

Riding Scheme Based on Fraction-weight Model

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Abstract. The difference of 0.01s is enough to decide the winner of a race. To push oneself to the limit through training is the meaning of sports, and to seize every second by helping athletes polish details and optimize tactics is the meaning of modern sports technology. This is evident in the Individual Time Trial (ITT), a race that focuses on tactics and details. In order to help riders achieve better results, it have created optimization methods for each type of rider and course, taking into account energy reserves (stamina), rider power and course conditions. So, several models are developed in this paper: scoring weighted regression model and energy-power model based on Skiba model.

Keywords: Skiba model; Energy-power model; Critical power.

1. Introduction

With the development of computer simulation and computing technology, mathematical optimization is becoming more and more integrated with sports. The use of modeling and quantitative analysis to optimize athletes' movements and race strategies has become an indispensable training process today.

Individual road cycling time trials stipulate that each rider should individually follow the same route, with the shorter time winning. Each category of cyclist has its own unique power curve, and both brief overpowering and fatigue can affect actual performance, which can be combined with course data to optimize the power scheme during the ride.

Several models are developed in this paper: scoring weighted regression model; energy-power model based on Skiba model [1].

In order to find sufficient data, recorded the data of many riders from Strava, a professional cycling website. However, most of the real people are not exactly Time Trial Specialist (TTS) or puncheur riders, but more of a neutralization of both. To characterize both types of riders with their data, used a weighted linear regression.

Modeled the optimal riding strategy in terms of both energy and power. First, by introducing the Skiba model and the critical power derived from the power curve, we modeled the energy of the rider. It also specified the mechanism of dividing the course into different sections. By studying the power output data of outstanding riders in different sections, we obtained the power output strategy of riders. Combining the above three, we obtained the energy-power model of the rider and optimized the optimal power output strategy of the rider, i.e., except for the initial acceleration phase, the rider should ride in a power mode that does not exceed his or her power curve as much as possible.

2. Model Assumptions and Notation

2.1. Assumptions

- Assume that the race will take at most one hour.
- Assume that the rider's energy output per unit time is limited and less than his own total energy.

2.2. Notations

Important notations used in this paper are listed in Table 1.

Table 1. Notations

SYMBOL	MEANING
W'_{bal}	the balance of W'
τ_W	the time constant for reconstituting W'
v	the velocity of the rider
C_r	the resistance coefficient
s	upward slope
η	drivetrain efficiency
a	acceleration
c_W	coefficient of wind resistance
m	total mass of the vehicle with driver

3. Model construction and solving

3.1. Define the Power Profiles

3.1.1 Scoring Weighted Regression Model

Pedal power, the power output of a rider that we can measure with a power meter, is an important output indicator in cycling because it reflects pedal speed, an important factor affecting cycling power [2].

A rider's power curve represents the amount of time a rider can maintain a given amount of power. Individual riders can produce different levels of power for different lengths of time, and the amount of power a rider can produce for a given amount of power and the length of time for a given amount of power varies greatly between riders. Generally speaking, TTS is better at long periods of low power output and Puncher is better at short periods of high power output power. Due to the differences between people, even among the same type of riders, the power curve of each person is more or less different. In order to calculate the power distribution applicable to most people for the benefit of the group, we built the optimal power curve model by selecting 20 male and female riders on Strava (average weight 65kg for men and 60kg for women, average height male 175cm, female 170cm) to obtain their cycling event data. We set up a scoring mechanism for the top five events in which each of them placed best, and the top 20 scorers were categorized as TTS and the other 20 were categorized as Puncher. In the top five events with the best ranking is shown in Table 2.

Table 2. In the Top Five Events with the Best Ranking

[1] ITT Tournament Percentage	[2] Score
[3] 20%	[4] 1
[5] 40%	[6] 2
[7] 60%	[8] 3
[9] 80%	[10] 4
[11] 100%	[12] 5

The influence factor of the riding data is then calculated based on the rider's category and performance δ , and the higher the performance in the corresponding category the higher the δ .

$$\delta \sim s_c + d \quad (1)$$

The most generalized power curve was obtained by multiplying the influence factor δ with the player's power curve by a linear regression equation.

$$X_{in} = \delta X_{in}' \quad (2)$$

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \dots + \beta_p X_{ip} + \varepsilon_i \quad (3)$$

$$\hat{\beta} = (X^T X)^{-1} X^T y \quad (4)$$

3.1.2 Results

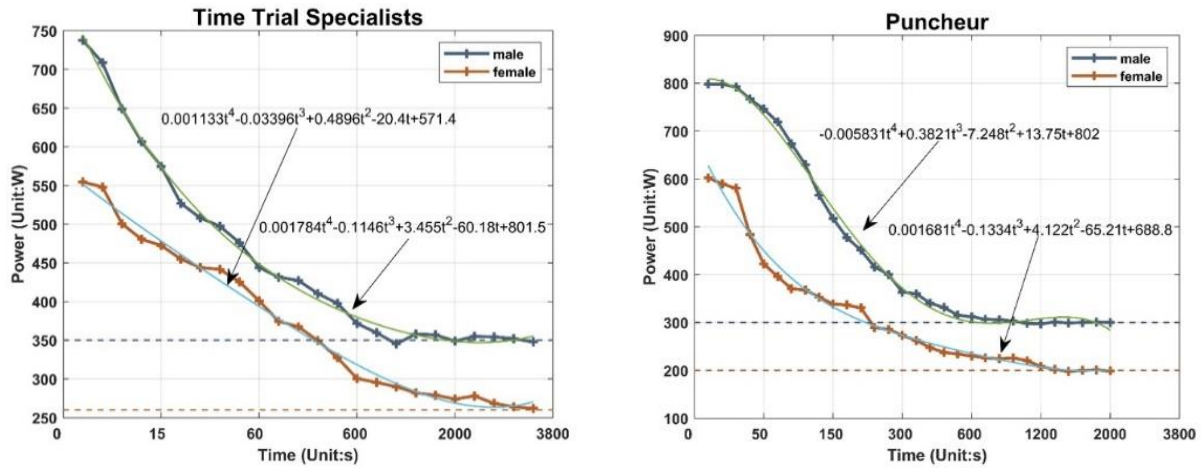


Fig 1. the power profiles of the two types of riders

Fig. 1 shows the power profiles of the two types of riders.

4. Power Application Strategies in Competition

4.1. Energy-Power Model

4.1.1 Critical Power and Skiba Model

Critical power (CP) [3], the maximum power that an individual can sustain for prolonged exercise without failure, is the power output of the body at the lactate threshold, and the work that can be done above it is W' (in kJ) [4]. W' can vary widely. W' for endurance athletes is usually around 9-15kJ for men and 6-10kJ for women [5]. The relationship between it and CP is as follows:

$$P = \left(\frac{W'}{t} \right) + CP \quad (5)$$

From the formula can be obtained, as in Fig. 2, in the power curve (PC), the time tends to infinity, the power value to the line of power that is the critical power.

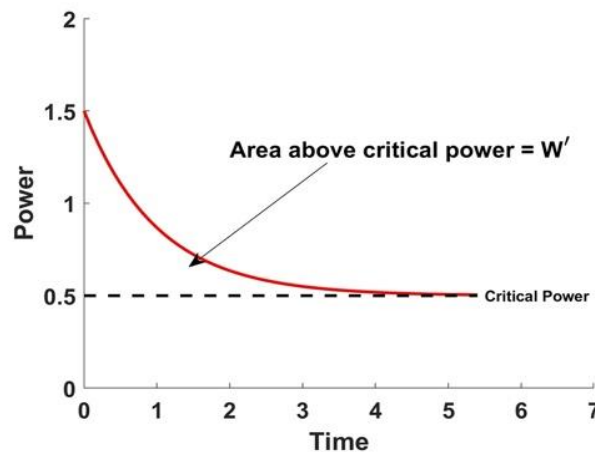


Fig 2. Power Curve

Furthermore, after a study by Alexander D. Springer [6], an equation can be formulated describing the balance of $W'(W'_{bal})$ where some amount of W' was expended (W'_{exp}):

$$W'_{bat} = W' - \int_0^t W'_{exp}(t) e^{-\frac{(t-u)}{\tau'_w}} dt \quad (6)$$

$$\tau'_w = 546e^{-0.01D_{cp}} + 316 \quad (7)$$

To put it simply, the energy in the human body is like a wine bottle with a built-in throttle ball. The moment the bottle is turned upside down, the energy flows out rapidly, but the process does not last long - the throttle ball drops to the top of the bottle and begins to reduce the flow rate of energy. Slowly, the flow rate of energy decreases due to the presence of the throttle ball and tends to a stable constant value, and this stable flow rate is what we call critical power.

At the same time, because of the ‘teleoanticipation’ mechanism [7], athletes will still retain a certain amount of physical strength after the competition to participate in subsequent activities or events.

4.1.2 Analysis of the Riding Habits of Outstanding Riders

Through our extensive research on the behavior of outstanding riders, we found that riders tend to be higher than output on uphill, less than output on downhill, fluctuate output above and below CP on flat roads, and stop output on curves. We use this as the base riding strategy in our model.

To facilitate the calculation, simplified the riding process into the following four states.

When riding at a constant speed on a flat road, the resistance is [8]

$$F_{totalflt} = (F_{wind} + F_{roll}) \div \eta \quad (8)$$

When riding uphill at a constant speed, the resistance is

$$F_{totalup} = (F_{wind} + F_{roll} + F_{slope}) \div \eta \quad (9)$$

When accelerating on a flat road, the resistance is

$$F_{total} = (F_{wind} + F_{roll} + F_{accel}) \div \eta \quad (10)$$

When riding downhill at a constant speed, the resistance is

$$F_{totalup} = (F_{wind} + F_{roll} - F_{slope}) \div \eta \quad (11)$$

Where

$$F_{wind} = \frac{\rho c_w A v_{relat}^2}{2} \quad (12)$$

$$F_{roll} = c_r mg \quad (13)$$

$$F_{slope} = smg \quad (14)$$

$$F_{accel} = ma \quad (15)$$

At any moment, the power is calculated as

$$P = F_{total} \times v \quad (16)$$

From this, the problem is transformed into the ODE problem, hence we can use the following formula [9].

$$\max \int_0^{T_j} v(t) dt \quad (17)$$

$$\begin{cases} \frac{dx}{dt} = v(t) \\ \frac{dv}{dt} = \frac{u(t)}{c_3 v(t)} - \frac{c_1 v(t)^2}{c_3} - \frac{c_2}{c_3} \\ \frac{da}{dt} = u(t) - CP \end{cases} \quad (18)$$

$$st.x(0) = 0, v(0) > 0, a(0) = 0, a(T_f) = W. \quad (19)$$

$$H(x, u, l) = v(t) + l_1(t)v(t) + l_2(t) + \left[\frac{u(t)}{c_3 v(t)} - \frac{c_1 v(t)^2}{c_3} - \frac{c_2}{c_3} \right] + l_3(t)(u(t) - CP) \quad (20)$$

4.1.3 Classification of Course Section Types

Segmented the courses into the for four types (flap road, uphill, downhill and curves) and matched the corresponding riding strategies. To simplify the model, assume that riders can only choose to ride at one type of power in each stage.

4.1.4 Results

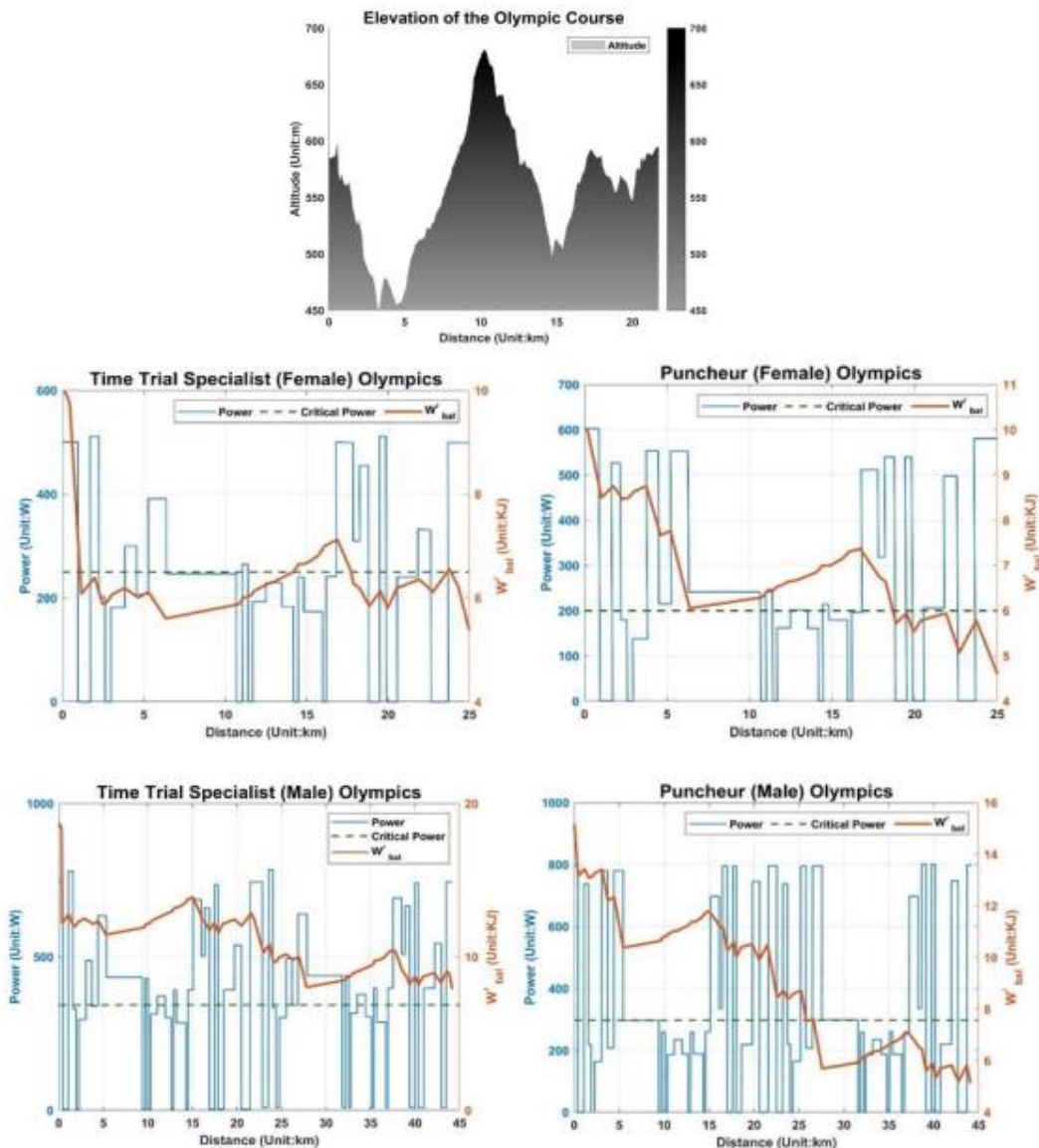


Fig 3. 2021 Olympic Time Trial course in Tokyo, Japan

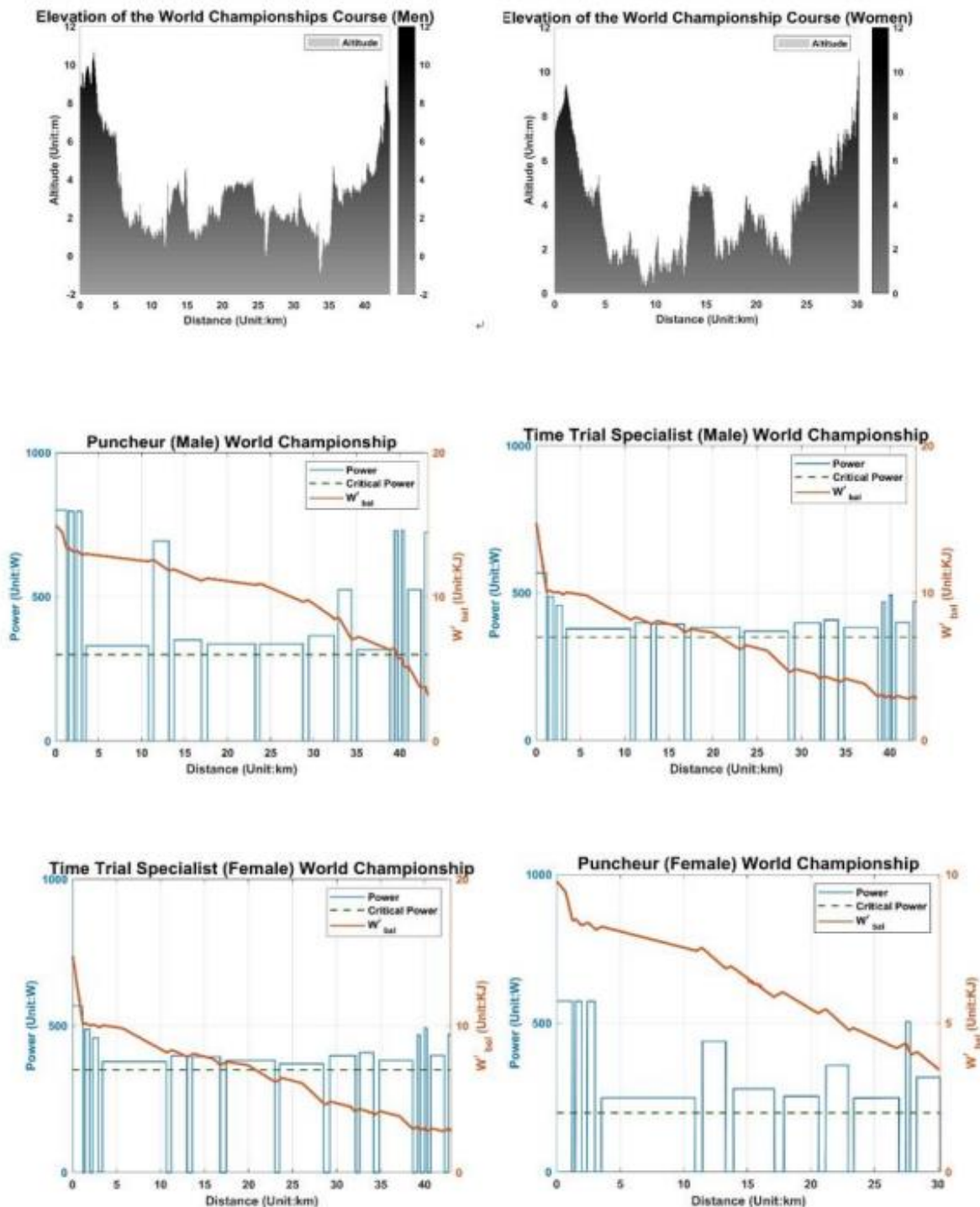


Fig 4. 2021 UCI World Championship time trial course in Flanders, Belgium

By calculating the above, obtain the power distribution of the rider. Using Google Maps, we obtained the elevation data of the corresponding courses elevation map was made. 2021 Olympic Time Trial course in Tokyo, Japan is shown in Fig. 3. 2021 UCI World Championship time trial course in Flanders, Belgium is shown in Fig. 4.

At the same time, designed our own course (the first 33.8km was a warm-up section), containing several corners, and a long uphill (special section): as shown in Fig. 5.

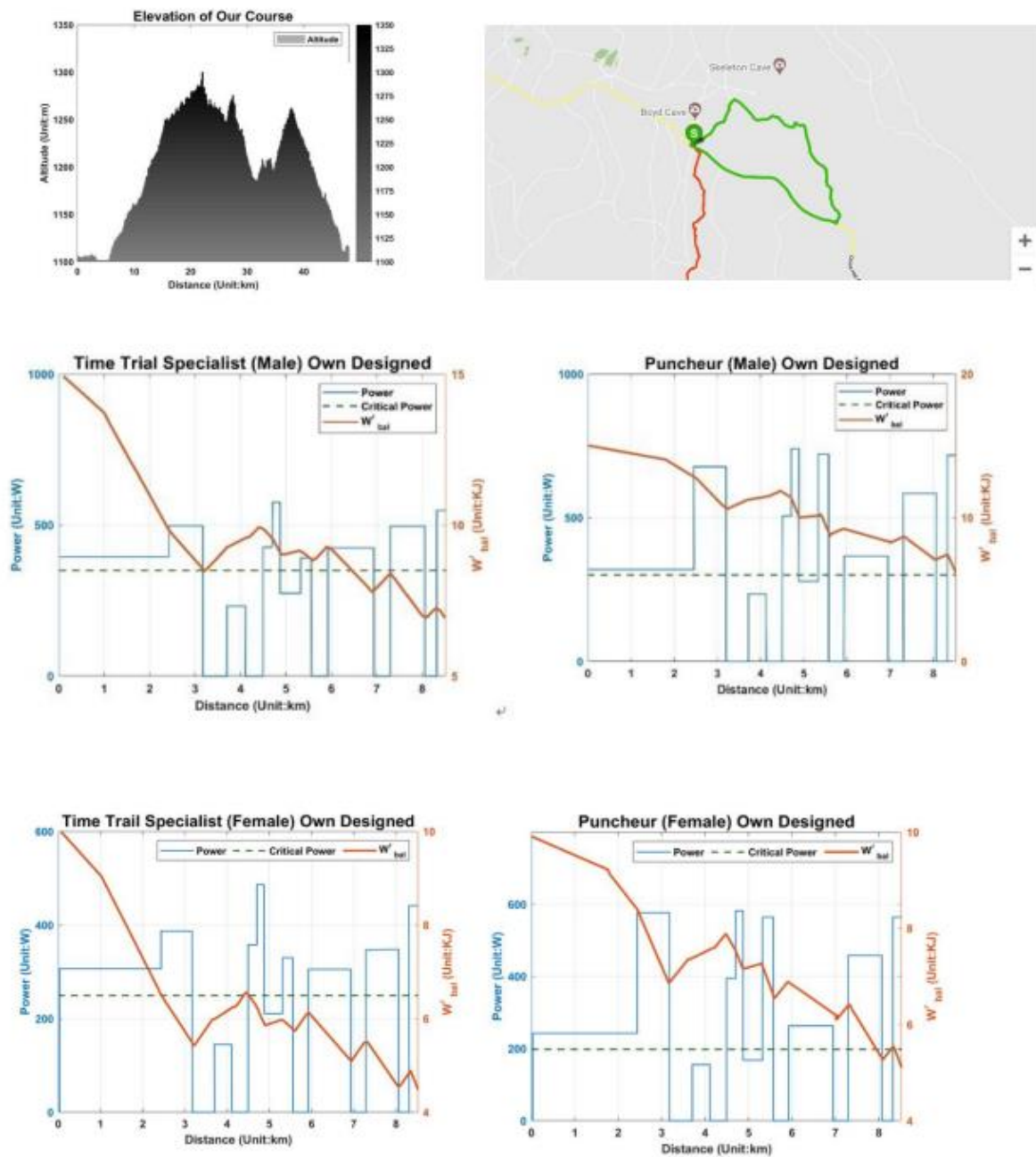


Fig 5. Our Course

The results obtained from the calculations are in general agreement with our observation:

1. Maintaining CP output on flat roads
2. Higher than CP output on uphill to maintain speed at critical speed
3. Less than CP output on downhill to maintain speed at critical speed
4. Stop output for a few seconds during sharp turns

4.2. Optimization of Formation Riding

4.2.1 Aerodynamic Analysis

ASSUMPTION: In a certain speed range, the power of the rider in the same position is reduced by the same percentage.

Wind speed does not affect the rider as much as climbing [10], but it does require careful consideration in long distance racing events. In the study [11] an aerodynamic analysis of uphill drafting in bicycle riding was carried out by WT measurements and CFD simulations. The drag reduction was calculated and a power model was used to calculate the potential energy savings.

Similar power reductions were found for all remaining cyclists on the same. Sensitivity analysis of the input variables showed that team riding saves energy for different bike, cyclist, road and environmental characteristics.

In the event of a crosswind, the formation of the team should be adjusted accordingly to maximize energy savings. Effects of crosswinds is shown in Fig. 6.

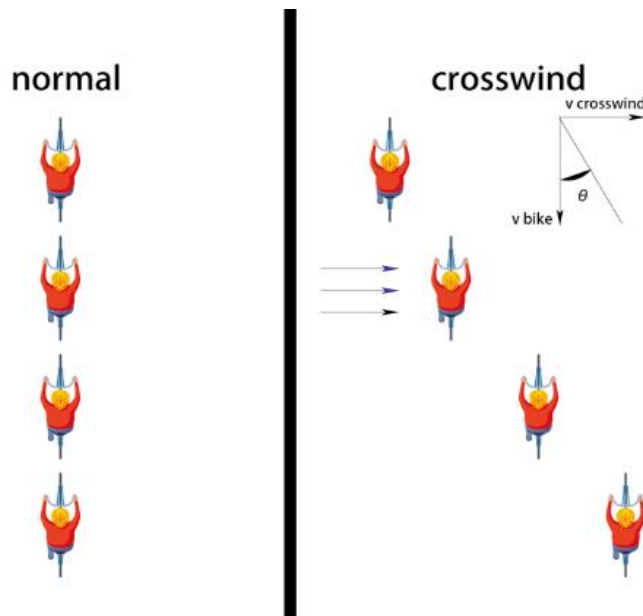


Fig 6. Effects of crosswinds

According to the model calculation, we let the distance between riders keep around 0.98 feet, the second cyclist's power at CP value, the first cyclist 's power maintained at CP/0.52, and the rotation time depends on the power curve, the first cyclist can maintain the time at CP/0.52 power, in such a case, the rider can obtain an average speed of 1.24 times compared to when alone.

When the leader's energy consumption reaches the limit time, he stops pedaling, decelerates by air resistance, and enters the rear of the group when the defender overtakes. When the rider is in the leader's position, he consumes energy quickly, and recovers it slowly in the third, fourth, fifth and sixth positions, with the same time for each stage. Schematic Diagram of Formation Riding is shown in Fig. 7.

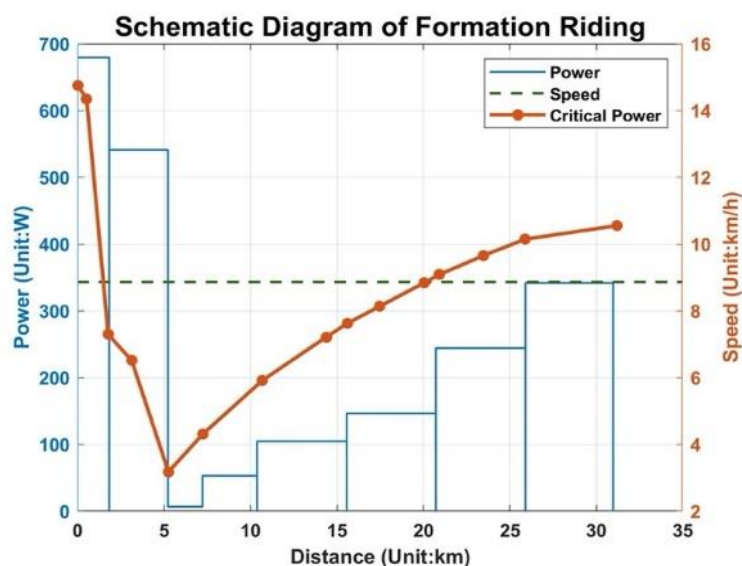


Fig 7. Schematic Diagram of Formation Riding

4.3. Sensitivity Analysis

4.3.1 Sensitivity Analysis to Wind and Environmental Factors

4.3.1.1 Wind Direction and Wind Speed

When building Model 2, wind resistance is the most important drag in our model, so sensitivity analysis of wind speed and wind direction is crucial. We set the initial values of wind for the Olympic track (single lap) and the World Championships track to 20km/h and the initial value of wind direction to 90 degrees. Controlling one of these quantities constant and the other varying from -10% to 10%, the corresponding race times are shown in the Fig. 8 below.



Fig 8. the corresponding race times

Considering also that the Olympic course is in mountainous terrain and the World Championships track is in plain terrain, for from the figure it can conclude that our wind sensitivity is weaker and the wind direction is more sensitive in plain areas and less sensitive in mountainous areas. From this suggest that riders riding in the plains pay attention to the wind direction of the course and prepare in advance.

Suggest that riders riding in formation pay attention to adjusting the formation with the wind direction to reduce wind resistance as much as possible.

4.3.1.2 Environmental Factor

Humidity is the one of main environmental factors to be considered in sports [12]. still use the example of the expert (male), so that the relative humidity changes from -10% to 10%

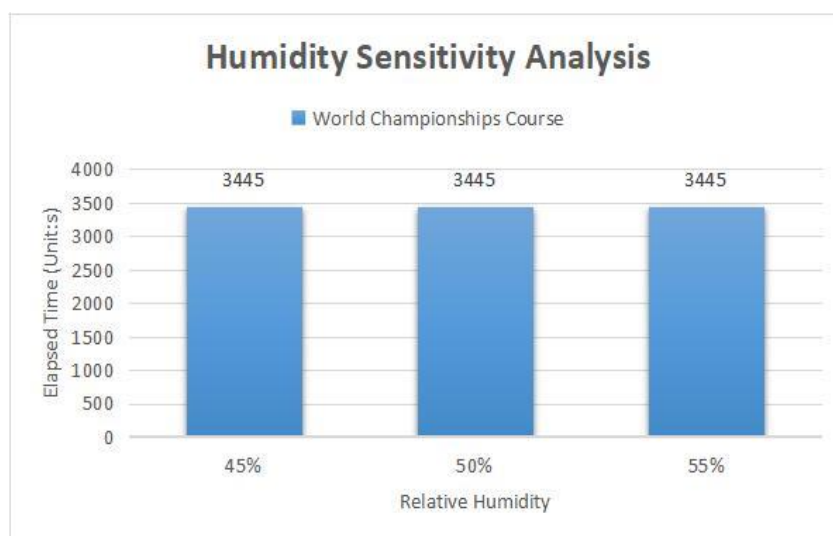


Fig 9. Humidity Sensitivity Analysis

From the Fig. 9, it can be seen that the sensitivity of relative humidity is weak.

4.3.2 Sensitivity Analysis to Rider Deviations

Competition conditions are often rapidly changing and coaches and riders often have their own ideas, making it difficult to strictly follow our optimal strategy, so we need to test the sensitivity of the deviation scenario. Take Specialist (male) in Olympic course (single lap) as an example, the corresponding race time is shown in the Fig. 10.

Where a indicates a 0-10% offset in power on sharp turning sections, b indicates that the optimal strategy is strictly followed, and c indicates a 0-10% offset on short uphill sections.



Fig 10. Rider Deviation

From the figure, the sensitivity of the rider's deviation is related to the power of the road section where it is located, specifically:

- 1.The smaller the difference between this power and the critical power, the lower the sensitivity.
- 2.The sensitivity of the power greater than the critical power is higher than that of the power less than the critical power.
- 3.The more the actual output time of the power is greater than the time it can be maintained in the power curve, the higher the sensitivity.

As a result, recommend that riders practice as much as possible to try to ride with the best strategy during the race and to correct any deviations immediately.

4.3.3 Strengths and Weakness

Innovative use of the Skiba model and the energy tank concept for better adaptation to real-world conditions. Build an energy dynamics model to facilitate the solution of the best strategy so that cyclists can get to the finish line the fastest.

There are some idealistic assumptions that make the results not match with the reality. The indicators estimated using the Skiba model have some subjective influence and a sensitivity analysis is required to calculate the reliability of the model.

5. Conclusion

In this paper, develop riding programs on different trails based on the riding data of the best riders. In Strava software, riding data of multiple riders are collected based on image recognition and course matching, etc. An evaluation model was created. The data will be scored, weighted, and analyzed according to the importance of their riding sections, and then the best ride plan for each section will be determined. Innovative use of the Skiba model and the energy tank concept for better adaptation to real-world conditions. Build an energy dynamics model to facilitate the solution of the best strategy so that cyclists can get to the finish line the fastest. There are some idealistic assumptions that make the results not match with the reality.

References

- [1] Skiba P F, Clarke D C. The W' Balance Model: Mathematical and Methodological Considerations[J]. International Journal of Sports Physiology and Performance, 2021, 16(11): 1561-1572.
- [2] Martin J C, Davidson C J, Pardyjak E R. Understanding sprint-cycling performance: the integration of muscle power, resistance, and modeling[J]. International journal of sports physiology and performance, 2007, 2(1): 5-21.
- [3] Vanhatalo A, Jones A M, Burnley M. Application of critical power in sport[J]. International journal of sports physiology and performance, 2011, 6(1): 128-136.
- [4] Poole D C, Burnley M, Vanhatalo A, et al. Critical power: an important fatigue threshold in exercise physiology[J]. Medicine and science in sports and exercise, 2016, 48(11): 2320.
- [5] Bailey S J, Vanhatalo A, Dimenna F J, et al. Fast-start strategy improves VO₂ kinetics and high-intensity exercise performance[J]. Medicine and science in sports and exercise, 2011, 43(3): 457-467.
- [6] Springer A D. Optimizing cycling power[D]. Massachusetts Institute of Technology, 2016.
- [7] Faria E W, Parker D L, Faria I E. The science of cycling[J]. Sports medicine, 2005, 35(4): 285-312.
- [8] Rainer Pivit. Drag Force Formulas [M]. German: Radfahren, 1990: 44-46
- [9] De Jong J, Fokkink R, Olsder G J, et al. The individual time trial as an optimal control problem[J]. Proceedings of the Institution of Mechanical Engineers, Part P: Journal of Sports Engineering and Technology, 2017, 231(3): 200-206.
- [10] Gordon S. Optimising distribution of power during a cycling time trial[J]. Sports Engineering, 2005, 8(2): 81-90.
- [11] van Druenen T, Blocken B. Aerodynamic analysis of uphill drafting in cycling[J]. Sports Engineering, 2021, 24(1): 1-11.
- [12] Peiser B, Reilly T. Environmental factors in the summer Olympics in historical perspective[J]. Journal of Sports Sciences, 2004, 22(10): 981-1002.