Research on UAV formation based on UAV positioning technology and iterative algorithm

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Abstract. Nowadays, UAVs play a very important role in various fields of life, so the flight problem and formation problem of UAVs in real life is one of the important directions to study UAVs, and the positioning function has quite an important role in the formation problem, so this paper studies the formation problem of UAVs based on positioning. A mathematical model is established to study UAV formation based on UAV positioning technology and iterative algorithm.

Keywords: UAV positioning technology, iterative method, error correction, recursive idea, partitioning idea.

1. Introduction

The positioning function of UAV depends on the UAV transmitting electromagnetic waves outward. Each aircraft has its own number, and the position of the aircraft in the formation remains unchanged during the flight, and the direction information of the UAV that receives another UAV transmitting signals is the angle between these aircraft. This chapter establishes a mathematical model to solve the following problems: this paper uses a division of the situation of the location of the UAV. (1) The different combinations of positioning UAVs are divided into four cases in total, and then the positions of the deviating UAVs are divided, and the different combinations of positioning UAVs are classified according to the position relationship of the deviating UAVs, which are one case inside the corner and three cases outside the corner: upper left, lower left and lower right. Then the positioning models of the deviated UAVs in each case are established according to the angle relationship and the sine and cosine theorem. (2) Secondly, this paper adopts increasing the number of positioning UAVs with unknown numbers one by one to prove whether there exists a unique point that can be represented by these individual UAVs. (3) Using the iterative method, the position of the UAVs is corrected. (4) Finally, a positive triangle is determined and the lower UAV positions are corrected step by step according to this triangle.

2. Model building

2.1. Positioning of the deviated UAV by calculating θ and R for different cases

2.1.1 Delineate the position relationship between the combination of UAVs with different numbers and where the points are located

(1) Take F0→F1 as an example (the rest of the cases are just rotating F0→F1) and draw all the combinations of drones.

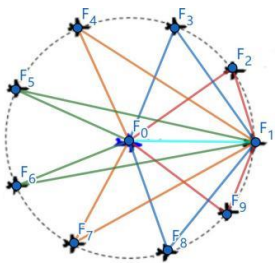


Figure 1. Full portfolio

The various combinations of UAVs can be found in a symmetrical up-and-down relationship, so they can be divided into four categories (the four groups in the upper half of the table below are examples) [1].

(2) Through a large number of experiments, it can be found that the UAV has a different position relationship with the three positioning UAVs in terms of the circular center angle, and the positioning model sought is also different. So basically, it can be divided into four cases, which are one case inside the corner and three cases outside the corner: upper left, lower left and lower right (as shown in the figure 2 below) [3].

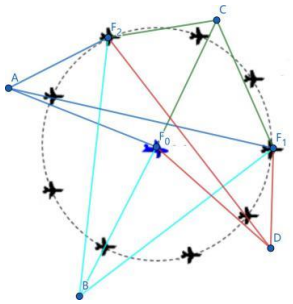


Figure 2. Four types of location map

2.1.2 Positioning model in different cases according to the position of the UAV and the other three UAVs

(1) The situation is divided according to the position of the UAV in relation to the circular center angle of the other three UAVs [2].

According to the previous step, the division of four UAV combinations regarding the position relationship between the deviating UAV and the other three UAVs in terms of the circular center angle can be obtained (as follows).

a). When the drone combination is (FY00, FY01, FY02), it can be found that all the aircraft are located outside $\angle F_1F_0F_2$, so at this time there is only one case that the drone is located outside the corner.

b). When the combination of drones is (FY00, FY01, FY03), it can be found that there are deviated aircraft inside $\angle F_1F_0F_3$ and there are deviated aircraft outside the corner, so there are four distributions of deviated aircraft Fn, which are 1<n<3,3<n<6,5<n<8,7<n<=9.

c). When the UAV combination is (FY00, FY01, FY04), it can be found that there are deviated aircraft located inside $\angle F_1F_0F_4$, and there are also deviated aircraft located outside the corner, so there are four distributions of deviated aircraft Fn, which are 1<n<4, 4<n<6, 5<n<8, 7<n<=9.

d). When the combination of UAVs is (FY00, FY01, FY05), it can be found that there are deviated aircraft inside $\angle F_1F_0F_5$ and there are deviated aircraft outside $\angle F_1F_0F_5$, so there are two distributions of deviated aircraft Fn, which are located inside and outside the corner.

In summary, we can find that the combination (FY00, FY01, FY02) is two cases; the combination (FY00, FY01, FY03) and (FY00, FY01, FY04) are four cases and the classification is basically the same, so they can be combined together in the following calculation; the combination (FY00, FY01, FY05) is two cases.

(2) Establish the positioning model in the case of the number combination (FY00, FY01, FY02).

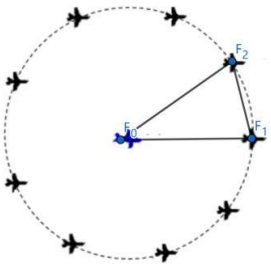


Figure 3. (FY00, FY01, FY02) Combination

Based on the analysis of the situation in the previous step, we can conclude that there is only one case at this point (in the figure below, $\angle F_2F_3F_0 = \alpha_1$, $\angle F_1F_3F_0 = \alpha_2$).

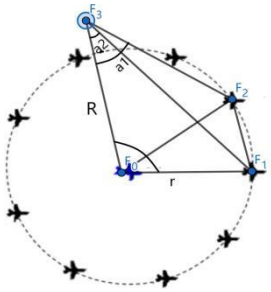


Figure 4. (FY00, FY01, FY02) Combination

At this point, a system of equations can be formulated based on the relationship between α1, α2 and the sides using the sine theorem.

$$\begin{cases} \dfrac{r}{sin\alpha_2} = \dfrac{R}{sin\left(\theta + \alpha_2\right)} \\ \dfrac{r}{sin\alpha_1} = \dfrac{R}{sin\left(\theta + \alpha_1 - \dfrac{2\pi}{9}\right)} \end{cases} \tag{1}$$

R is obtained by substituting θ into the system of equations, where (R, θ) is the polar coordinate of the deviated aircraft.

$$\frac{sin\alpha_1}{sin\alpha_2} = \frac{sin\left(\theta + \alpha_1 - \dfrac{2\pi}{9}\right)}{sin\left(\theta + \alpha_2\right)} \tag{2}$$

(2) Through a lot of experiments and calculations, we can find that the classification of (FY00, FY01, FY03) and (FY00, FY01, FY04) groups are similar, so we establish the localization model in the case of numbered combinations (FY00, FY01, FY03) and (FY00, FY01, FY04) (the following figure).

The distribution of the localized UAVs for the two combinations is shown.

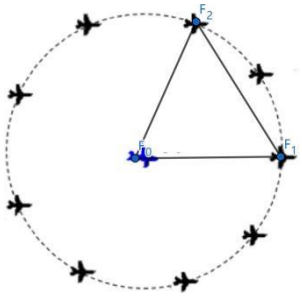


Figure 5. (FY00, FY01, FY03)

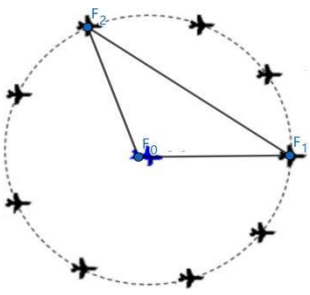


Figure 6. (FY00, FY01, FY04)

Therefore, the positioning models can be obtained when the UAV Fn with deviated position is located within $\angle F_1F_0F_3$ (1<n<3), when the upper left (3<n<6), when the lower left (5<n<8), and when the lower left (7<n<10), and finally it can be found that the positioning models of the two combinations are consistent, and the difference lies in the range of n values, so the two combinations can be combined together..

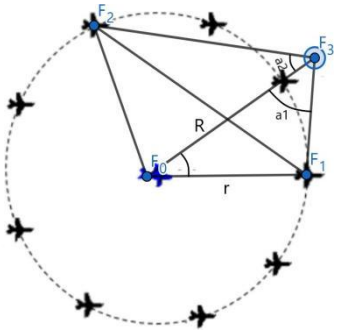


Figure 7. Example of (FY00, FY01, FY04) combination (1<n<3)

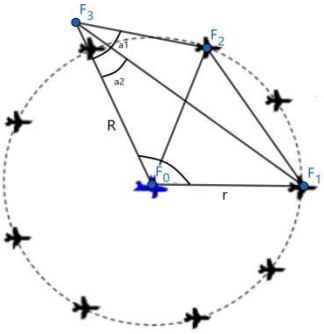


Figure 8. Example of (FY00, FY01, FY03) combination (3<n<6).

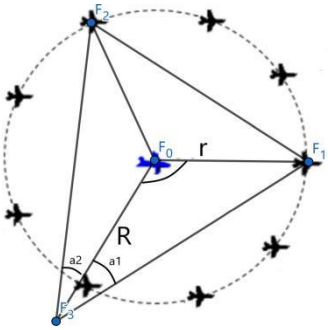


Figure 9. Example of (FY00, FY01, FY04) combination (5<n<8)

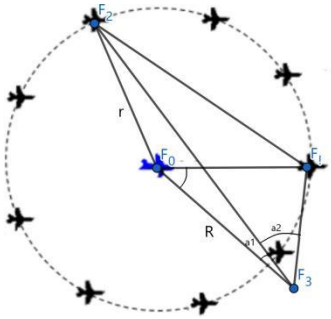


Figure 10. Example of (FY00, FY01, FY04) combination (7<n<10)

(3) Build the positioning model in the case of number combination (FY00, FY01, FY05).

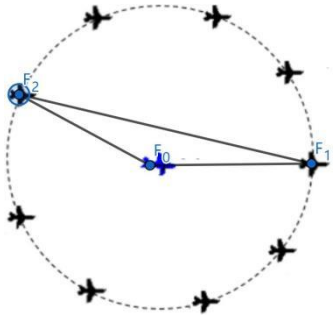


Figure 11. (FY00, FY01, FY05)

When the deviated UAV Fn lies within ∠F1F0F3 (1<n<5), as shown in the figure 12, and when the deviated UAV Fn lies within ∠F1F0F3 (6<n<=9), as shown in the figure 13.

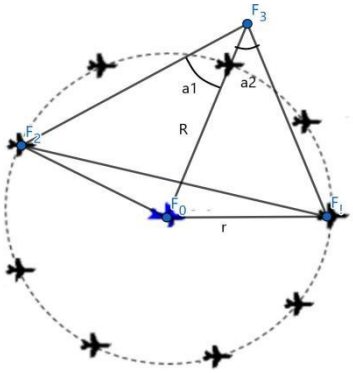


Figure 12. (FY00, FY01, FY05)

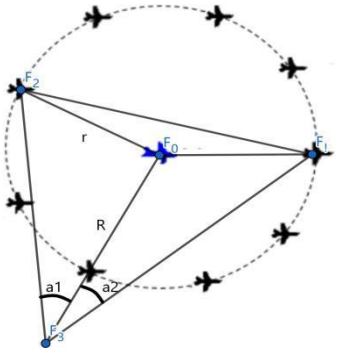


Figure 13. (FY00, FY01, FY05)

2.2. Selecting any drone located on the circumference and calculating whether it is uniquely determined by stepwise incremental selection

2.2.1 When an additional drone is selected, calculate whether the drone is uniquely determined

1. Select 1 additional drone, at which point the drone distribution map is

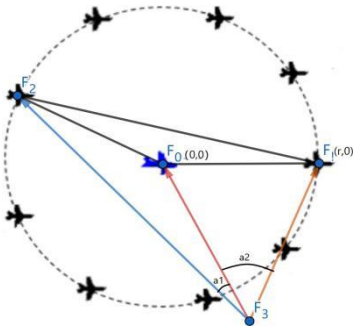


Figure 14. The case of 1 drone

2, By the cosine formula of vectors:

$$cos\alpha = \frac{\vec{a}\cdot\vec{b}}{\left|\vec{b}\right|\cdot\left|\vec{a}\right|} \tag{3}$$

From the above equation we have:

$$cos\alpha = \frac{F_3F_2\cdot F_3F_0}{\left|F_3F_2\right| ? \left|F_3F_2\right|} \tag{4}$$

By substituting α1,α2,θ,r,R etc. into the above equations, the following system of equations can be obtained.

$$\begin{cases} cos\alpha_1 = \dfrac{R(R-r)+\theta^2}{Rrsin\theta} \\ cos\alpha_2 = \dfrac{R(R-r)+\theta^2+\dfrac{2}{9}k\pi\theta}{Rrsin\theta\left(\dfrac{2}{9}k\pi\theta-\theta\right)}\cdot sin\alpha_2 \qquad k = 2,\ 3,\ \ldots\ldots,\ 9 \end{cases} \tag{5}$$

Although θ and R are known, we still cannot find a specific point because the phenomenon of symmetry points may occur at this point, as in figure 15 follows

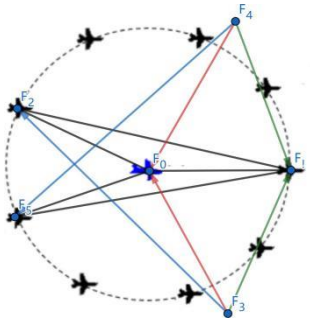


Figure 15. The distribution location of the drones at this time

So at this point we still need to be taking a location drone.

2.3. When two additional UAVs are selected, it is calculated whether the UAV is uniquely determined

(1) due to the existence of the phenomenon of pair formation, we can not determine whether it is located in the upper half of the region or in the lower half, so at this time should choose another drone line positioning. At this point the situation is as in figure 16 follows [4]

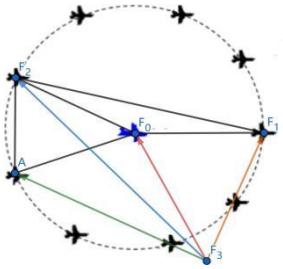


Figure 16. Location map of the two selected drones

(2) When a drone is selected again, an equation for the cosine of the vector is also created, and a set of θ and R can be found by this equation.

(3) Two sets of (R1, θ1), (R2, θ2) can uniquely determine the position of a drone. In summary, only two more UAVs need to be taken to achieve effective positioning of UAVs.

2.4. Adjustment of deviating UAV positions by iterative method until they are close to the respective ideal positions

2.4.1 UAV for positioning and correction method

(1) Since there is a certain range of aircraft deviation, it is assumed here that one UAV cannot deviate into the area where another UAV is located, so a UAV error range is established here as a rectangle [5].

(2) Select a step value, take the points in the error rectangle one by one and calculate the angle θ1 between the point and the other two UAVs, note that the angle between the ideal point and the other two UAVs is θ2, calculate $\Delta\theta = |\theta 2 - \theta 1|$, and select the point with the smallest Δθ as the corrected point of the UAV [6][7].

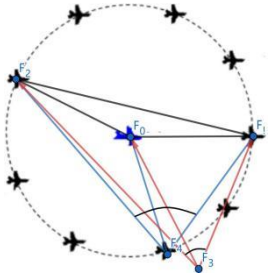


Figure 17. Correction of error angle

2.5. Selecting three UAVs as positioning points for positioning and correction of other deviated UAVs

(1) When positioning and correcting the position of the UAVs, a step value of 0.3 is selected, and the distance of all the UAVs except FY00 and FY01 from the ideal point is calculated as dn(n=2,3,...,9)[8], and the sum of the deviation distances after each adjustment is S(set the coordinates of the ideal point as (n=2,3,...,9). ,9), and the sum of the deviation distance after each adjustment is S (let the coordinates of the ideal point be (xin ,yin)[9], and let the coordinates of the actual point be (xn,yn)):

(2) Firstly, the dn calculated in (1) is used to select the point with the smallest dn as the first positioning drone, and the positioning drones are FY00, FY01 and FY02.

(3) According to the three selected positioning drones, each drone is corrected, and the distance between the corrected point and the ideal point is calculated.

The value of S can be controlled within the range of 0.1(m) by iterating continuously.

(4) Through iteration, we can find that when the number of iterations becomes higher, the other positioning drone is basically FY03, and the coordinates of FY03 are (99.991234, 79.999898), and the sum of deviation distance before adjustment is S=52.0826. Therefore, only the selection scheme of positioning drone with 10 iterations is given below [10].

Table 1. Sum of deviation distances

Selected UAV	Sum of deviation distancesS(m)
0, 1, 2	21.4241
0, 1, 6	2.7140
0, 1, 9	4.4586
0, 1, 8	1.1318
0, 1, 3	0.6515
0, 1, 3	0.9082
0, 1, 3	1.0228
0, 1, 3	0.6783
0, 1, 3	0.9193
0, 1, 3	0.9326

(5) Through the above selection scheme we can get the adjustment scheme, in the first 5 times of adjustment, respectively 2,6,9,8,3 as the third drone to correct the position of the rest of the drones[11], and after 5 times basically FY03 as the third drone for correction, and S has met the restriction of less than 1, so we can think that continue to iterate, the drone can be corrected to the correct position, we made three adjustments, and finally get the polar coordinates of the corrected drone (retain four decimal places) as follows.

Table 2. polar coordinates

UAV number	polar coordinates (R,◦)
0	(0.0,0.0)
1	(100.0,0.0)
2	(100.0481,40.1336)
3	(99.9912, 79.9999)
4	(100.1179, 119.9629)
5	(100.1783, 159.9146)
6	(99.9790, 199.9978)
7	(99.9688, 239.9970)
8	(99.8997, 279.9977)
9	(100.1173, 320.1627)

(6) When the step value is set to 0.5, since the intermediate process of iteration is similar to (1) (5), we will not elaborate too much here, and the convergence values of S for both cases can be obtained by conducting 100 iterations for the step value of 0.3 and the step value of 0.5 respectively.

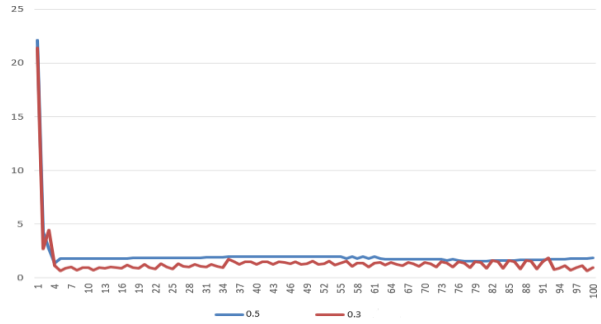


Figure 18. Comparison of convergence values of S

The above comparison shows that the smaller the step value is, i.e., the more precise the accuracy is, the smaller the convergence value of S we can obtain, except that when the step value is 0.3, its convergence value of S is already basically less than 1 to meet the conditions we have restricted and the computer requires the most reasonable running time, so we will not iterate on the case of obtaining a smaller step value [12].

3. Correction of the lower UAV position

3.1. Establish a positive triangle according to the distribution of UAVs

(1) First, take out two of the UAVs in the group noted as A and B, and from them, determine the position of another UAV C. The adjustment method is to move C to the position of ∠ACB = 60 (as in figure 19 follows) [13].

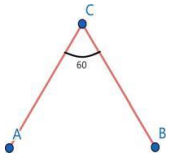


Figure 19. AB to position C

(2) Then the signals from A and C are emitted to position D. The adjustment method is to move D to the position of ∠ADC=0.

(3) Then D, B, and C emit signals to position A. The adjustment method is to move A to the position of ∠BAC=0, and keep the rest of the angle unchanged, so A can only move on A→D.

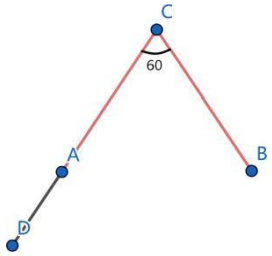


Figure 20. AC to locate D

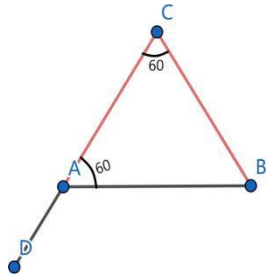


Figure 21. BC, D to locate A

In summary, we can now take three drones to form a square triangle.

3.2. Using the above-mentioned square triangle to locate the UAVs in each layer downward layer by layer

(1) Then A, B, C to the next layer of E to locate, positioning method is to move E to ∠AEC=30, ∠BEC=30.

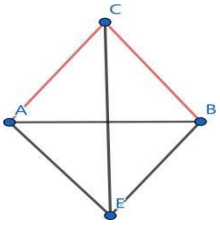


Figure 22. A, B, C to locate E

(2) By this method we can locate the drones on each level layer by layer

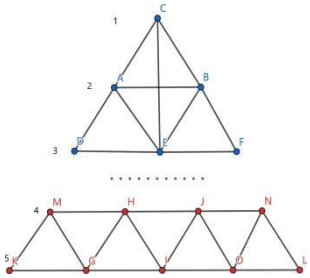


Figure 23. Layer-by-layer positioning

The red point in the figure is the requested point, and its positioning is the same as that of D, E and F in the third layer, so the UAV can be restored to the conical formation by positioning layer by layer [14].

4. Conclusion

(1) The iterative method code runs efficiently because the time only increases due to the increase in the number of cycles and there is no additional space overhead; the iterative method can greatly simplify the amount of operations and the calculated values can increase with the number of iterations; the iterative method can reduce the workload and greatly save the computation time.

(2) The model can be applied to more situations by building the positioning model by situation.

(3) The layer-by-layer downward positioning and correction of the UAV position can reduce the number of corrections and make the results more accurate.

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