Forest management system based on carbon sequestration

Shaopu Zhang [a, #], Yuxin Jiang [b, #], Jiangxu Gu [*, #]

Hohai University, Nanjing, China

* Corresponding Author Email: gjx220831@163.com, [a]1829691278@qq.com, [b]2422820877@qq.com

[#]These authors contributed equally.

Abstract. It is a new challenge for forest managers to balance the relationship between the forest and its derivatives to maximize the overall benefits of the forest. Based on the growth pattern of trees and statistical theory, we developed a carbon sequestration model with a normal distribution at different ages of the forest and derived the amount of carbon sequestered by the forest and its products in a certain period of time. In order to balance the ecological and social values of the forest, we firstly selected 12 indicators from three aspects: climate benefit, ecological benefit and social benefit to build a forest decision model. Secondly, the original indicators were downscaled to three principal components by cluster analysis and principal component analysis and the benefit scores were obtained for each year. Finally, a linear fitting method was used to derive the relationship between the forest area and the composite score.

Keywords: Forest carbon sequestration, CASA model, Principal component analysis, gray prediction.

1. Introduction

In recent years, as people become more environmentally conscious, topics such as carbon emissions and carbon neutrality have become increasingly popular. Studies have shown that in addition to the forest itself, the wood products derived from it can also be carbon sequestered, and the combination of forest and forest products will sequester more carbon than the traditional carbon sequestration method to a certain extent [1]. It is a new challenge for forest managers to balance the relationship between forests and their derivatives in order to maximize the overall benefits of forests.

In this paper, establish a carbon sequestration model and determine the conditions under which the forest will sequester the greatest amount of carbon.

Besides, develop a forest decision model to determine the final forest management strategy based on problem 1, considering various factors, including the appropriate scope of management and transition point settings.

Overall, the model constructed in this paper provides a set of scientific and effective solutions for decision making, and at the end, the rationality and advantages and disadvantages of the model are analyzed.

2. Modeling and solving for carbon sequestration

2.1. Carbon sequestration model description

In the process of photosynthesis, forests convert carbon dioxide and water into biomass and release oxygen, thus absorbing large amounts of carbon dioxide, an effect known as the carbon sequestration effect of forests [1]. The aim of this paper is to develop a carbon sequestration model to quantify the specific amount of carbon sequestered by forests.

Usually, the growth cycle of trees is divided into the growth phase and the maturity phase, both of which are defined as follows: for the growth phase of trees, the uptake and release of carbon are unbalanced, and the effective area of carbon dioxide absorbed by trees increases with time; for the maturity phase of trees, since their biomass basically stops growing, their uptake and release of carbon are basically balanced, and the effective area of carbon dioxide absorbed by trees is almost constant

[2]. If the mature trees are made into forest products, the carbon sequestration capacity of mature trees will be better utilized. The carbon sequestration capacity of a typical forest product is 10% of that of a tree in the growing season [3]. Therefore, it is only necessary to calculate the effective area of carbon dioxide absorbed by the forest in the growing season and the amount of carbon sequestered by the forest products made from mature trees to obtain the specific amount of carbon sequestered by a forest with relative accuracy.

First, assume that the total area of the forest in year *X* is S0, where the number of trees of different ages obeys a normal distribution, and let the percentage of trees in the growing season be P_1 and the percentage of trees to be cut down each year be P_2. Then, after *t* years.

$$P_1(t) = \int_t^{t_1} \frac{1}{\sqrt{2\pi}\sigma} exp\left[-\frac{(z_1-u)^2}{2\sigma^2}\right] dz_1, t < z_1 < T, \sigma > 0 \tag{1}$$

$$P_2 = \int_{T-1}^{T} \frac{1}{\sqrt{2\pi}\sigma} exp\left[-\frac{(z_1-u)^2}{2\sigma^2}\right] dz_2, T-1 < z_2 < T, \sigma > 0 \tag{2}$$

Second, since the uptake and release of carbon from trees during the maturity period are essentially equal [1], the effective area of the forest is assumed to be SY(t). Let the growth rate of the tree species relative to the forest area be V. The area of the growing season forest after t years from year X is.

$$S_Y(t) = S_0 \cdot [(1-P_2)\cdot(1+V)]^t \cdot P_1(t) \tag{3}$$

Assuming that the annual carbon dioxide uptake per 10,000 ha of forest is q, and that th magnitude of q depends on the specific major tree species planted in that forest, the amount of carbon sequestered by the forest in year t, Q_1 is.

$$Q_1 = \int_0^t S_Y(t) \cdot q \, dt \tag{4}$$

Considering that the carbon sequestration capacity of forest products is 10% of the carbon sequestration capacity of trees in the growing season, it is assumed that the annual carbon dioxide uptake of forest products made from mature trees is $0.1q$ per 10,000 ha. Then the amount of carbon sequestered by forest products in year t, Q_2 is.

$$Q_2 = 0.1q \cdot \left(tS_0P_2 - (t-1)S_0{P_2}^2 + \cdots + (-1)^t 2S_0{P_2}^{t-1} + (-1)^{t+1}S_0{P_2}^t\right) \tag{5}$$

Then the final amount of carbon sequestered by the forest, Q is.

$$Q = Q_1 + Q_2 \tag{6}$$

2.2. The best forest management strategies for CO2 absorption

Based on the developed carbon sequestration model, assuming that the forest manager manages a certain forest from year t, the expanded value $\Delta S_Y(t,K)$ of the effective area $S_Y(t)$ of trees in the growing season in time period K is obtained as

$$\Delta S_Y(t,K) = S_Y(t+K) - S_Y(t) \tag{7}$$

$$\Delta S_Y = S_0 \cdot \begin{Bmatrix} [(1-P_2)\cdot(1+V)]^{t+K} \cdot P_1(t+K) \\ -[(1-P_2)\cdot(1+V)]^t \cdot P_1(t) \end{Bmatrix} \tag{8}$$

Among different forest management strategies, maintaining the stability of the forest ecosystem can maintain a relatively high rate of CO2 uptake [4]. Therefore, the forest management strategy proposed in this paper includes two components, namely planting and cutting. In the forest management strategy that is most suitable for CO2 absorption, the effective area of the forest should be made almost constant in order to maintain the stability of the ecosystem.

The forest management strategy is analyzed as follows.

Planted area: $S_p = \Delta S_Y(t, K)$ the area of planted trees after K years from year t.

Felled area: $S_p = \Delta S_Y(t, K)$ the area of felled trees after K years from year t, with the following equation.

$$\Delta S_Y = S_0 \cdot \begin{Bmatrix} [(1-P_2)\cdot(1+V)]^{t+K} \cdot P_1(t+K) - \\ [(1-P_2)\cdot(1+V)]^{t} \cdot P_1(t) \end{Bmatrix} \tag{9}$$

The advantage of this is that the forest always absorbs CO2 at a relatively high rate and produces a fixed amount of wood products over a fixed period of time.

2.3. Validation of carbon sequestration model by CASA model

The CASA model is a classical parametric model, which can accurately find out the net primary productivity of local vegetation by using relevant mathematical formulas, taking environmental regulated factors and relevant resource data as input variables, given the environmental conditions of the ecosystem as well as the characteristics of the vegetation itself. Net primary productivity is the remaining portion of the total organic matter produced by photosynthesis on a unit area of vegetation per unit time after deducting autotrophic respiration, and is the value of energy that producers can use for growth, development and reproduction, reflecting the efficiency of plants in fixing and converting photosynthetic products, as well as the material basis for the survival and reproduction of other living members of the ecosystem [5]. The central idea of the model is that the magnitude of vegetation net primary productivity is determined by the combination of light energy utilization and photosynthetically active radiation, with the following equation.

$$NPP = \varepsilon \cdot APAR \tag{10}$$

In the above equation, NPP indicates the net primary productivity $(g\ C/m^2)$ per unit area of vegetation per year in this ecosystem. ε denotes light energy utilization, and $APAR$ denotes photosynthetically active radiation (MJ/m^2).

ε is influenced by factors such as temperature and moisture, and ε is given by the following equation.

$$\varepsilon = T_{\varepsilon_1} \cdot T_{\varepsilon_2} \cdot W_\varepsilon \cdot \varepsilon_{max} \tag{11}$$

Where, T_{ε_1} denotes the optimum temperature stress constraint, T_{ε_2} denotes the average temperature stress constraint; W_ε denotes the moisture stress constraint; and ε_{max} denotes the maximum vegetation light energy utilization. ε_{max} is one of the key factors of $CASA$, which is closely related to the location of the study area, vegetation species, and input data scale. ε_{max} can be used from previous studies [6], as shown in Table 1.

Table 1. Maximum light energy utilization of different vegetation

Primary terrestrial ecosystem type	Secondary terrestrial ecosystem types	Maximum light energy utilization
Forest	Evergreen broad-leaved forest	0.808
	Deciduous broadleaf forest	0.585
	Evergreen coniferous forest	0.378
	Deciduous coniferous forest	0.434
	Broadleaf-coniferous mixed forests	0.461
	Dense vegetation	0.421
	Shrubs	0.389
	Sparse vegetation	0.421

APAR is determined by the proportion of photosynthetically active radiation absorbed by vegetation and total solar radiation.

$$APAR = k \cdot FPAR \cdot SOLAR \tag{12}$$

Where, k is the ratio of solar effective radiation available to vegetation to total solar radiation, and k is equal to 0.5 [6]; FPAR is the proportion of incident photosynthetically effective radiation absorbed by the vegetation canopy; and SOLAR is the total solar radiation for one month (MJ/m^2). The CASA model flow chart is shown in Figure 1.

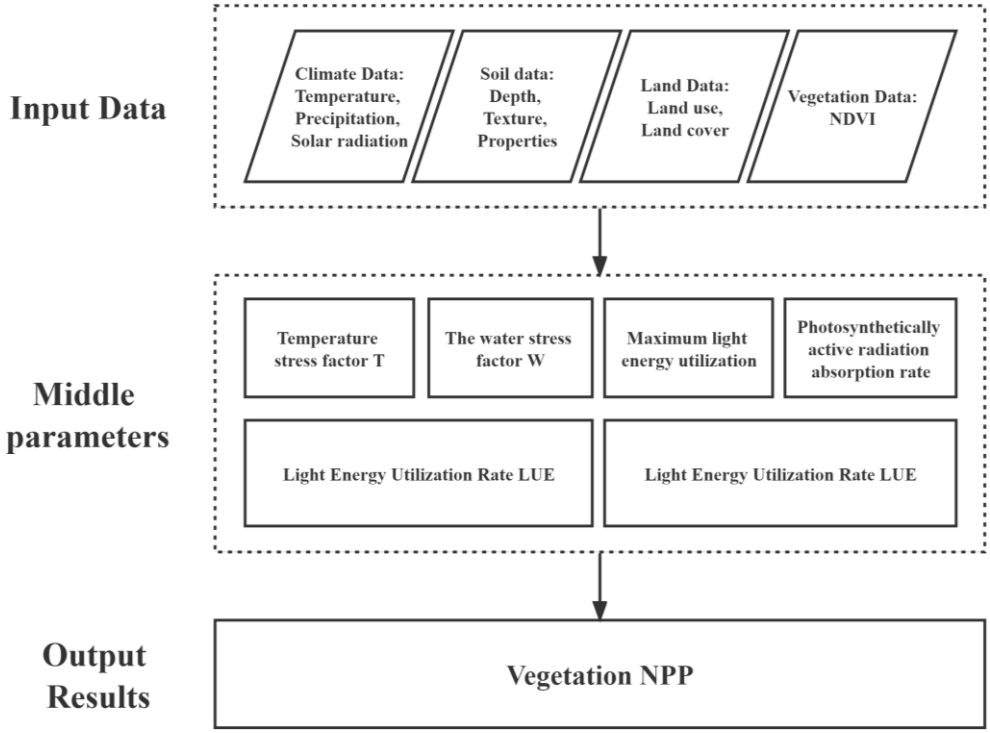


Figure 1. Flow chart of CASA model

Studies have shown that 1 kg of elemental carbon is equivalent to 2.2 kg of organic matter [7], by the chemical equation of photosynthesis in green vegetation:

$$6CO_2 + 6H_2O(\text{Photosynthesis}) \quad C_6H_{12}O_6 + 6O_2 \tag{13}$$

It is known that vegetation can fix 1.63 kg of CO_2 and release 1.19 kg of O_2 for every 1.00 kg of organic matter produced, and 1 kg of CO_2 contains 0.27 kg of elemental carbon. According to this relationship, the annual carbon sequestration of the ecosystem can be obtained from the total amount of vegetation NPP.

$$W_{co_2} = NPP \times 1.63 \times 2.2 \tag{14}$$

$$W_C = W_{CO_2} \times 0.27 \tag{15}$$

Where, W_{CO_2} denotes the amount of fixed CO_2 per unit area of an ecosystem (g/m^2), and W_C denotes the corresponding amount of carbon sequestered per unit area of the ecosystem (g/m^2).

In summary, the amount of carbon sequestered by a forest, W_C, can be expressed through the CASA model as

$$W_C = 0.48 \times T_{\varepsilon_1} \cdot T_{\varepsilon_2} \cdot W_\varepsilon \cdot \varepsilon_{max} \cdot FPAR \cdot SOLAR \tag{16}$$

In order to verify the accuracy of the carbon sequestration model in this paper, the parameters of Wuyishan forest were selected for calculation.

Some of the data in the above equation are too complicated, so we analyzed them with the help of ArcGIS software. The CASA model of the region was calculated by ArcGIS software, and the NPP values of the vegetation in the region were calculated to obtain the forest carbon sequestration W_C in the region.

Firstly, we input the specific data of the region, including the NVDI value, average annual precipitation, average annual temperature, vegetation type, and solar radiation, and visualize the data, and the obtained data part is shown in Figure 2.

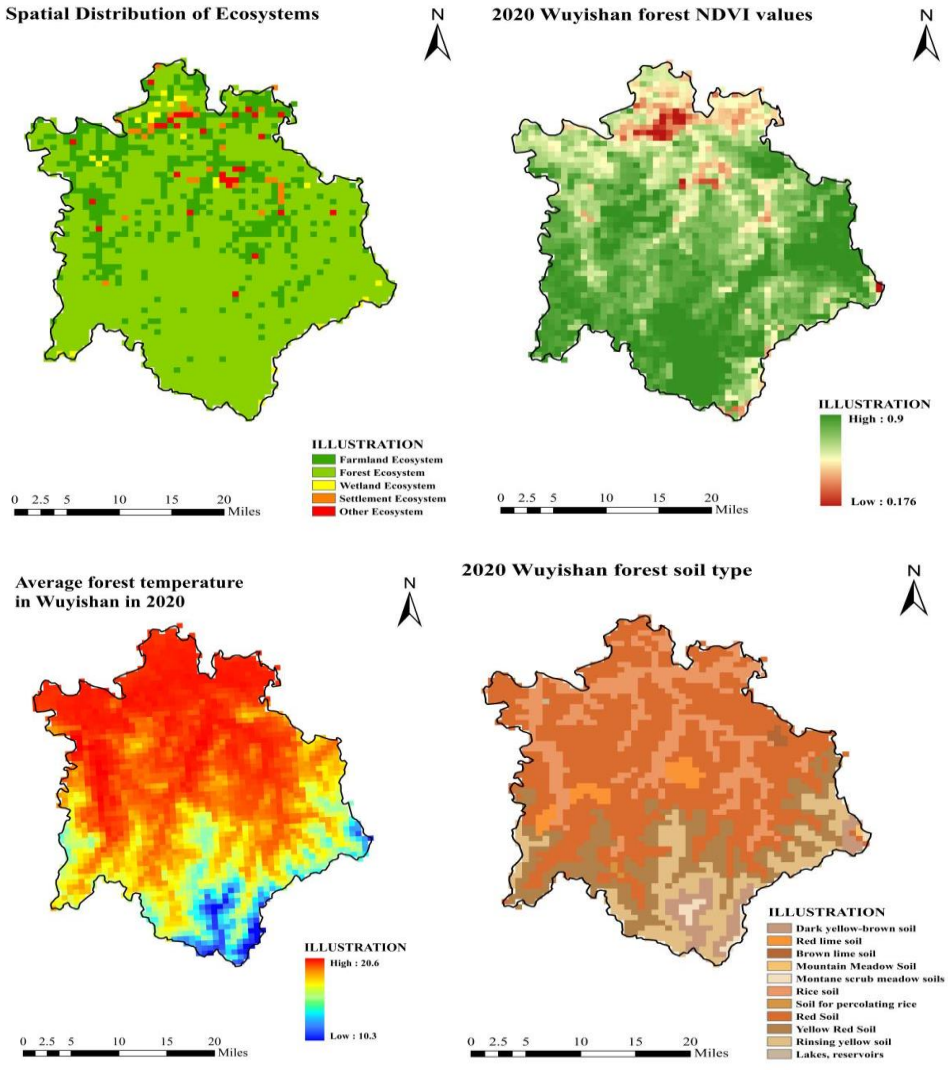


Figure 2. Partial visualization data of Wuyishan forest

Secondly, $T_{\varepsilon_1}, T_{\varepsilon_2}, W_\varepsilon, \varepsilon_{max}, FPAR$ and $SOLAR$ were calculated from the input data respectively, which finally resulted in the NPP of the region, and the carbon sequestration W_C of the forest in the region was obtained by multiplying with the scale factor, as shown in Figure 3.

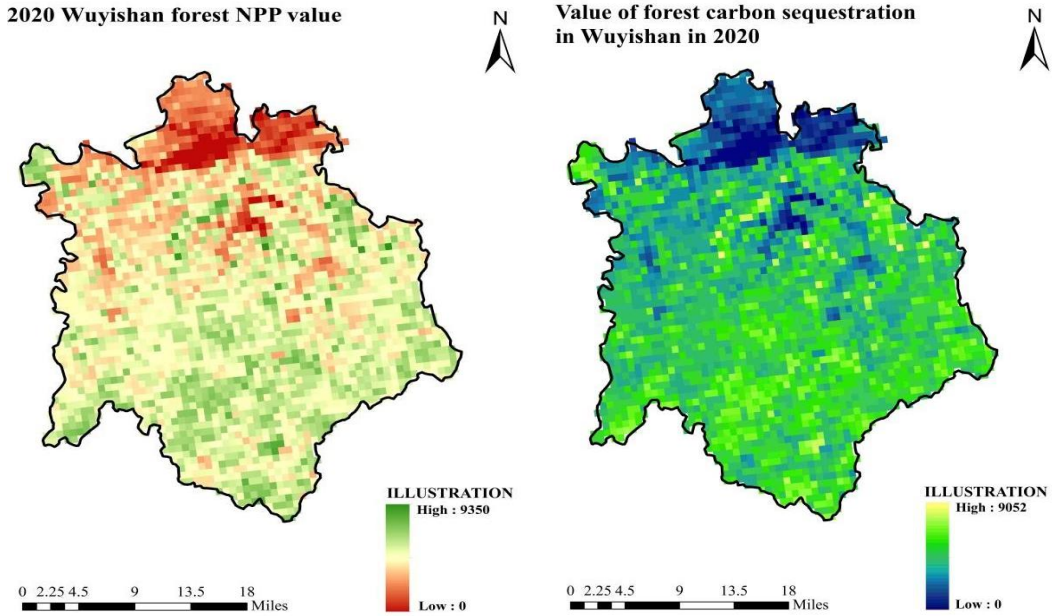


Figure 3. NPP and carbon sequestration values of Wuyishan forest

The mean value of $W_{c_{ave}}$ of W_c per 10,000 ha of forest is found by integration with the following equation.

$$W_{c_{ave}} = \frac{\iint W_{c_{ave}}(x,y)dxdy}{s_{sum}} \tag{17}$$

Where $W_c(x,y)$ is the value of W_c for each image element; s_{sum} is the total area of the region. The solution yields an average value of 11.667 million tons.

Applying the carbon sequestration model of this paper to the Wuyishan forest, the carbon sequestration equation is.

$$Q = \int_0^t S_Y(t) \cdot q\, dt + 0.1q \cdot \begin{pmatrix} tS_0P_2 + \cdots + (-1)^t 2S_0{P_2}^{t-1} + \\ (-1)^{t+1}S_0{P_2}^t \end{pmatrix} \tag{18}$$

The calculated carbon sequestration volume of Wuyishan forest in 2020 is 10.352 million tons. The error between the two is 11.2%, which shows that the carbon sequestration model established in this paper is still relatively accurate.

3. Forest decision model based on R-type cluster analysis method and principal component analysis method

3.1. Forest Decision Model

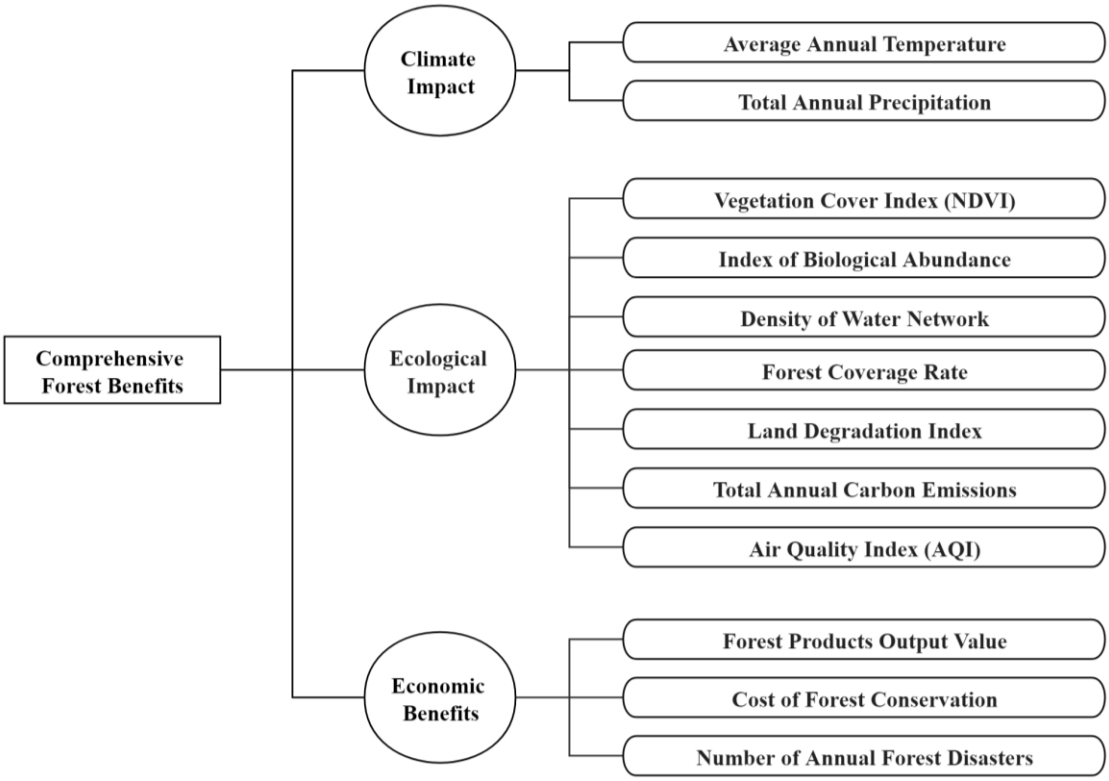


Figure 4. Forest Decision Model Indicator System

Forest decision model indicator system is shown in Figure 4. From the biological point of view, forest ecosystems have many influencing factors on the environment and society, and their classifications are also various. In this paper, the multiple influencing factors are classified into three major categories: climate influence, ecological influence and social influence, and the influencing factors are optimized and reorganized through cluster analysis and principal component analysis.

For climate impact, this paper sets the indexes of total annual precipitation and average annual temperature; for environmental impact, this paper sets the indexes of vegetation cover index, biological abundance index, water network density, forest cover, land degradation index, total annual carbon emission and air quality index; for social impact, considering the economic and ecological benefits of forest products, this paper sets the indexes of forest product output value, forest conservation cost and the number of forest disasters.

In order to solve the forest decision model quantitatively, the forest model in this paper is selected from Xishuangbanna tropical rainforest.

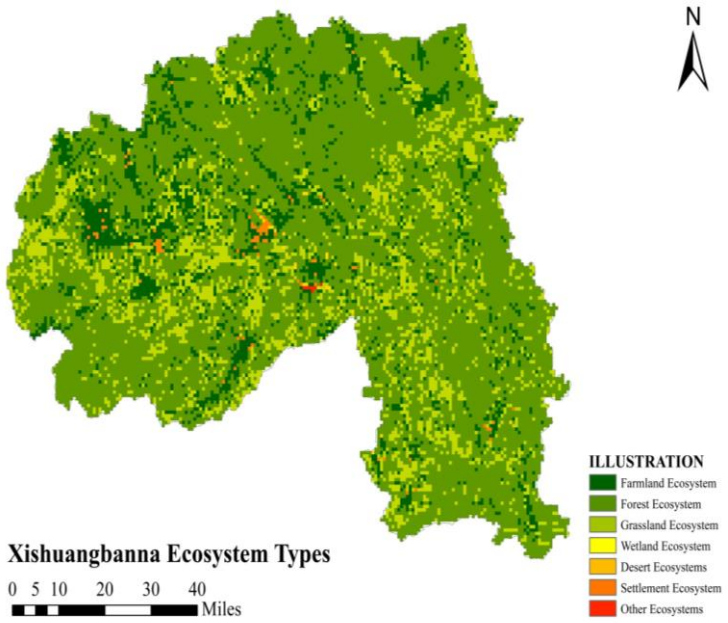


Figure 5. Map of ecosystem types in Xishuangbanna

From the Figure 5, it can be clearly seen that most of Xishuangbanna is covered by forest, which is very suitable for the site selection of forest decision model. In this paper, the data of each index of Xishuangbanna tropical rainforest for 12 years from 2010 to 2021 were selected [8], as shown in Table 2.

Table 2. Each index of Xishuangbanna tropical rainforest for 12 years

Year	Average annual temperaturex_1	Total annual precipitationx_2
2010	20.7	568.4
2011	19.0	869.1
2012	19.2	565.8
2013	18.3	695.0
2014	20.5	802.1
2015	20.5	804.7
2016	21.7	1078.2
2017	20.9	1150.2
2018	21.6	1186.4
2019	21.3	1085.2
2020	21.5	840.2
2021	19.3	1057.4

Year	Forest product valuex_{10}	Forest conservation costsx_{11}	Number of forest disastersx_{12}
2010	37021	401	61
2011	36263	416	63
2012	36514	445	62
2013	37438	451	64
2014	36464	417	65
2015	36876	462	62
2016	36418	485	62
2017	35767	476	64
2018	36354	484	63
2019	35629	503	61
2020	35357	495	64
2021	34954	513	61

Examining 12 evaluation indicators in 3 aspects regarding the benefits of Xishuangbanna tropical rainforest, it can be seen that there may be strong correlations between certain indicators, such as land degradation index and water network density. To verify this idea, Matlab software was used to calculate the correlation coefficients among the 12 indicators.

In this paper, the 12 indicators are R-type clustered according to their correlations, and then representative indicators are selected from each class.

3.1.1. Cluster analysis solution process

In the cluster analysis model of this paper, there are the following parameters: $i = 1,2,\ldots,12$ denotes the time from 2020 to 2021; $x_j(j = 1,2,\ldots,12)$ denotes the indicator; a_{ij} denotes the value of the jth indicator variable in the $i = 1,2,3,\ldots,12$ years of the jth indicator variable; μ_j denotes the sample mean of the jth indicator variable; s_j denotes the sample standard deviation of the jth indicator variable; and y_j denotes the standardized indicator variable.

First, the data were standardized.

$$b_{ij} = \frac{a_{ij}-\mu_j}{s_j}, i = 1,2,\ldots,12; j = 1,2,\ldots,12 \tag{19}$$

$$\mu_j = \frac{1}{12}\sum_{i=1}^{12} a_{ij}, s_j = \sqrt{\frac{1}{11}\sum_{i=1}^{12}(a_{ij}-\mu_j)^2} \tag{20}$$

The standardized indicator vectors are.

$$y_j = \frac{x_j-\mu_j}{s_j}, j = 1,2,\ldots,12. \tag{21}$$

Next, the correlation coefficient is calculated. Note that the variable x_j takes the values $[x_{1j}, x_{2j}, \ldots, x_{mj}]^T \in R^m(j = 1,2,\ldots,p)$. Then the sample correlation coefficients of the two variables x_j and x_k can be used as their similarity measures, i.e.4

$$r_{jk} = \frac{\sum_{i=1}^{m}(x_{ij}-\overline{x_j})(x_{ik}-\overline{x_k})}{[\sum_{i=1}^{m}(x_{ij}-\overline{x_j})^2 \sum_{i=1}^{m}(x_{ij}-\overline{x_k})^2]^{\frac{1}{2}}}, j,k = 1,2,\ldots,p, \tag{22}$$

Then, 12 classes are constructed and each class contains only one sample point, and the platform height of each class is zero; the two closest classes are merged into a new class, and the distance value between these two classes is used as the platform height in the clustering graph; finally, the clustering graph is drawn, and the number and class of the desired decision classes are selected. The clustering tree diagram is shown in Figure 7.

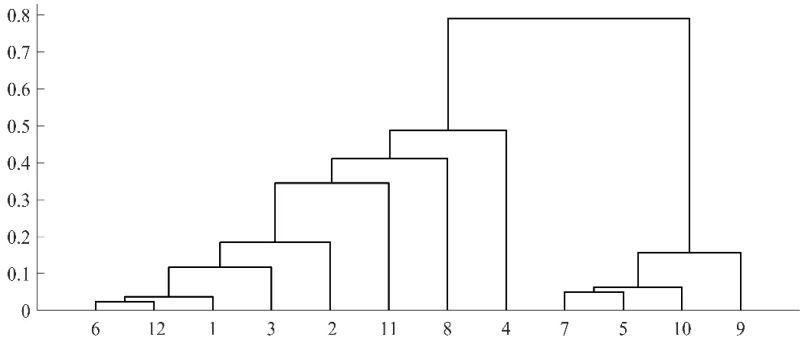


Figure 7. clustering tree diagram

From the clustering diagram, it can be seen that 6, 12, 1 and 5, 7, 9, 10 have high correlation between them and are clustered together by each. The ten indicators in the clustering diagram were divided into six categories, and the indicators with high correlation coefficients were selected from their respective categories to form a new indicator system. In general, indicators with correlation less than 0.1 were not selected [7], so the five indicators 6, 12, 1, 7, and 5 were excluded, and the remaining seven indicators formed the new evaluation system.

Due to the large number of indicators, it is troublesome to examine forest benefits in a multidimensional space. In order to overcome this difficulty, it is necessary to carry out dimensionality reduction, that is, to replace the original more variable indicators with a few less comprehensive indicators, and make these less comprehensive indicators can reflect as much as possible the information reflected by the original more variable indicators, and at the same time they are independent of each other, so the forest decision model in this paper selects the principal component analysis method for analysis[9].

3.1.2. Principal component analysis solving process

First, the original data is standardized. The value of each indicator a_{ij} is converted into a standardized indicator $\widetilde{a_{ij}}$, with.

$$\widetilde{a_{ij}} = \frac{a_{ij}-\mu_j}{s_j}, i = 1,2,\dots,12, j = 1,2,\dots,7, \tag{23}$$

In the formula:

$\mu_j = \frac{1}{12}\sum_{i=1}^{12} a_{ij}; s_j = \sqrt{\frac{1}{12-1}\sum_{i=1}^{12}(a_{ij}-\mu_j)^2}, j = 1,2,\dots,7$, that is, μ_j, s_j are the sample mean and sample standard deviation of the jth indicator. Correspondingly, it is said that.

$$\widetilde{x_j} = \frac{x_j-\mu_j}{s_j}, j = 1,2,\dots,7 \tag{24}$$

Are Standardized indicator variables.

Next, calculate the correlation coefficient matrix R. The correlation coefficient matrix $R = (r_{ij})_{7\times7}$,then.

$$r_{ij} = \frac{\sum_{k=1}^{12}\widetilde{a_{ki}}\cdot\widetilde{a_{kj}}}{12-1}, i,j = 1,2,\dots,7, \tag{25}$$

Where: $r_{ii} = 1; r_{ij} = r_{ji}, r_{ij}$ is the correlation coefficient of the ith indicator with the jth indicator.

Next, the eigenvalues and eigenvectors are calculated. The eigenvalues $\lambda_1 \geq \lambda_2 \geq\cdots\geq \lambda_7 \geq 0$,and the corresponding normalized eigenvectors $u_1, u_2, \dots, u_7$,where $u_j = [u_{1j}, u_{2j}, \dots, u_{7j}]^{\mathrm{T}}$ of the correlation coefficient matrix R are calculated, and seven new indicator variables are formed from the eigenvectors.

$$\begin{aligned} y_1 &= u_{11}\widetilde{x_1} + u_{21}\widetilde{x_2} + \cdots + u_{71}\widetilde{x_7}, \\ y_2 &= u_{12}\widetilde{x_1} + u_{22}\widetilde{x_2} + \cdots + u_{72}\widetilde{x_7}, .. \\ y_7 &= u_{17}\widetilde{x_1} + u_{27}\widetilde{x_2} + \cdots + u_{77}\widetilde{x_7}, \end{aligned} \tag{26}$$

Where: y_1 is the 1st principal component; $y_2$is the 2nd principal component; ...; y_7 is the 7th principal component. The $p(p \leq 7)$ principal components are selected to calculate the comprehensive evaluation value. First, the information contribution rate and cumulative contribution rate of the eigenvalues $\lambda_j(j = 1,2,\dots,7)$ are calculated. Called.

$$b_j = \frac{\lambda_j}{\sum_{k=1}^{7} \lambda_k}, j = 1,2,\dots,7 \tag{27}$$

Is The information contribution of the principal component y_j; and it is said that

$$\alpha_p = \frac{\sum_{k=1}^{p} \lambda_k}{\sum_{k=1}^{7} \lambda_k} \tag{28}$$

Is The cumulative contribution of the principal components $y_1, y_2, \dots, y_p$. When α_p is close to 1 ($\alpha_p = 0.85,0.90,0.95$), the first p indicator variables $y_1, y_2, \dots, y_p$ as the p principal components instead of the original 7 indicator variables, so that the p principal components can be analyzed comprehensively. Then the composite score is calculated as follows.

$$Z = \sum_{j=1}^{p} b_y y_j, \tag{29}$$

Where: b_j is the information contribution rate of the jth principal component, and the evaluation can be made based on the integrated score value.

According to the above algorithm flow, the data of each index of Xishuangbanna tropical rainforest was analyzed by principal component analysis using Matlab.

3.1.3. Linear fitting solution process

First, given a linearly independent function system $\{\varphi_k(x)|k = 1,2,\dots,m\}$,if the fitted function is in the form of its linear combination

$$f(x) = \sum_{k=1}^{m} a_k \varphi_k(x) \tag{30}$$

Then $f(x) = f(x, a_1, a_2, \dots, a_m)$ is the linear function with respect to the parameters $a_1, a_2, \dots, a_m$.appears, for example.

$$f(x) = a_m x^{m-1} + a^{m-1} x^{m-2} + \cdots + a^2 x + a_1, \tag{31}$$

Or

$$f(x) = \sum_{k=1}^{m} a_k cos\,(kx), \tag{32}$$

Then, substituting equation (1) into $\sum_{n=1}^{n} (f(x_i) - y_i)^2$, the objective function $J = J(a_1, a_2, \dots, a_m)$ is a multivariate function about the parameters $a_1, a_2, \dots, a_m$.

$$\sum_{j=1}^{m} [\sum_{i=1}^{n} \varphi_j(x_i)\varphi_i(x_i)]\, a_j = \sum_{i=1}^{n} y_i \varphi_k(x_i), k = 1,2,\dots,m. \tag{33}$$

Thus equation (33) forms a linear system of equations about $a_1, a_2, \dots, a_m$, called the regular system of equations.

$$\boldsymbol{R} = \begin{bmatrix} \varphi_1(x_1) & \cdots & \varphi_m(x_1) \\ \vdots & \ddots & \vdots \\ \varphi_1(x_n) & \cdots & \varphi_m(x_n) \end{bmatrix}, \boldsymbol{A} = \begin{bmatrix} a_1 \\ \vdots \\ a_m \end{bmatrix}, \boldsymbol{Y} = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}, \tag{34}$$

The specific equation of this function is.

$$f(S) = \frac{p_1 \times S^4 + p_2 \times S^3 + p_3 \times S^2 + p_4 \times S + p_5}{S_2 + q_1 \times S + q_2} \tag{35}$$

The specific values of each coefficient are shown in the following Table 3.

Table 3. The specific values of each coefficient

Coefficient	Coefficient value	95% confidence interval
$\boldsymbol{p_1}$	-1.097e-05	(-0.0202, 0.02018)
$\boldsymbol{p_2}$	0.04091	(-1.471e+09, 1.471e+09)
$\boldsymbol{p_3}$	-35.24	(-5.484e+12, 5.484e+12)
$\boldsymbol{p_4}$	874.4	(-4.723e+15, 4.723e+15)
$\boldsymbol{p_5}$	3.339	(-1.172e+17, 1.172e+17)
$\boldsymbol{q_1}$	3055	(-1.34e+14, 1.34e+14)
$\boldsymbol{q_2}$	13.02	(-4.095e+17, 4.095e+17)

The extreme value of this function was solved by Matlab, and it was concluded that the composite forest benefit index was highest when the forest area was 1903 thousand hectares.

3.2. Forest management strategies balancing various factors

Based on the linear fitting curve above, it is concluded that for any forest of $f(s)$, there is always a forest area $f(s)$ that maximizes the combined forest benefits.

For a given forest, the forest management strategy provided in this paper includes cutting and planting. The assumptions of the carbon sequestration model above are extended: the initial area of the forest is S_0; the growth rate of the main tree species in this forest is V relative to the forest area; and the trees are cut down once they reach the age of T years. Let the planted area be S_p and the deforested area be S_c, the effective forest area $S_Y(t)$ after t years is[10].

$$S_Y(t) = S_0 \cdot [(1 - p_2)(1 + V)]^t \cdot P_1(t) \tag{36}$$

For forest managers, the first step should be to determine the relationship between the magnitude of $S_Y(t)$ and S_f.

1) $\boldsymbol{S_Y(t) > S_f}$

In this case, the effective forest area is larger than the optimum forest area and should be cut down as soon as possible and planted in an appropriate amount, and there is no transition point in the forest management strategy in this case. In the first year of the forest management strategy, the area of forest cut down $S_{c_1}(1)$ is $\frac{S_Y(t) - S_f}{P(t)}$, and the area of plantation $S_{p_1}(1)$ is 0. In the subsequent year t of the forest management strategy, the area of forest cut down $S_{c_1}(t)$ is:

$$S_0 \cdot \begin{Bmatrix} [(1 - p_2)(1 + V)]^{t+1} \cdot P_1(t+1) - \\ [(1 - p_2)(1 + V)]^t \cdot P_1(t) \end{Bmatrix} \tag{37}$$

The planted area $S_{p_1}(t)$ is equal to the felled area.

2) $\boldsymbol{S_Y(t) \leq S_f}$

In this case, the effective forest area is smaller than the optimum forest area, deforestation cannot be carried out, and there is an urgent need to plant a large number of trees, and the forest management strategy has a transition point in this case. In the first year plan of the forest management strategy, the planted forest area is $S_{p_2}(1) = \frac{S_f - S_Y(t)}{P(t)}$. Since the forest planted in the first year needs to grow to produce an effective forest area, assuming Z years of growth, $\Delta S_Y(t, Z) = S_{p_2 1}$, which is.

$$S_0 \cdot \begin{Bmatrix} [(1-p_2)(1+V)]^{t+Z} \cdot P_1(t+Z) - \\ [(1-p_2)(1+V)]^{t} \cdot P_1(t) \end{Bmatrix} = \frac{S_f - S_Y(t)}{P(t)} \quad (38)$$

The specific value of Z can be found from the above equation. Planting $(Sf - S_Y(t))$ area of forest in the first year can be the fastest way to reach the effective forest area of S_f. After the first planting, a transition period of Z years is required before the forest can be planted and deforested.

In the specific analysis of a particular forest, it is considered that different forests have different situations, which are reacted in the model of this paper as the difference in the growth rate V of the main tree species and the difference in the optimum area S_f. Therefore, for different forests, a specific Z-year transition period for that forest can be calculated.

4. Conclusion

Validation of carbon sequestration model by CASA model. In order to verify the accuracy of the carbon sequestration model in this paper, the parameters of Wuyishan forest were selected for calculation. And with the help of ArcGIS software analysis, the CASA model of the region was calculated by ArcGIS software.

The decision model of the forest. A combination of cluster analysis and principal component analysis was used for forest carbon sequestration assessment, and a normal distribution function was introduced.

Secondary factors were ignored. In the process of building the model we only selected a small number of factors for evaluation, and ignored some other minor factors. The proportion of these factors is relatively small, but the lack of these factors will cause errors in the model, thus not accurately reflecting the actual carbon sequestration effect of the forest.

We established a carbon sequestration model of the forest based on the normal distribution and verified the accuracy of the carbon sequestration model by combining the CASA model. Second, we obtained the forest decision model by cluster analysis method and principal component analysis method

Based on the model we provided, there are three different management strategies

Management strategy when the carbon sequestration capacity is the strongest.

Management strategy when social benefits are considered.

Management strategy when the harvesting cycle changes.

References

[1] Wang Yan, Liu Xing'e. Progress of wood carbon sequestration research [J]. World Forestry Research,2009, 22 (01): 54 - 58.

[2] Li Zhong. Study on the relationship between tree species and forest age and forest carbon sequestration [J]. Forestry Survey and Design, 1999 (01): 21 - 22.

[3] Wu Wisdom. Analysis of the relationship between life cycle of wood and carbon sink [J]. Furniture, 2015, 36 (02): 1 - 6+43.

[4] YING Yuqi, XIANG Tingting, LIN Weilei, WU Jiasen, YANG Jie, JIANG Peikun. Sequestration characteristics of phytosilica carbon in the apoplastic layer of five forest stands in subtropical China [J]. Forestry Science, 2015, 51 (03): 1 - 7.

[5] Wen Yuyue, Sun Qiang, Yan Yuchao, Xiao Minzhi, Song Weiwei, Yang Jian. Impacts of terrestrial ecosystem evolution on carbon sequestration and oxygen release services in the Guangdong-Hong Kong-Macao Greater Bay Area [J]. Journal of Ecology, 2020, 40 (23): 8482 - 8493.

[6] WANG Yanyan, YANG Mingchuan, PAN Yaoqing, ZHU Wenquan, LONG Zhonghua, LIU Xuzao, GU Xiaohe. Remote sensing estimation of organic matter value of terrestrial vegetation ecosystem production in China [J]. Ecological Realm, 2005 (04): 455 - 459.

[7] Wang, X. K., Liu, W. W. Factors affecting carbon sequestration in forests [J]. Forestry and Ecology, 2021, (03): 40 - 41.

[8] LI Wei, HUANG Mei, ZHANG Yuandong, GU Fengxue, GONG He, GUO Rui, ZHONG Xiuli, YAN Changrong. Spatial and temporal dynamics of carbon storage and carbon sequestration rate in Chinese national forest parks[J]. Journal of Applied Ecology, 2021, 32 (03): 799 - 809.

[9] Cai Y, Zhou J-T. Research progress of carbon sequestration estimation methods in forest ecosystems [J]. Green Technology, 2020, (18): 48 - 50.

[10] Liu, K. E., Song, W. C. Study of average forest carbon sequestration and ton carbon footprint in China based on forest ecosystem[C]//. Proceedings of the 2021 Annual Scientific and Technical Conference of the Chinese Society of Environmental Sciences (I)., 2021: 91 - 96.