Study of UAVs in attempted formation flight based on pure azimuth passive positioning

Yizhou Liu *

School Of Control and Computer Engineering, North China Electric Power University, Beijing, China, 100096

* Corresponding Author Email: 2499429785@qq.com

Abstract. As a great crystallization of human technology in the past hundred years, many western countries have applied high technology to the research and development of UAVs since the 1990s. In this paper, the pure azimuth passive positioning problem in attempted formation flight of UAVs is explored through the intersection of different straight lines to determine the position of the target aircraft, and the location information of the predicted and accurate points is used to optimize the target analysis. And the conclusion is drawn in minimizing the emission signal scenario of electromagnetic waves. Then, according to the knowledge of graph theory, the UAV array is fitted to a graph, a coordinate system is established with the position of one of the UAVs, and a database with standard positions is created for comparison.

Keywords: Passive localization, Error triangle, Least squares, Greedy algorithm Bit.

1. Introduction

As a great crystallization of human science and technology in the past hundred years, since the 1990s [1], many Western countries have applied high technology to the research and development of drones, and the development of drones has entered an unprecedented stage of rapid development [2-3]. In the 21st century, UAVs are used in all aspects of human society [4], and UAV technology will show its superiority in more fields in the future [5]. In this paper, we try to solve the problem of pure orientation passive positioning in the attempted formation flight of UAVs by exploring the pure orientation passive positioning problem in the attempted formation flight of UAVs, combined with our known geometric knowledge [6-7].

2. Model Building

2.1. UAV position modeling

In order to visualize the position of each UAV relative to the UAV FY00 and to describe it better, we first establish a two-dimensional plane coordinate system with the coordinates of the location of FY00[8]. As in Figure 1, we can assume that the angular measurement error obeys Gaussian distribution in the actual measurement, and we can use the C++ program to simulate the error to get an error triangle area, in which the predicted target position can be approximated by using different measurement methods [9-10]. By simulating with the C++ program, we can get that the Steiner method has a better accuracy in the case given in the question. The localization triangle is established from the angles of the three signal transmitters. The three-point cross coordinates can be calculated according to the following formula. The results are shown in the figure 1.

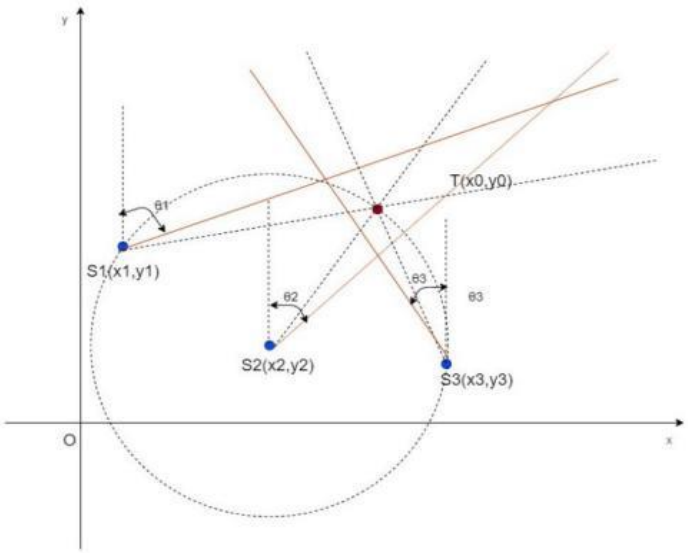


Figure 1. The results of three-point cross coordinates

After determining the coordinates of the three vertices of the locus triangle, use the center of gravity method, the inner method, and the Steiner point method to calculate the required locus point:

1) to obtain S, which is given by

$$S=|x_{12}y_{13}+x_{13}y_{23}+x_{23}y_{12}-x_{12}y_{23}-x_{23}y_{13}-x_{13}y_{12}| \quad (1)$$

2) Find the three sides of the triangle, the formula is the same as above;

3) Calculate the value of k_i with the formula as:

$$k_1=\frac{\sqrt{3}}{2}(c^2+b^2-a^2)+S$$

$$k_2=\frac{\sqrt{3}}{2}(c^2+a^2-b^2)+S \quad (2)$$

$$k_3=\frac{\sqrt{3}}{2}(a^2+b^2-c^2)+S$$

4) Calculation d:$d=\frac{1}{\sqrt{3}}(k_2+k_2+k_3)$

5) Calculate the coordinates of the Steiner point

After obtaining the above points by the formula, simulate the error using random number seeds in C++ environment for each of the following four cases, and calculate the distance between the predicted point and the calculated point, and obtain the optimal solution by comparing the calculation error of the position of the points derived from the three point finding methods.

The statistical average of the four runs is shown in the table1:

Table 1. Statistical average of the four runs

Steiner error	Barycenter error	Inner error
0.1523	0.2174	0.2098

From this, we can tentatively conclude that under the circumstances given in the question, the Steiner method has the smallest average error and is the most accurate method among the three methods, which can be used as the optimal solution to the question.

2.2. Solving in an electromagnetic silent situation

The question asked us to keep the electromagnetic silence and the number of added drones as small as possible, so based on the original question, we first added one drone to do the test, using a single-station simulation to get a system of equations to solve and simulate the location of the predicted points through a C++ program. Subsequently, on the basis of three UAVs, try to add another UAV,

write multiple sets of equations for the trajectory of the circle by the equality of the angles of the circumference of the circle to which the chord lengths are applied, and use a two-station crossover simulation to obtain an approximate solution by writing the equations and using a form of least squares to compare the data obtained by three UAVs with four UAVs and their errors, and choose a better prediction effect.

From the information of the topic can be learned that the location of the UAV that emits the signal is precise, and the location of the UAV needs to be predicted with a slight error, and within the error allowed, the triangle is constructed and the angle and distance between every three aircraft is calculated by the sine theorem, which facilitates the prediction analysis of the subsequent model. The results are shown in the figure 2.

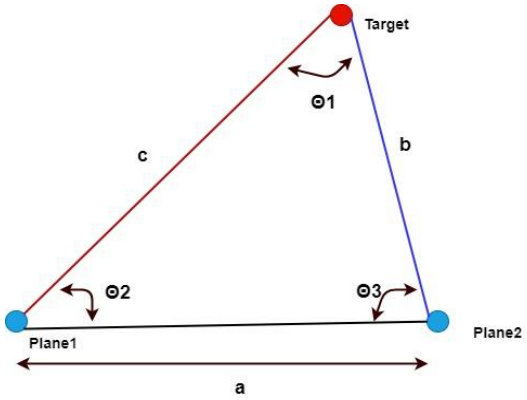


Figure 2. The results of three aircraft calculation

In the simulated triangle constructed above, the position of the aircraft at the center of the circle (Plane1) is known and the position of the other aircraft (Plane2) is known and there is no deviation in position, so the distance c and the distance b can be calculated according to the sine theorem.

We first use the model of three points to predict the position of the plane, as shown in the figure. The results are shown in the figure 3.

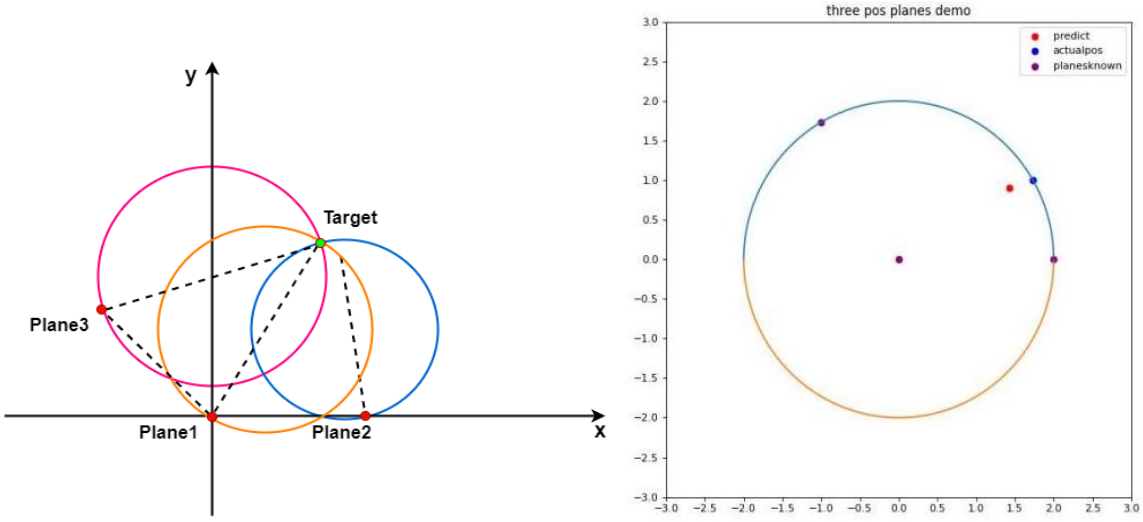


Figure 3. The results of predicting the position of the plane

As shown in the figure, we can perform a computational simulation to find an approximate solution based on the assumption of coordinates of three known points in the plane. Three circles intersecting two by two can yield two sets of solutions, and the average of them is taken into the calculation. Assume that the coordinates of the plane located at the center of the circle are: P1(x_1, y_1), Another known aircraft coordinates P2(x_2, y_2), The coordinates of the third drone transmitting the signal are P3(x_3, y_3). The point to be determined is noted as (x, y), Let the radius of each of the three circles be ρ_1, ρ_2, ρ_3, So the equation can be written:

$$\begin{cases} x^2 - 2x_1x + y^2 - 2y_1y = \rho_1^2 - x_1^2 - y_1^2 \\ x^2 - 2x_2x + y^2 - 2y_2y = \rho_2^2 - x_2^2 - y_2^2 \\ x^2 - 2x_3x + y^2 - 2y_3y = \rho_3^2 - x_3^2 - y_3^2 \end{cases} \tag{3}$$

Presenting this equation as a matrix:

$$\mathrm{Ac} = \mathrm{b} \tag{4}$$

Among them

$$\mathrm{A}=\begin{bmatrix} 2(x_2 - x_1) & 2(y_2 - y_1) \\ 2(x_3 - x_2) & 2(y_3 - y_2) \end{bmatrix} \tag{5}$$

$$\mathrm{c}= \begin{pmatrix} x \\ y \end{pmatrix} \tag{6}$$

$$\mathrm{b}=\begin{pmatrix} \rho_1^2 - \rho_2^2 + x_2^2 - x_1^2 + y_2^2 - y_1^2 \\ \rho_2^2 - \rho_3^2 + x_3^2 - x_2^2 + y_3^2 - y_2^2 \end{pmatrix} \tag{7}$$

By writing the matrix equation in columns, we can write the matrix equation by c=$A^{-1}b$The general solution coefficients are obtained, and the coordinates of our solution point are now given. The simulation using python is given in the figure above.

The model for position prediction by adding two aircraft is given below: using the principle that equal chords on a circle correspond to equal centroid angles, construct the circle of the target UAV's motion trajectory, solve for the coordinates of the intersection points of different circles, obtain the coordinates of the solutions of multiple different groups of points, optimize using the least squares method, and combine mathematical knowledge so as to obtain the coordinates of the point with the smallest theoretical error, and the effect is optimized and improved after using simulation analysis. The results are shown in the figure 4.

Schematic diagram of two additional drones for position prediction simulation:

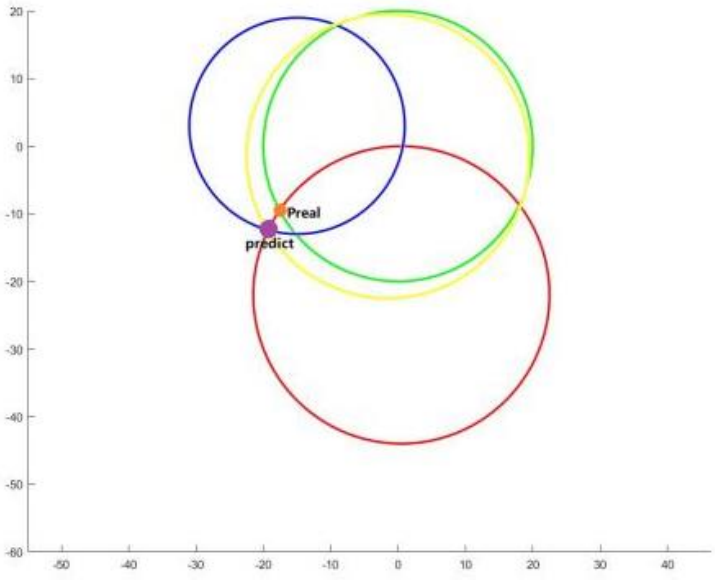


Figure 4. The results after using simulation analysis

We used matlab for simulations in four different cases and obtained more reliable results, the relevant results are shown in the figure above.

To make the error range small, the sine theorem can be used to approximate the side lengths of each triangle by listing a system of equations and obtaining the optimal solution by comparing the errors. The other two aircraft choose a total of four permutations and combinations, so the calculation method of two-station cross-positioning is used, and four different possible permutations are listed and written, resulting in different equations.

Table 2. Four different possible permutations are listed and written

Classification	Drone correspondence number		
1	FY00	FY01	FY0A
2	FY00	FY01	FY0B
3	FY00	FY0A	FY0B
4	FY01	FY0A	FY0B

The solution of this problem can be solved by the least squares method, by which four possible approximate solutions are obtained, and the optimal approximate solution of the UAV is the coordinates of the point at the location where the sum of the distances to the points in the set of four solutions is smallest.

To simplify the calculation, we can obtain the optimal solution of the problem by solving for the Fermat point. When the four approximate solutions form a convex polygon, the coordinates of the intersection of the two diagonals are the coordinates of the point. The results are shown in the figure 5.

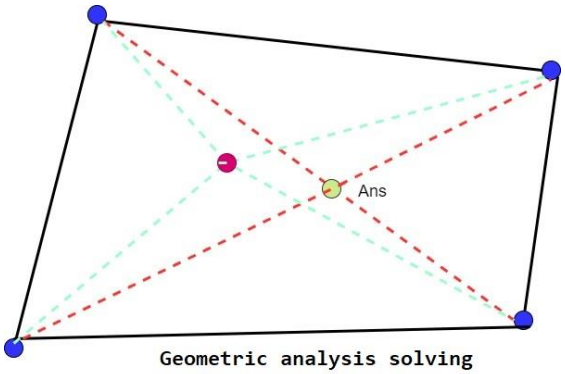


Figure 5. The results of geometric analysis solving

When the polygon is a concave polygon, the corresponding concave point (B) is the desired point. The results are shown in the figure 6.

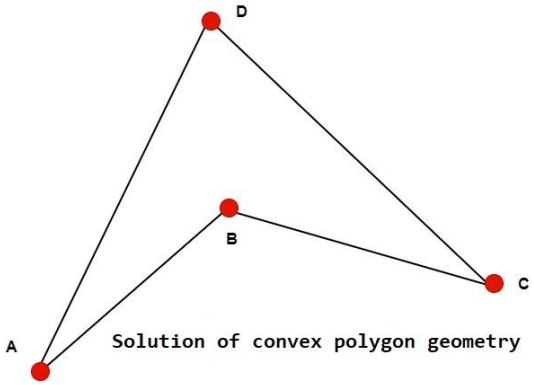


Figure 6. The results of solution of convex polygon geometry

Through matlab comparison, we found that although two additional frames can play a more excellent prediction effect, but the improvement is not significant compared with the effect of single station prediction. Therefore, in order to keep electromagnetic silence, we still choose single-station prediction analysis.

2.3. Part of the aircraft is known

Since the standards of FY00 and FY01 are known, in order to maintain electromagnetic silence, we choose FY00 and FY01 as fixed vertices for initial positioning each time, and take another point for random prediction, i.e., select the UAV with the smallest prediction error in the current set to

obtain a certain circumference close to the standard circumference, and use the known data to compare the real position of the UAV with the standard position when selecting this point, calculate the amount of error, and select the optimal solution from it to make the total error minimum. The amount of error is calculated by comparing the real position of the UAV with the standard position at this point, and the optimal solution is selected from it to minimize the total error. The prediction is performed by using the Steiner method and the optimal solution is obtained by comparing the errors. The basic position solving equation is:

$$x_{sp} = \frac{k_1 k_2 k_3}{2\sqrt{3}Sd}\left(\frac{x_{12}}{k_1} + \frac{x_{13}}{k_2} + \frac{x_{23}}{k_3}\right) \tag{8}$$

$$y_{sp} = \frac{k_1 k_2 k_3}{2\sqrt{3}Sd}\left(\frac{y_{12}}{k_1} + \frac{y_{13}}{k_2} + \frac{y_{23}}{k_3}\right) \tag{9}$$

3. Formation arrays using breadth-first traversal

We coordinate the drone to facilitate programming according to the graph structure. We first determine the four points around the origin of coordinates so that they emit signals to each other, which can be obtained from the standard rhombus with equal angles corresponding to two by two. After this four standard positions are adjusted, after analysis, 3 points are located, and a unique circle is constructed with these 3 points. At the same time, we know that the angles of circumference corresponding to equal chord lengths are equal in size, and we note that the angles of circumference corresponding to the standard team shape are α_0, Connecting the points in the system to the ends of the chord, the resulting angle is α_1, if $\alpha_1 > \alpha_0$, Then the point is inside the circle at this time. The results are shown in the figure 7.

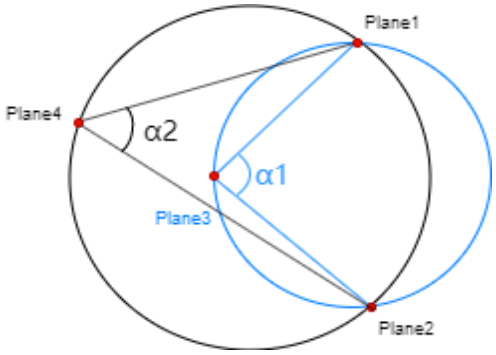


Figure 7. The results of the point is inside the circle

Specifically, we rotate the formation to create the coordinate system shown in the figure 8 as follows:

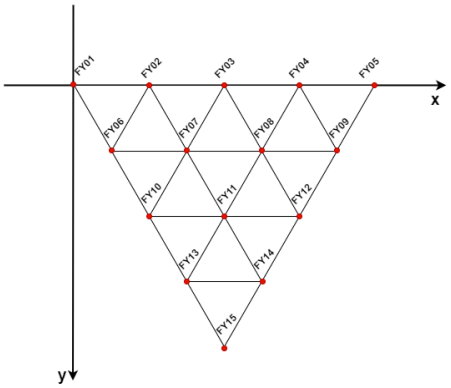


Figure 8. Rotating the formation to create the coordinate system

According to the coordinate system and angle and length requirements as shown in the figure, a standard database is created with a straight line distance of 50m between the points and a connecting angle of 60°. In the actual programming process, random errors are first added by setting C++

temporal random number seeds for all but the four aircraft in the initial state, and are adjusted using a C++ program for simulation. Through the method of breadth-first traversal, first make the four selected UAVs around the origin transmit signals to each other, according to the angle between the relative two UAVs and the adjacent two UAVs are the same, respectively, to construct a standard rhombus for judging the position of the measured UAVs relative to the circumference, and then write a circular program to gradually adjust according to the method described above, so that the UAVs can fall precisely on the requested circumference to ensure their In the same way, the distance between the points can be kept at 50m after the cycle against the standard database, which meets the requirements of the topic. In the subsequent signal transmission process, only two UAVs are allowed to transmit signals at a time to ensure maximum electromagnetic silence, and each point is adjusted twice to ensure the final formation standard. And so on, the UAVs in the conical array are gradually adjusted to the predetermined position to complete the passive positioning of the UAV attempted formation array.

4. Conclusions

In this paper, we thought of using three signals intersecting at a point to get the exact target location point of the receiver, so we obtained the error triangle region by three straight lines and attaching the measurement error. In this region, the error analysis is simulated by combining four different arrangements to determine the error of the center of gravity method, the inner method, and the Steiner method in different arrangements, and finally the conclusion that the Steiner method has the lowest error is obtained.

In the case of minimizing the number of electromagnetic wave transmitters for UAV positioning, we propose to add one UAV to use the single-station prediction model first by comparison, and use a C++ program to simulate the location of the prediction point to obtain a three-transmitting UAV passive positioning model. Subsequently, the total number of signal transmitting UAVs is increased to four, and the trajectory of the equations of motion of multiple groups of P is constructed by equalizing the corresponding circular angles of the same arc, and better prediction results are obtained by the least squares method, increasing the set of equations and comparing the data obtained by three UAVs and four UAVs and their errors to obtain different prediction effects, so that a positioning model with small error and high accuracy can be selected. From the final results for comparison we conclude that one more UAV is needed to achieve effective UAV positioning.

After the detailed UAV polar equations are given, the analysis shows that the polar path is temporarily unable to produce the exact position. We expand the previous method by using a C++ program using the greedy algorithm to analyze the number of the selected UAV for the cycle, and by comparing it with the standard point, the optimal solution is derived for each case. The final result analysis is based on FY00 and FY01, supplemented by a UAV of other sorties fitted with more standard position points, and the solution set is expanded step by step, and the final results are shown in the question-answer section.

While still considering the case of pure azimuthal passive localization, we try to transform the passive localization into an active localization problem by first properly rotating the formation of the queue when the target formation is known, and building a system on top of that to obtain an abstract graph structure using graph theory knowledge. We first target the initial four UAVs to signal each other, so that the initial four UAVs maintain standard positions. We build the set of standard points for comparison, and gradually find the adjusted UAVs in the fleet by breadth traversal, and adjust the unadjusted UAVs according to the adjusted UAVs. A triangle of two adjusted UAVs and one unadjusted UAV is constructed, and signals are emitted from two points at a time to ensure maximum electromagnetic silence, and then a small step size is set, which is cycled in C++ to adjust the angle of the received signals. After that, we use the same method to make the points meet the relationship between the angles, and then gradually correct the distance to get the conical array required by the question.

References

[1] Organizing Committee of National University Student Mathematical Modeling Contest, Format Specification of the National University Student Mathematical Modeling Contest Paper, Gaoxue Cup, Beijing, 2020.

[2] Yang Wei. Bayesian estimation based multi-station pure azimuth passive localization and optimization [D]. Nanjing University of Technology, 2015.

[3] Mathematical Modeling, Higher Education Press Qiyuan Jiang

[4] Ren Lichao. Accuracy Analysis of Passive Positioning Algorithm for Communication Radiation Source [D]. Xi'an University of Technology 2021. doi: 10.27398/d.cnki.gxalu.2021.000968.

[5] Liu Jinbao, Zhang Chunmei, Gan Linhai, Wang Yingchun. Linear correction model for target tracking based on pure orientation information [J]. Command Control and Simulation, 2016, 38(01): 1-3+12.

[6] Xin Shao, Chen Ke, Song Zhenlin, Gui Xinying, Qi Guoqing. A cooperative target tracking algorithm for multiple UAVs based on pure orientation [J]. Electronic Design Engineering, 2021, 29(24): 18-23.

[7] Yu T. Resolution method of pure azimuthal target motion parameters for fixed single station [J]. Journal of radio wave science, 2014, 29(04): 634-638+652.\

[8] Li YB, Gao Q, Zhang WT. Application of PPK-assisted UAV photogrammetry in road survey [J/OL]. Railway Survey: 1-6 [2022-10-16]. http://kns.cnki.net/kcms/detail/11.5182.U.20221012.1437.002.html

[9] Liu Ze-Q, Xu Y, Du Bo-Jun, Wang Yalin. Research on high-speed camera calibration method based on airborne marking targets [J/OL]. Advances in Laser and Optoelectronics:1-14[2022-10-16]. http://kns.cnki.net/kcms/detail/31.1690.TN.20221009.1715.010.html

[10] Yang Rui, Zhao ZJ, Lan TX, Zou Hongyan. Design of an automatic power change system for unmanned aircraft [J]. Mechanical Engineering and Automation, 2022(05): 189-191+194.