Relationship Between Cauchy Integral Theorem and Residue Theorems

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Abstract. Cauchy integral theorem belongs to an extremely important part of complex functions, which is a fundamental bridge, and people can derive Cauchy integral theorem from the residue theorem. Cauchy's integral theorem is generally applied in many higher mathematics, is an important theorem concerning path integrals of fully pure functions. It claims that if there are two different paths from a point to another point and the function is fully pure between these two different paths, then it can be derived that the two path integrals of the function are equal. It is widely believed that the a generalization of Cauchy integral theorem and Cauchy integral formula is just like the residue theorem, and the flexible use of the residue theorem can easily solve some difficult problems in complex functions. Therefore, this paper will use the definition and derivation process of Cauchy residue theorem and integral theorem to demonstrate the specific connection between them and their practical application.

Keywords: Fundamental bridge, equal, solve, connection, pure.

1. Introduction

Ref [1] showed that some academic knowledge of the writer Kodaira and Kunihiko, it involves eight chapters and talked about the knowledge about Cauchy's Theorem.One of its chapters called Cauchy's Theorem was helpful for humans to understand how to use Cauchy's Theorem and its function. And Ref [2] showed that the academic book called Complex Analysis was written by Serge Lang, it was be used in graduating texts in mathematics. It includes nineteen chapters, having very extensive and important knowledge, and people used the chapters called Cauchy's Theorem in function ways, Applications of using Cauchy's Integral Formula and Calculus of Residues and some other chapters to help people to understand relevant knowledge about Cauchy integral theorem and Residue theorems better. In addition, Ref [3] showed that how to use Cauchy's theorem and Residue theorem and some function about it. Some useful chapters of it are definition and Evaluation of Residue, Application of Calculus of Residue to Special Function and Augustin-Louis Cauchy. These chapters provided people some opinions to find the connection between Cauchy integral theorem and Residue theorems. Ref [4] it was shown that in the mathematical field of complex analysis, the contour integral is a way to evaluate certain integrals along a path in the complex plane. The contour integral is more relevant to the calculation of residuals. Ref [5] Moreover, it is shown in that Goursat, in two memoirs entitled Acta Mathematica and in the Monthly Notices of the American Mathematical Society, formulated the Cauchy integral theorem, which assumes no continuity of the derivatives in the closed region bounded by the integral curve C. A summary of these memoirs can be found in the Bulletin of the Society.

Cauchy integral theorem, is an important theorem about the path integral of a fully pure function on the complex plane. Cauchy integral theorem states that if there are two different paths from one point to another and the function is a powerful tool used to calculate an analytic function's integral along a closed curve's path, and can also be used to calculate a real function's integral. The Cauchy integral theorem and Cauchy integral formula is a generalization. It is an effective method to study the calculation of definite integrals, especially for the real and improper integrals whose original

functions are not easy to be found directly. Ref [6] showed that the Cauchy-Goursat Theorem is about the integration of 'holomorphic' functions on triangles. A holomorphic function is basically an extension of what differentiability means, but for complex numbers. Basically, people want the following expression to converge as h tends to zero.However, as people are dealing with complex functions (i.e. a function which takes a complex number as an input, and a complex number as an output), the conditions are a little stronger than for real functions. That's because the complex numbers are a plane: a 2-dimensional object. So our limit can come from any direction: left, right, up, down, spiral, people name it. Whereas in normal calculus, people can only approach a number, say 3, in a one-dimensional manner So 'holomorphism' is a stronger requirement, but it has some wondrous consequences. The subsequent list in Ref. [7] enumerates many of the primary alterations or additions to the first edition.1.The connection between the Cauchy-Riemann equations and real-differentiability. Also, Ref [8] showed a little more in-depth analysis.This is an extremely useful and beautiful section of mathematics that serves as the foundation for numerous methods utilized in numerous fields of physics and mathematics. The familiar concepts of derivatives and integrals from calculus will be applied to complex functions of a complex variable by the researchers.As a result, people will encounter analytical functions, which are the course's central focus.In fact, the study of analytical functions constitutes a significant portion of complex analysis.Analyticity and numerous examples of analytic functions will be discussed first, following a brief overview of complex numbers as points in the complex plane.The generalized Cauchy Integral Formula and a few of its applications will be the culmination of the researchers' discussion of complex integration.The residue calculus and power series representations of analytic functions are then discussed, enabling complex integration to quickly and easily compute infinite sums and many real integrals. In addition, Ref [9] showed that The idea that all quadratic equations with real coefficients "ought" to have solutions is where complex numbers get their start. Since it "ought" to have a solution, one can just say that there is "some sort of number," denoted by i, whose square is 1. The quadratic equation x^2+1 = 0 has no real solution. Ref [10] showed that an overview of analytic functions; integrals, derivatives, conformal mappings, and complex numbers Theorem of Cauchy; Laurent series, power series, singularities, and analytic continuation;surfaces by Riemann;special purposes. Further, Ref [11] showed that Ref [3] is a first course in the theory of complex variables. An introductory theory of complex variable functions. Cauchy's integral theorem and its application.

Next, Ref [12] showed that the present book is intended as a textbook for an advanced undergraduate or early graduate course in the theory of functions of complex variables. There are, of course, numerous older and newer books on this subject; see, for example, our reviews of Muir, Bak and Newman, Sasane and Sasane, Howie, and Ahlfors. The book generally covers many of the same topics as those books: basic definitions and topology in the field of complex numbers such as the Cauchy-Riemann equation, analytic and harmonic functions, power series, conformal maps, complex integrals, singularities and residues, uniform convergence, analytic continuation, and a study of Riemann surfaces. In contrast, the presentation of this book sets it apart from other works. Whether these features are improved, however, depends largely on the tutor's preferences and goals.Ref. [13] also demonstrated that the second-year Center modules of science programs cover material that is essential to many later points.This kind of information is important for many combined programs and helps the development of mathematics in all three major streams offered by degrees with only one subject.Not only do these modules cover the fundamentals, but they also cover applications and methods that all students will be able to use in a variety of contexts later on Ref [14] also showed that this course can be used as an introduction to complex variables for students of engineering, physics and mathematics. The course includes analytic functions, Cauchy integral formulas, Taylor and Laurent series, residue theory and its application in evaluating real integrals, and angle-preserving mappings and their application in solving physical problems. Some of the topics covered in this course are properties of complex numbers, analytic functions and Cauchy-Riemann equations, complex logarithms, exponential and trigonometric functions, Cauchy integral formulas, complex power series, applications of the residue theorem and definite integral calculus.

In addition, Ref. [16] demonstrated that the positive orientation contour and the interior of f are holomorphic. Nap in the interior. f(n)(a) exists if n = 1, 2,..., and f(n)(a) = n!. Next Ref. [17] demonstrated that Cauchy's Integral Theorem is a powerful tool for a number of reasons, including the fact that numerous mathematical methods can be used to estimate and evaluate integral quantities and that it allows the function f(z) to be represented as a contour integral of relatively nice functions. Cauchy's Integral formula and the Fourier and Laplace transform can be used to show analytic functions as integrals over simple functions such as exponentials. In this case, the "simple function" over which f(z) is shown to be an integral is. This property is useful because, with the exception of and, any straightforward contour integral around z will return the constant value, making it analytic everywhere else. Bootstrapping arguments can also be used to prove that the analytic function f has strong properties because it appears as the weighting function in the Cauchy integral rather than as a transform of it.This is because the New integral includes the function f. Additionally, [Ref.]18] demonstrated that analytic functions on the unit disc with a point-wise, distributionally integrable boundary value function can be investigated using the Cauchy representation formula.The formula is shown to be valid when the values of the distributional boundary are present, and examples are provided to show when this might not be the case.The researchers also show that for functions with distributional boundary values, the maximum principle applies to point-wise boundary values. It was also proved in [19] that the Cauchy integral formula for a matrix function cannot be extended to its higher order derivative form, since the derivative of a matrix function of order n over a complex field is usually defined as a matrix of order n2. With the new definition of derivatives in this paper, it is shown that the Cauchy integral formula is valid for matrix functions in higher order derivative form. Examples of the use of the Cauchy integral formula and its higher order derivative form to determine the value of a matrix function are also given.

2. Method

According to Ref. [20], Cauchy's Integral Formula is a fundamental result in complex analysis. Let denote the region's interior, which is bounded by, for any in the interior of the region bounded by C, it is a complex-differentiable function on and is a subset of the complex plane containing a simple counterclockwise loop and the region bounded by, thenLet's denote a straightforward loop that moves counterclockwise around its radius. There is some such that exists for everyone because the interior of the region that is bounded by is an open set. utilizing the application of Cauchy's Integral Theorem for these values of

Because it can be distinguished, anyone can select a reasonable small amount so that at any time. Parameterize as, for now. Since and can simultaneously become arbitrarily small, it follows (again based on Cauchy's Integral Theorem) that is equivalent to the desired theorem. Researchers discover that the derivative of it is through induction. The derivative is especially present for all. To put it another way, a function can be infinitely differentiable inside a region if it is complex-differentiable in one area.

Given that the derivative typically exists, it follows that it is continuous. This isn't true for actual variable functions! For instance, even though the real function can be differentiable everywhere, its derivative does not seem to be continuous atThe mystery vanishes in complex analysis: Because the function has a fundamental singularity at, no derivative can be established there.

The theorem can be used to estimate a function or its derivative at that point by looking at how the function behaves around a point. For instance, the theorem makes it clear that holomorphic functions are analytical. Finally, Reference [21] demonstrated that Cauchy's integral formula represents the culmination of our knowledge of contour integration, holomorphic functions, and contours: an impactful and unexpected result that is unrelated to the real world in calculus. Application and comprehension of its numerous effects on holomorphic functions will take up the remainder of the class.

For all wB(b,R), where t)=b+ride on [0,2], one may have f(w)=12if(z)zwdz for all zC:|zb|RD, for some bD and some R>0 in Fig 1 and 2.

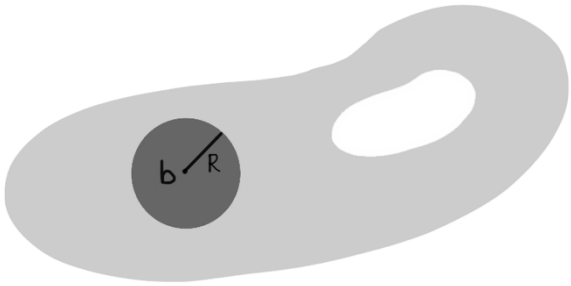


Fig 1. It adjusts web (b, R). For r>0, let r have the form r(t)=w+wide on [0,2]. All points with |zb|R are contained in D. If r is small enough, it is contained in B (b, R).

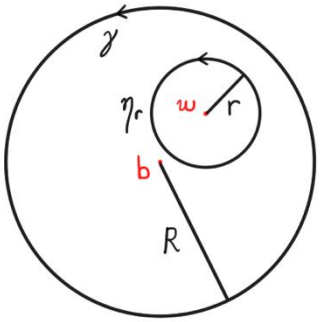


Fig 2. It shows that the contour r is contained in D for sufficiently small r. There exists a holomorphic function h(z) = f(z)zw on Dw. Cauchy's theorem states that h = rh for any sufficiently small r for several contours. The limit on the right-hand side is interpreted as r0.

Correction ">0". Since f is holomorphic in w, it is also continuous in w. Therefore, whenever "zw" appears, ">0, so |f(z)f(w)|" will appear. By the estimation theorem. rh=02f(w+reit)reitireitdt=i02f(w+reit)f(w)+f(w)dt and |i02f(w+reit)f(w)dt|2 when r, because in this case |f(w+reit)f(w)| for all 0t2A as a result, one may have |h2if(w)|2 when any r. Cauchy's integral formula is remarkable: it proves the rigidity of holomorphic functions in contrast to functions of single real variables. It shows that the output of F on the boundary of B (b, R) completely determines the output of the interior of B (b, R). In particular, the holomorphic function f: [c,d] has many differentiable functions with given outputs at c and d. According to the averaging property, f(b) is the average off (b + reit) as t changes from 0 to 2. This property is satisfied whenever D-C:|z-b|R-D. More precisely: as the researcher will see in the next section, any holomorphic function can be extended to a power series at almost any point in its domain! This is called a holomorphic function.

3. Result

First of all, when researchers are studying the relationship between them, researchers should first figure out what they are respectively. The Cauchy integral formula is the central proposition of the theory of functions of complex variables in mathematics it asserts that the value of a fully pure function defined on a disk is completely determinative. The integral formula for each derivative of the completely pure function is provided. Moreover, this formula is named after Augustin-Louis Cauchy. In complex analysis, the Residue theorem, a powerful tool used to calculate the integral of an analytic function along the path of a closed curve, can also be used to calculate the integral of a real function. It is a corollary of Cauchy integral theorem and Cauchy integral formula. Before discussing the theorem and formula of the Cauchy integral, let's understand what is a single connected region in Fig 3 and 4.

A connected region is called a single connected region and is also a point in region D if all interior points of a closed curve C are shown in region D. If a complex function f (z) is analytic and on a closed contour c in a simply connected region, and if z0 is any point in the middle of C, then

f(z0) =12πi∫c f(z)(z-z0) dz

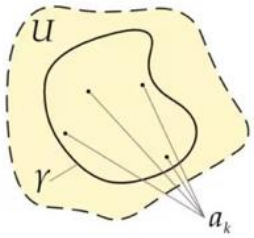


Fig 3. It shows a specific example to explain the idea of this paper.

Assuming that U is a single connected open subset in the complex plane, the a1, an is a finite number of points in the complex plane and f is a fully pure function defined in U ∣ a1, an. If γ is a summable long curve enclosing the entire subset but not passing through any ak and its starting point coincides with the end point, then:

$$\oint_\gamma f(z)\,dz = 2\pi i \sum_{k=1}^{n} \mathrm{I}(\gamma, a_k)\,\mathrm{Res}(f, a_k).$$

Fig 4. It shows the formula of the residue theorem.

Here, Res (fa, k) denotes the number of stays off at the point ak and the number of coils is an integer that describes the number of times the curve v wraps around the point a. If v moves around a in a counterclockwise direction, the coiling number is a positive number, and if y does not go around ak at all, the coiling number is zero.

The Cauchy integral formula is an important conclusion of complex analysis in mathematics, which shows that the value of any fully pure function on a closed region inside the region depends entirely on its value on the boundary of the region, and gives the way to calculate the integral of any order derivative at every point in the region. Cauchy's integral formula is the expression of the "differentiation is equivalent to integration" property of the full pure function in complex analysis. Such a result is completely impossible to achieve in real analysis. This formula was proved by Cauchy in 1831. It was first published on October 11 of the same year and was included in the book Exercices d'analyse et de physique mathématique, published in 1841.

4. Conclusion

In the absence of singularity, the Cauchy integral theorem holds true, in fact the case where the stay number theorem answers to deal with a single pole (the product function has only one pole of order z0), also the Cauchy integral formula for nth order derivatives is the case where the stay number theorem deals with a pole of order n+1. It can be any segmented solvable long simple curve starting at a and ending at b. The function F is called the (complex) original function of f or the inverse derivative function. The integral formula and the integral theorem of Cauchy are equivalent.The retention number theorem and the Cauchy integral formula can both be derived from the Cauchy integral theorem.

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