# The Comparison of the computing ability of quantum and conventional computer

Haoxin Li [1, †], Guangyu Zhao [2, *, †]

[1]North East Yucai Foreign Language School, Shenyang, China

[2]School of Physics, University of New South Wales, Sydney, Australia

[*]Corresponding author: guangyu.zhao@student.unsw.edu.au

[†]These authors contributed equally.

**Abstract.** Contemporarily, whether quantum computing performs better than conventional computers remain an unresolved issue. In this paper, we compare the time complexity between quantum and conventional computers for a specific type of issue. Theoretically, quantum computation is better in solving nonlinear calculations, the quantitative evaluation of the computing ability should be investigated. In this paper, the Fourier transform was applied in both classical and quantum logical circuits to calculate the theoretical time complexities respectively and make a comparison after that. According to the analysis, the ideal quantum computer performed really fast as the number of tasks increased, but the gate-level accurate quantum circuit, which was a simulated circuit was running slower. The paper shows that the current technology cannot perform the perfect quantum computer, but the future of it would be very promising. Overall, these results shed light on guiding further exploration of quantum computing.

**Keywords:** Quantum computation, Fourier transform, Qiskit.

## 1. Introduction

Quantum computer is a kind of machine that uses quantum laws to calculate and process files. The background of the quantum computing can be traced back to about 60 years ago. In 1959, Richard Feynman firstly pointed out the possibility of using quantum effects for computation. He gave his lecture which calls Feynman's Classic Caltech Lecture today. Before his lecture, a lot of people had succeeded in constructing electric motors the size of a small fingernail, and he emphasized the capability of the complex activity of the animal's organization and the human's organization, this capable of the manufacture of various substances, movement, and storage of information. Besides, he combined this with the quantum physics, and his theory finally turned to the success of the appearance of quantum computers, etc. Therefore, Richard Feynman was the pioneer of the quantum computing.

Compared to the traditional computers, quantum computers have a faster speed of 10 qubits, which means they can make 1024 times calculations in one time. In calculating and with a higher level of safety in processing the files and resolving problems, scholars usually use this way to process more information to ensure its precision of it. Therefore, the object of this research is very clear, quantum computing or the logic working way of quantum computing QFT [1].

As the quantum computer is applied more and more in the simulation of the experimental environment, it is necessary to have a certain degree of understanding of this concept. Subsequently, in 1993, an international group made up of six scientists proved that perfect teleportation was technically possible but in order to work the original thing being copied would have to be destroyed. From that time, the quantum physics began to be popular, which contributed to the appearance of the quantum computer.

In 2009, Yale university created the first solid-state quantum processor. In 2011, there was a breakthrough in quantum teleportation made by Japan and Australia. In the same year, D-Wave announced its first commercial quantum annealer, the quantum annealing processors can return low-energy solutions. So, it means a higher standard in quantum computing. In 2011, Google launched the Quantum AI Lab, firstly using the all-quantum computing equipment in one lab. In 2015, NASA established the D-Wave System, named after its inventor, and that made the quantum computing step

to a new height. Contemporarily, humans are still researching if there will be more breakthroughs in the quantum computing [2].

The IMB Quantum leads the world in quantum computing. In 2016, the IBM Quantum company firstly built up this system. This nascent technology is widely expected to solve valuable problems that cannot be solved by the most powerful and state-of-art classical supercomputers. The IBM Quantum Company aims to turn the world's most challenging problems into valuable opportunities [3]. The professors from the University of Science and Technology designed the first quantum computer in China which called China optical quantum computer, which has led China to enter the new world of the quantum computer in the world.

The rest part of the paper is organized as follows. The Sec. 2 will introduce principles of primary quantum computation and the basic fast and quantum Fourier transform, few functions and figures were included in this section to explain the principle clearer. The Sec. 3 will explain the method that applied to accomplish the aim of this paper, the exact use of tools was listed and explained. The Sec. 4 will present the result based on each step in the method section and give a relevant analysis of the results and conclusion. The Sec. 5 will conclude the whole paper and briefly look to the future of the application of quantum computer.

## 2. Principle

### 2.1 Quantum computer

A type of computation that harnesses the collective properties of quantum states (superposition, interference, and entanglement) to perform the calculation. A qubit can be 1 and 0 at the same time while a bit can only be 0 or 1. Moreover, A qubit (or quantum bit) is the quantum-mechanical analog of a classical bit [4].

Besides, it is necessary to demonstrate the phenomenon of the quantum entanglement. In quantum physics, only the properties of the overall system can be described instead of separately when several particles interact with each other, which is known as the quantum entanglement [5]. Thus, in the quantum computing, the quantum entanglement is mostly used to transform some information, which can secretly give your information to others. Assuming a composite system is composed of two subsystems A and B, and the Hilbert Space of the two subsystems A and B are respectively. Subsequently, the Hilbert space of the composite system is the tensor product, which can be described as:

$$H_{AB} = H_A \otimes H_B \tag{1}$$

### 2.2 Quantum circuit

A Quantum circuit is the combination of a classical circuit and many quantum-based elements such as some quantum operations on qubits. The system is constructed on a concurrent real-time classical computation with a series of quantum measurements, logic gates, and resets well-aligned, all potentially conditioned on and used by data from real-time classical computations [6]. An example of a quantum circuit of quantum teleportation is sketched in Fig. 1.

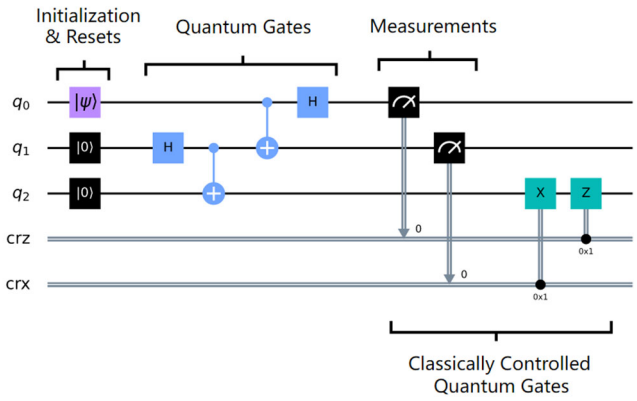



**Figure 1.** The Quantum circuit of quantum teleportation consists of three qubits and two classical bits.

### 2.2.1 Initialization and resets

The circuit starts with a well-defined quantum state at the very beginning. Initialization and reset elements can guarantee this goal. The resets can be carried out using a mix of single-qubit gates and concurrent real-time classical computation, which monitors if measurements have achieved the required state. Single-qubit gates are applied after initializing q0 into a specific state $|\psi\rangle$.

### 2.2.2 Quantum gates

In addition, to alter the three qubits in the teleportation algorithm's requirements, the utilization of a series of quantum gates is necessary. In this scenario, one just simply uses single-qubit Hadamard (H) and two-qubit Controlled-X ($\oplus$) to meet the requirements.

### 2.2.3 Measurements

Two of the three qubits are measured in the third step. The measurements of each qubit are interpreted as classical outcomes (0 and 1) and stored in the two classical bits by a classical computer.

### 2.2.4 Classically conditioned quantum gates

Subsequently, the circuit use single-qubit Z and X quantum gates on the third qubit. The results of the measurements recorded as the two classical bits are utilized to condition these gates. In this situation, the results of a classical computation are used in real-time within the same quantum circuit.

## 2.3 Quantum logic gates

There are many gates in a quantum circuit to express different operating modes. In this report, only a few gates are included in the introduction as space is limited. The most basic gates must be the Pauli gates, which are specifically classified as X, Y, and Z gates. These gates should be familiar to researchers with linear algebra backgrounds as they are all simple 2×2 matrices with only the bar and ket element coming from the quantum section [8]. For the X-gate, it is represented by the Pauli-X matrix:

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = |0\rangle\langle 1| + |1\rangle\langle 0| \tag{2}$$

To examine how the gate affects a qubit, just multiply the state vector of the qubit by the gate. The X-gate may be seen to alter the amplitudes of the states $|0\rangle$ and $|1\rangle$:

$$X|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |1\rangle \tag{3}$$

The X-gate could be considered of as a rotation around the x-axis of the Bloch sphere by π radians. The X-gate is also known as a NOT-gate, which refers to its classical counterpart. Similar to the X-gate, Y and Z-gates are both represented by Y and Z Pauli matrices:

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = -i|0\rangle\langle 1| + i|1\rangle\langle 0| \tag{4}$$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = |0\rangle\langle 0| - |1\rangle\langle 1| \quad (5)$$

They also rotate around the Bloch sphere's y and z axes by π respectively, which is expected in Ref. [7]. The next gate must be the Hadamard gate, it is a basic quantum gate also known as the H-gate. It enables the system to move away from the Bloch sphere's poles and generate a superposition of $|0\rangle$ and $|1\rangle$. It has the following matrix:

$$\boldsymbol{H} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad (6)$$

That performs the operation as below:

$$H|0\rangle = |+\rangle, H|1\rangle = |-\rangle \quad (7)$$

This can be interpreted as a rotation around the Bloch vector [1,0,1] (the line connecting the x and z axes) or as a transformation of the qubit's state between the X and Z bases.

### 2.4 FFT and QFT

Fast Fourier transform (FFT) is an efficient way to calculate disperse Fourier transform (DFT). FFT is one of the most basic methods in the analysis of the time-domain one-frequency domain transformation, where DFT is the basic solution to many digital signals. The one key point for FFT is the requirement is the number of bits. For DFT, the number of bits, which is defined as N can be any positive integer. However, FFT requires N to be 2 to the power of k, where k = 1, 2, 3…, n ($n \subset N_+$). With the development of the computer, more files need to be proceeded. The FFT algorithm can be divided into a time-based extraction algorithm and a frequency-based extraction algorithm, which first briefly introduce the basic principles of FFT. Starting with the DFT operation, the mathematical description of FFT can be given as:

$$X(k) = DFT(x) = \sum_{n=0}^{N-1} x(n) W_N^{kn}, k = 0,1, \dots, N-1 \quad (8)$$

$$X(n) = IDFT(X) = \sum_{k=0}^{N-1} X(k) W_N^{-kn}, n = 0,1, \dots, N-1 \quad (9)$$

Regarding to the Quantum Fourier Transform (QFT), in domains ranging from signal processing to data compression to complexity theory, the Fourier transform appears in many distinct variations. The quantum Fourier transform (QFT) is a quantum version of the discrete Fourier transform applied to a wavefunction's amplitudes. It's used in a variety of quantum techniques, including Shor's factoring and quantum phase estimation [7]. The discrete Fourier transform takes a vector ($x_0$, ..., $x_{N1}$) and transfers it to a vector ($y_0$, ..., $y_{N1}$) according to the formular:

$$y_k = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} x_j \, \omega_N^{jk} \quad (10)$$

where $\omega_N^{jk} = e^{2\pi i \frac{jk}{N}}$.

Similarly, the quantum Fourier transform acts on a quantum state $|X\rangle = \sum_{j=0}^{N-1} x_j |j\rangle$ and maps it to the quantum state $|Y\rangle = \sum_{k=0}^{N-1} y_k |k\rangle$ according to Eq. (10). Since the state's amplitudes were affected by this transformation, this can also be described as the map:

$$|j\rangle \mapsto \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \omega_N^{jk} |k\rangle \quad (11)$$

or expressed as the unitary matrix:

$$U_{QFT} = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} \sum_{k=0}^{N-1} \omega_N^{jk} |k\rangle\langle j| \quad (12)$$

## 3. Methodology

In this research, to compare the differences between the quantum computer and conventional computer in the ability of computing, fast Fourier transformation (FFT) and quantum Fourier transformation (QFT) were applied to study the time complexity in different numbers of bits and qubits. From this specific parameter, we are able to tell the advantage of a quantum computer in completing Fourier transformation and understand the difference in computing mechanism. In this research, QFT was studied based on the Qiskit, which is an open-source software development kit (SDK) for dealing with quantum computers at the pulse, circuit, and application module level.

1) The logical circuit of FFT and QFT were constructed.
2) The code of the logical circuit of FFT was built based on the code provided by the website GeeksforGeeks.
3) The code on the official website of Qiskit was applied to export the graph of the QFT logical circuit.
4) The relationship between the time complexity and the numbers of bits or qubits applied in two operations was calculated from the derivation of formulas respectively.
5) The N, which stands for the numbers of bits or qubits was started from 4 to 200 with a spacing of 2. The corresponding time complexity of FFT and QFT was then calculated based on the relationship derived in step 4). All data were collected and recorded in a table.
6) A figure of Log (Time complexity) vs Numbers of bits/qubits was plot for further analysis.

## 4. Results and Discussion

### 4.1 Set up logical circuits

In the building of logical circuit of QFT, many standard qiskit libraries was imported from NumPy, Qiskit and Ibm_quantum_widgets [9]. The sketch of circuits FFT and QFT are illustrated in Figs. 2 and 3, respectively. To be specific, Fig. 2 provides a 8 bits logical circuit at the website GeeksforGeesks, Fig. 3 illustrates the self-built python code of QFT logical circuit and outputted from the qc.draw command.

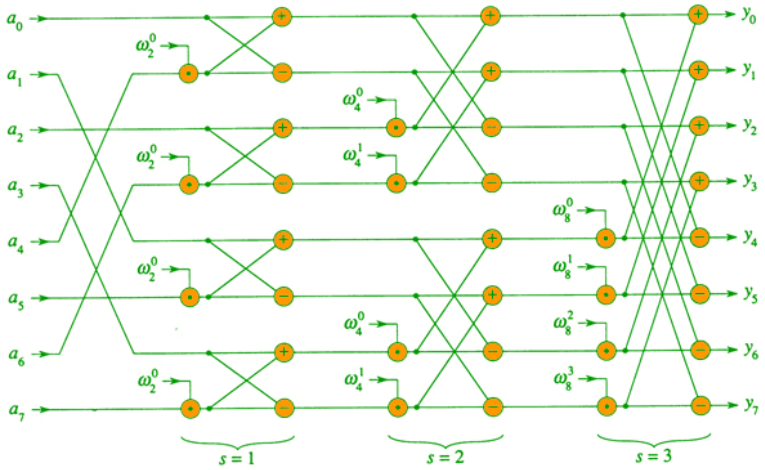



**Figure 2.** A sketch of logical circuit of FFT with N=8 from GeeksforGeeks.

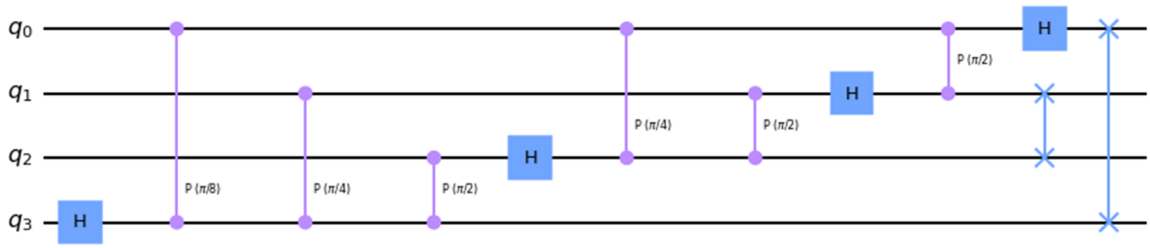



**Figure 3.** A sketch of the logical circuit of QFT with N=4 built based on the IBM Quantum

### 4.2 Time complexity of FFT & QFT

For FFT, by the Cooley-Tukey FFT algorithm, we can have FFT computing have the time complexity as O(NlogN), which is much better than the time complexity of DFT. The reason for such different as O(NlogN) compare with $O(N^2)$ is basically due to the reorganization of the first-step bits management [10]. FFT is also known as butterfly computation as it has a strict request function that $N = 2^M$ $(M = 0,1,2,3,\dots)$.

The FFT algorithm separate one N bits of FFT into two N/2 bits of FFTs, which is the basic mechanism of FFT and that is the reason why FFT was also known as butterfly computation. Since the quantum calculation of quantum Fourier transform was simulated on classical computer, it is impossible to get a completely accurate value of time complexity on the circuit. For first qubit, it will go through one H-Gate and N-1 controlled phase shift gates (written R-Gate) as in:

$$R_m = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\Phi} \end{bmatrix} \tag{13}$$

For some phase shift Φ. For second qubit, also go through one H-Gate but N-2 R-Gates. By parity of reasoning, the overall gates in calculation part would be $N(N\text{-}1)/2$ gates. Along with the SWAP calculation, the final overall gates would be $O(N^2)$ gates [10, 11]. The complexity for QFT is:

$$O(N^2) = O(\log_2^2 N) \tag{14}$$

### 4.3 Result of comparison between FFT and QFT

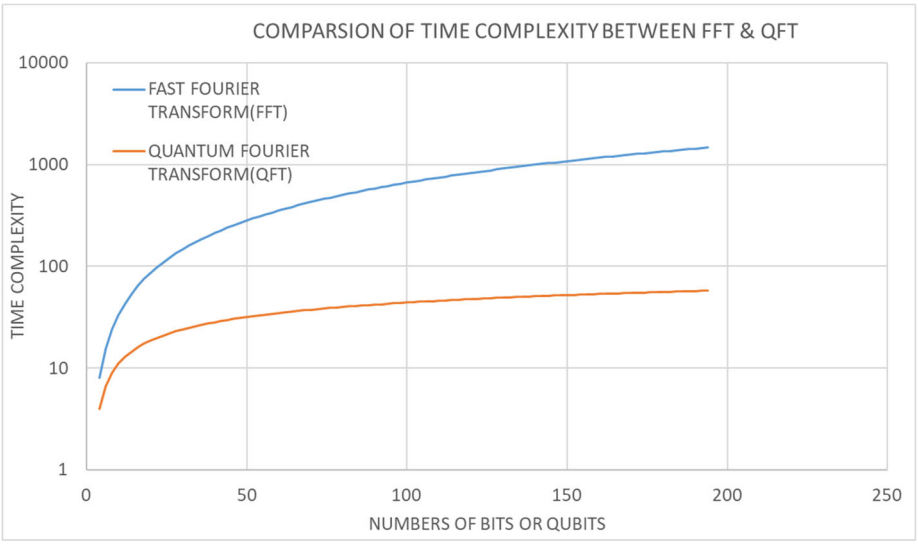



**Figure 4.** Log of time complexity as a function of the numbers of bits/qubits applied.

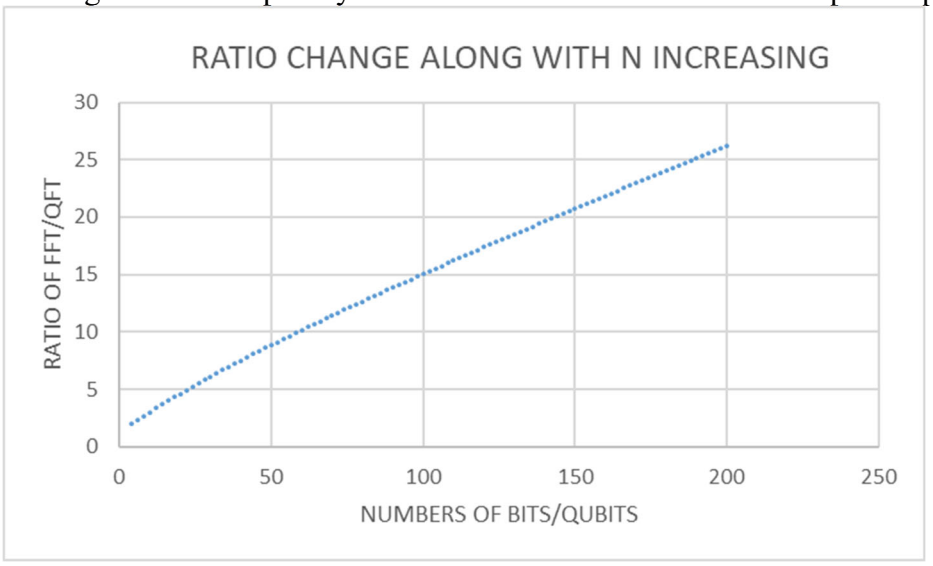



**Figure 5.** Ratio of time complexities between FFT and QFT as a function of quantity of bits/qubits

Fig 4 presented the change of both FFT and QFT along with increasing numbers of bits and qubits applied to each algorithm. In the diagram, the y-axis was taken log to its value as the change in QFT time complexity was much smoother and steadier than FFT. In other word, the gradient was too small for QFT trend to observe. According to the results, it is easy to find that the distance between two trends is becoming wider as the N increases, i.e., the time complexity difference became larger along x-axis. On this basis, as the task getting harder (increasing N), quantum computer is theoretically producing a better computational ability than conventional computer. Nevertheless, the quantum

computer is more like a theory proved prototype nowadays, the current technology ability would only build gate-level accurate quantum computer. In the case of this paper, the time complexity would be $O(N\log^2 N)$ [9]. It gives the current quantum computational ability on Fourier transform is no better than conventional computer, but represents a bright future of the quantum computer.

## 5. Conclusions

In summary, this paper investigates the computational ability difference between quantum and classical computers. Specifically, the ideal quantum computing is way much faster than classical ones. Based on the analysis, the Fourier transform implies the conclusion and also presents the boundedness due to the technology disability. The quantum circuit used in this paper was just a simulation of the algorithm that cannot be used to calculate the time complexity. To be more specific, it gives two different results in QFT, where one indicates the current technology ability in constructing quantum calculation while the other presents how fast the expected quantum computer would react and the great future in the upcoming new era. In the future, quantum technology can be applied to many areas (e.g., quantum annealing, quantum cloud). In this case, all kinds of nonlinear calculations are suitable for quantum computation, which gives quantum technology a promising future both in daily life and advance scientific development. Overall, these results offer a guideline for the basic understanding of quantum computation and the way to experience quantum calculation on ourselves.

## References

[1] Introduction to quantum computing-GeeksforGeeks, 2022. Information on https://www.geeksforgeeks.org/introduction-quantum-computing.

[2] D-Wave Systems | The Practical Quantum Computing Company, 2022. Information on https://www.dwavesys.com

[3] IBM Quantum, 2022. Information on https://quantum-computing.ibm.com

[4] Voorhoede, D, What is a qubit?. Information on https://www.quantum-inspire.com/kbase/what-is-a-qubit

[5] What It Means When Two Particles Are Entangled, 2022. Information on https://www.thoughtco.com/what-is-quantum-entanglement-2699355

[6] Omer, B. Quantum programming in QCL. Master's thesis, Institute of Information Systems Technical University of Vienn na, 2000.

[7] Nielsen, M. A., Chuang, I. L. Quantum computation and quantum information. Phys. Today, 54(2), 2001, 60.

[8] Williams, C. P., Clearwater, S. H. Explorations in quantum computing. Santa Clara: Telos, 1998.

[9] Aleksandrowicz, G., Alexander, T., Barkoutsos, P., Bello, L., Ben-Haim, Y., Bucher, D., et al. Qiskit: An open-source framework for quantum computing. Accessed on: Mar, 16, 2019.

[10] Musk, D. R. A comparison of quantum and traditional Fourier transform computations. Computing in Science & Engineering, 22(6), 2020, 103-110.

[11] Houhou, O., Aissaoui, H., Bougroura, H. Time complexity and gate complexity of the quantum Fourier transform. In AIP Conference Proceedings, Vol. 1444, No. 1, 2021, pp. 465-468. American Institute of Physics.