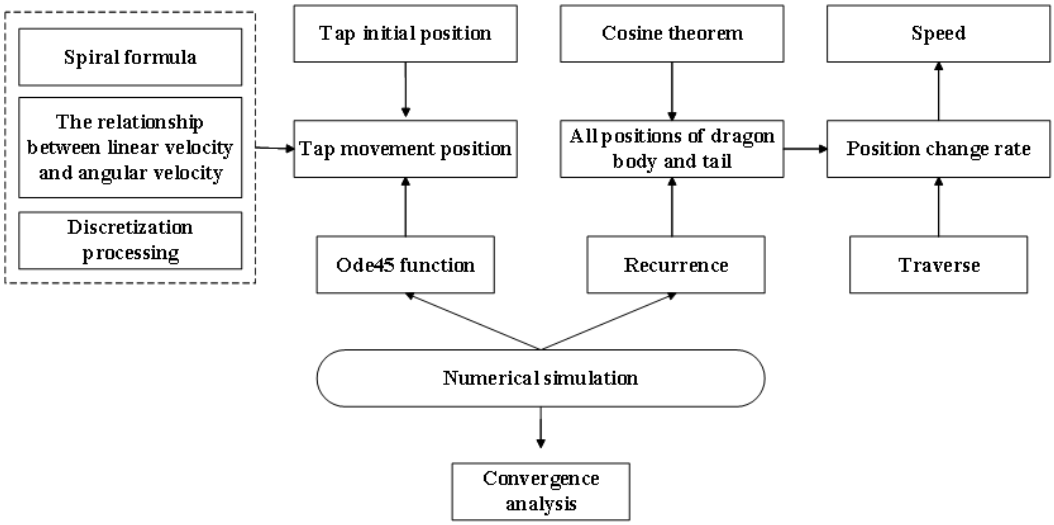


Fig 3. Flowchart of bench dragon motion modeling and solving.

Bringing the pitch p into (1), and since the dragon dance team is gradually approaching the origin from the 16th lap with point A (see Fig. 2), the solenoid equation is rewritten as

$$\rho = \rho_0 - \frac{p}{2\pi}\theta \tag{12}$$

According to the coordinate transformation relationship between polar coordinates and right-angle coordinate system, under the right-angle coordinate system with the center of the solenoid as the origin, the equation is obtained as

$$\begin{cases} x = (\rho_0 + a\theta)\cdot\cos\theta \\ y = (\rho_0 + a\theta)\cdot\sin\theta \end{cases} \tag{13}$$

3.1.2. Faucet front handle position

At the initial moment, the position coordinates of the faucet front handle are calculated to be $(880, 0)$. According to Eq. (10), take the time interval Δt, discretize the motion model, and calculate to get the polar coordinates of the faucet front handle after Δt time to represent

$$\begin{cases} \rho_1 = \rho_0 - \frac{p}{2\pi}\omega\Delta t \\ \theta_1 = \theta_0 - \omega\Delta t \end{cases} \tag{14}$$

Among them, $\omega = \frac{v}{h}$, θ_0 is the polar angle at the initial moment.By recursion one can obtain the polar coordinates of the leading handle after any n time intervals Δt expressed as

$$\begin{cases} \rho_n = \rho_{n-1} - a\frac{v}{\sqrt{a^2 + (\rho_0 + a\theta_{n-1})^2}}\cdot\Delta t \\ \theta_n = \theta_{n-1} - \frac{v}{\sqrt{a^2 + (\rho_0 + a\theta_{n-1})^2}}\cdot\Delta t \end{cases} \tag{15}$$

Translated into coordinates under the Cartesian coordinate system is expressed as

$$\begin{cases} x_n = \rho_n \cos\theta_n \\ y_n = \rho_n \sin\theta_n \end{cases} \tag{16}$$

According to the initial moment position of the faucet front handle, the position coordinates of the faucet front handle at any time point during the movement can be obtained by taking the time interval of Δt and using the recursive formula.

3.1.3. Location of the dragon's body and tail

Take two points on the screw line as a bench before the handle and after the handle, according to the distance between the two points for the distance between the two holes and known, through the cosine theorem of the triangle to calculate the front and rear handle and the center of the screw line line angle, so as to get the recursive formula between the two points, and then get the dragon body and the dragon tail before the handles and the rear handles of the position. As shown in figure 4

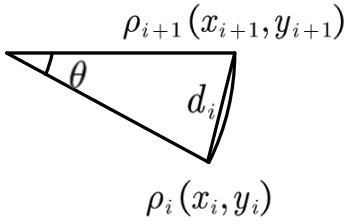


Figure 4. Two points on the solenoid

According to the coordinate equation under the right-angle coordinate system, there are

$$\begin{cases} x_i = \rho_i \cos\theta_i; & y_i = \rho_i \sin\theta_i \\ x_{i+1} = \rho_{i+1} \cos\theta_{i+1}; & y_{i+1} = \rho_{i+1} \sin\theta_{i+1} \end{cases} \tag{17}$$

According to the cosine theorem of a triangle, and the distance between two points is equal to the distance d_i between two small holes in the bench, the formula is

$$\cos\theta = \frac{\rho_{i+1}^2 + \rho_i^2 - d_i^2}{2\rho_i \rho_{i+1}} \tag{18}$$

Using the coordinate equation and the solenoid equation, solving this equation gives you the angle of pinch θ, which gives you the coordinates of the position of the next handle, and still using the idea of recursion, you can get the coordinates of all the dragons' bodies and tails.

3.2. Movement speed

For a particular handle on a particular section of the dragon's body, the forward speed of the handle can be obtained from the rate of change of the handle's position based on the rate of change of that handle's position for position (x_t, y_t) at moment t and position (x_{t+1}, y_{t+1}) at moment $t+1$, as shown in Figure 5,

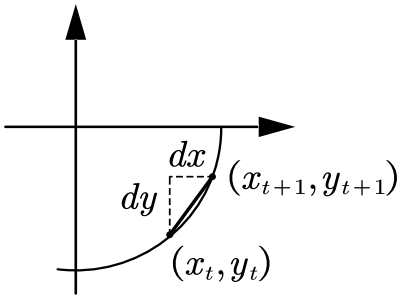


Fig 5. Schematic diagram of speed calculation.

For a certain handle n, the rate of change of its position (x_n, y_n) is

$$\begin{cases} \frac{dx_n}{dt} = \frac{x_{t+1} - x_t}{dt} \\ \frac{dy_n}{dt} = \frac{y_{t+1} - y_t}{dt} \end{cases} \tag{19}$$

By using the formula for the rate of change of bench handle position, the velocity of bench n is obtained as

$$v_n(t) = \sqrt{\frac{dx_n}{dt}^2 + \frac{dy_n}{dt}^2} \tag{20}$$

By iterating using the velocity of the front handle of the dragon head and the position of each bench relative to the front handle of the dragon head, MATLAB[9] programming solves for the velocity of the entire dragon dance team.

3.3. Steps for solving the model

Step1 Initialize parameters

Use the parameters given in the title to assign values to the variables, set the maximum number of iterations to 1000, and take 0.01 time step.

Step2 Solve the initial position

Use for loop to solve the initial position of the dragon dance team, use numerical simulation to recursively solve the initial coordinates of the dragon head, use the cosine theorem to iteratively calculate the angle in the loop, and use the solenoidal equation to update the polar path after each iteration, so as to get the initial position of all points.

Step3 Solve the motion position

Take the time step dt, discretize the continuous motion, define the fun function to represent the motion of the front handle of the dragon head, and use the fourth-order dragon-geekutaode45 function [10] with higher accuracy (the truncation error is about $O(h^5)$) for the solution, and still use the method of Step2 for the solution of the dragon body and dragon tail coordinates.

Step4 Solve the motion velocity

According to the position coordinates obtained in the above steps, using the position change rate of the coordinates, traverse the coordinates of each section through a for loop to get the velocity of the handle before and after each section per second.

4. Results

Suppose a bench dragon consists of 223 sections of benches, of which the first section is the dragon's head, the next 221 sections are the dragon's body, and the last section is the dragon's tail. The length of the dragon's head is 341cm, the length of the dragon's body and tail is 220cm, and the width of all the benches is 30cm. there are two holes in each section of the bench, with the diameter of the holes (hole diameter) of 5.5cm, and the center of the holes is 27.5cm away from the nearest head of the bench.

Through the position of the handle in front of the dragon head at the initial moment, the position of the dragon dance team at the initial moment is obtained recursively as shown in Fig. 6; according to the model established in this paper, the position of the dragon dance team at the time of 300s is plotted as shown in Fig. 7

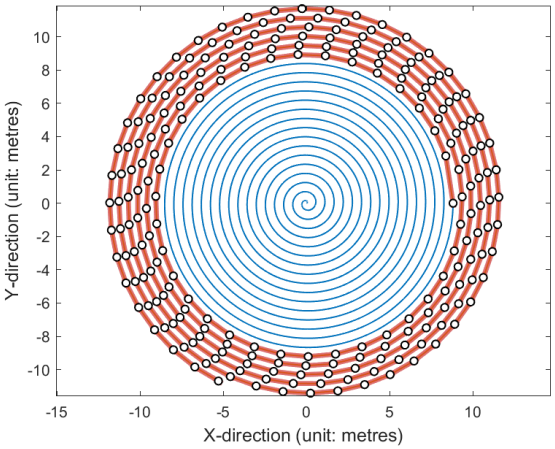


Fig 6. Initial moment position.

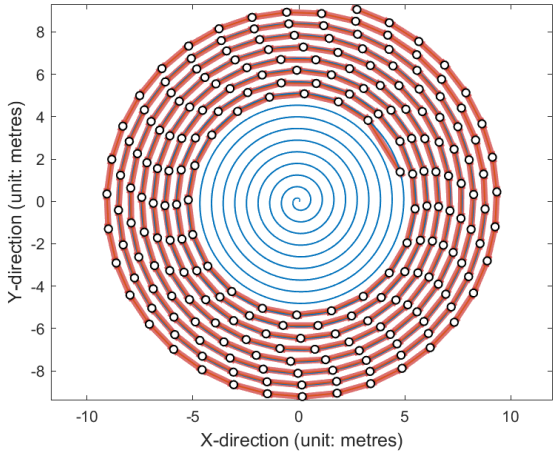


Fig 7. 300s moment position.

Observe Fig. 6, at the initial moment the front handle of the dragon's head is located at the starting point, and the center of each handle is at the 16th-21st circle of the spiral; observe Fig. 7, when the front handle of the dragon's head advances at a speed of 1m/s and moves for 300s, all the handles are moving towards the center of the spiral along the spiral and the closer to the center of the spiral, the greater the degree of the bench's deviation from the spiral.

The position data of the front handle of the dragon's head, the rear handle of the dragon's tail, and the front handle of the dragon's body in the 1st, 51st, 101st, 151st, and 201st sections behind the dragon's head at the time of the 0th, 60th, 120th, 180th, 240th, and 300th are shown in Table 1;

Table 1. Position of some of the handles at the relevant time.

	0s	60s	120s	180s	240s	300s
Mixer x(m)	8.800000	5.792952	-4.093304	-2.952754	2.580986	4.424471
Mixer y(m)	0.000000	-5.77724	-6.298877	6.099874	-5.363021	2.312063
Section 1: The Dragon's Body x(m)	8.360669	7.457351	-1.445399	-5.235975	4.818437	2.458558
Section 1: The Dragon's Body y(m)	2.836175	-3.439148	-7.405898	4.360965	-3.565602	4.403017
Section 51: The Dragon's Body x(m)	-9.520232	-8.686718	-5.543207	2.892082	5.977488	-6.30144
Section 51: The Dragon's Body y(m)	1.331116	2.538783	6.377897	7.248619	-3.831592	0.464759
Section101:The Dragon's Body x(m)	2.923726	5.688251	5.362001	1.897075	-4.920973	-6.238306
Section101:The Dragon's Body y(m)	-9.915533	-8.000592	-7.557595	-8.471981	-6.377034	3.935107
Section151:The Dragon's Body x(m)	10.860119	6.68125	2.388684	1.006900	2.969686	7.040182
Section151:The Dragon's Body y(m)	1.838754	8.135431	9.72743	9.424548	8.398151	4.393929
Section201:The Dragon's Body x(m)	4.545808	-6.620786	-10.627221	-9.288437	-7.460044	-7.458052
Section201:TheDragon'sBody y(m)	10.729143	9.024761	1.359772	-4.245068	-6.17717	-5.264266
Dragon's tail (back) x(m)	-5.296406	7.365625	10.974343	7.385137	3.245375	1.783981
Dragon's tail (back) y(m)	-10.681154	-8.797112	0.843548	7.491126	9.467813	9.301376

The velocity data of the front handle of the dragon's head, the rear handle of the dragon's tail, and the front handle of the dragon's body of the 1st, 51st, 101st, 151st, and 201st sections behind the dragon's head at the time of the 0s, 60s, 120s, 180s, 240s, and 300s are shown in Table 2:

Table 2. Velocities of some handles moving at relevant times.

	0s	60s	120s	180s	240s	300s
Mixer (m/s)	1.000000	1.000454	1.003589	1.008225	0.990008	0.992453
Section 1: The Dragon's Body (m/s)	0.999927	0.999106	1.001971	1.006203	0.987444	0.992060
Section 51: The Dragon's Body (m/s)	0.999697	0.998806	1.001561	1.005609	0.98653	0.990428
Section 101: The Dragon's Body (m/s)	0.999530	1.000595	1.00129	1.005245	0.986028	0.989671
Section 151: The Dragon's Body (m/s)	0.999402	0.998441	1.001098	1.004999	0.98571	0.989233
Section 201 Dragon Body (m/s)	0.999302	0.998323	1.000954	1.004821	0.98549	0.988949
Dragon's tail (back) (m/s)	0.999264	0.998279	1.000902	1.004758	0.985414	0.988853

From the calculation results, it can be analyzed that under the condition that the speed of the handle in front of the given tap is constant at 1m/s, the speed of the handle in front of and behind each section of the dragon's body is close to 1m/s and the range of change is small. This corresponds to the actual situation and proves the reasonableness of the speed obtained by programming.

As the numerical approximation of the analytical solution can be subject to some error, the convergence of the angle θ and the polar diameter ρ can be studied to obtain the iteration diagram shown in Figure 8 below.

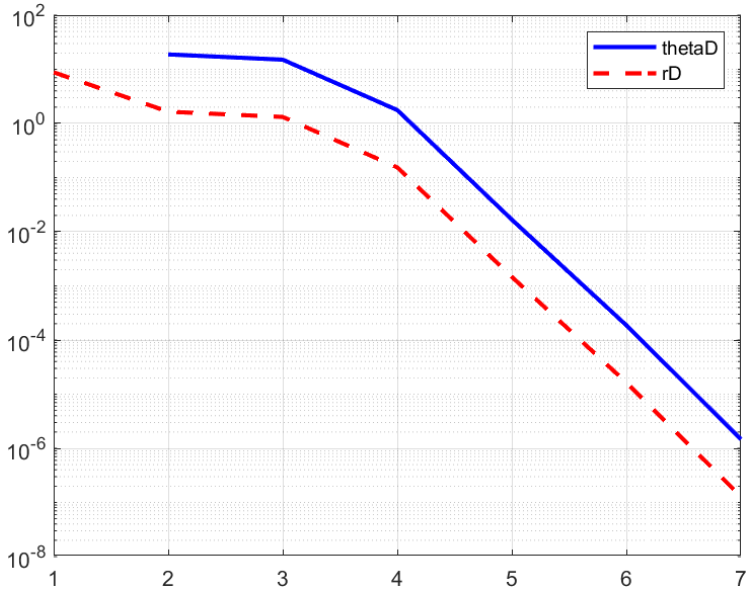


Fig 8. Convergence analysis graph.

From the Figure 8, it can be seen that with the increase of the iteration number, the angle θ and the polar diameter ρ tend to a stable value, which indicates that the numerical solution methods such as recursive and fourth-order Longecourt method can describe the changes of the parameters in the motion model well, and can simulate the motion of the dragon dance team better.

5. Conclusions

In this paper, the dynamic behaviour of bench dragon motion is systematically investigated through mathematical modelling and numerical simulation methods. In this paper, the mathematical model of the dragon's head motion is established, and the positions of the dragon's body and tail are calculated recursively by using the method of microelements and the cosine theorem, and the velocity equation is further constructed. The model was solved numerically using the ode45 function of MATLAB to obtain continuous motion data up to 300 seconds, including the position and velocity information of the dragon head, body and tail. This study not only expands the application scope of multi-body system dynamics theoretically, but also provides a scientific basis for the optimisation of the bench dragon performance at the practical level, which enhances the artistry and spectacle of the performance. This study involves a large amount of parameter setting and calculation work to ensure the accuracy of the results and the robustness of the model. The data and models obtained in this study can be directly applied to actual bench dragon performances, providing practical guidance to enhance the normality and artistry of the performances. Future work can combine the research results with modern technologies such as virtual reality to enhance audience immersion and interactive experience. In addition, the methodology of this research is also expected to be applied to other fields, such as robotic motion planning and animation production, to achieve interdisciplinary innovation and application.

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