Plant Competition Model Based on Self-Inhibition and Drought

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Abstract. Continued global warming has led to frequent droughts in many parts of the world. Drought has a large impact on plant communities in ecosystems everywhere, and therefore the adaptation of plant communities to drought will be critical to their future development and survival. This paper focuses on two key questions: first, how plant community populations change under the influence of dry weather and how plants adapt to dry environments. Second, how the succession of dominant populations affects the biodiversity within that plant community and what the biomass is when the community state is stable. In this paper, a grassland community dynamics system model based on population numbers and environmental resources is established, while precipitation is selected as a key indicator to study the effects of dry weather. Finally, the differential equation for the change of plant population with dry weather is derived. Through analyzing the relationship between total biomass, critical available resources and number of species, the optimal number of species when the plant community is stable is obtained.

Keywords: Drought adaptation, Dynamical System, Monod Model, Population competition, Self-Inhibition.

1. Introduction

Drought [1], as one of the most destructive natural disasters in the world, is expected to change significantly in intensity and frequency in some regions in the context of global warming. In this paper, a temperate grassland in a certain area is used as the research object, focusing on the influence of climatic factors (drought) on the production of plant communities in the local ecosystem [2] and predicting the future development trend of plant communities in the area. In this process, the studies of Wang Shuai [3] and Wang Zilong et al [4] on the adaptation of plant communities to environmental changes have provided important assistance in this paper in terms of ideas. The application and research results of Huang et al [5] and Lansun Chen et al [6] on dynamical systems, and the studies of Kortessis Nicholas et al [7] and Barre et al [8] on intra- and interspecific plant competition have contributed to the research methods and modeling of this paper.

In summary, this paper combines the dynamical system and the classical resource competition model (Morneau model), and introduces the self-inhibiting competition mechanism at the same time to optimize the model and carry out a series of analysis and research based on it.

2. Fundamentals of grassland community dynamics system model

2.1. A one-dimensional model of the grassland community dynamics system

This paper develops an optimized Monod Model [9] by introducing a self-inhibition competition mechanism and adding the precipitation factor to discuss the relationship between the number of species over time under the influence of irregular weather (dry weather). The main factors affecting the number of species are shown in Figure 1 below.

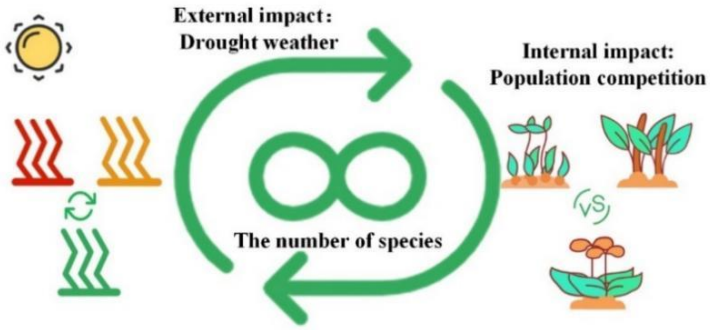


Figure 1. Factors affecting the number of species

First, the population density of individual species in a given community and the relationship between available resources and temporal changes are discussed.

$$\frac{dN_1}{dt} = G_1 - M_1 \tag{1}$$

N_1 denotes the population density of a single species, G_1 denotes the growth rate of the species, M_1 denotes the mortality rate of the species.

Meanwhile, the Monod equation is introduced to quantify the remaining resources of this community.

$$\mu(R) = \frac{\mu_{1max} R}{R + R_{1h}} \tag{2}$$

$\mu_{1max}$is the $\mu(R)$ upper limit when $R \to \infty$,R is the value of available resources, R_{1h} is the value of R when the growth rate up to $0.5\mu_{1max}$. Therefore, the relationship of the remaining resources with time can be obtained as follows.

$$\frac{dR}{dt} = S - LR - \mu(R)\mathrm{h}_1 \tag{3}$$

Where R is the value of available resource, L is the resource self-consumption rate, $R \bullet L$ is the resource consumption rate, $\mu(R)$ is the Monod equation.

The relationship between species size and growth rate is not considered in Monod Model. For this purpose, the variation factor $\delta(N)$ of the growth rate with N_1 and the variation factor $\mu(R)$ of the growth rate with the available resources value R are taken into account.

Furthermore, considering dry weather as the main exogenous cause of species population decline and ignoring the effects of other weather conditions, this article introduces the growth rate variation factor $\lambda(h)$ with precipitation to reflect the degree of drought by the amount of precipitation.

Precipitation and growth rate are positively correlated, the initial population size grows significantly with the growth of precipitation, with further increase in precipitation, the growth rate gradually decreases, and the capacity of plants to absorb and utilize water gradually tends to saturate to λ_{max}.

The optimized single species N_1 growth rates are as follows.

$$\begin{aligned} g_1(R, N_1, t) &= \emptyset_1 \times \delta(N_1) \times \mu(R) \times \lambda(h) \\ &= \emptyset_1 \times \frac{\delta_{1max} N_1}{N_1 + N_{1h}} \times \frac{\mu_{1max} R}{R + R_{1h}} \times \frac{\lambda_{1max} h}{h + h_{max}} \end{aligned} \tag{4}$$

It can be seen that the growth rate saturates with the increase in population density, which $\emptyset_1$ is the limiting value when $g_1(R, N_1, t)$ at the $R \to \infty, N_1 \to 0, t \to \infty$ time, i.e., the maximum environmental holding capacity, $\mu_{1max}$is the upper limit of $\mu(R)$ when the available resource $R \to \infty$, δ_{1max} is the upper limit of $\delta(N)$ when population density $N_1 \to 0$, $\lambda_{1max}$is the maximum amount of water absorbed by the species; N_{1h} is the growth rate up to $0.5\delta_{1max}$ the value of N_1,

R_{1h} is the growth rate up to $0.5\mu_{1max}$ the value of R, h_{max} is the maximum precipitation in the area, bringing equation (5) into (3) and (4), the following results are obtained.

$$\frac{dN_1}{dt} = N_1 g_1(R, N_1, h) - N_1 m_1 \tag{5}$$

$$\frac{dR}{dt} = S - LR - N_1 g_1(R, N_1, h) u_1 \tag{6}$$

Where N_1 denotes the population density of species1 in (4), $N_1 g(R, N_1, h)$ denotes the growth rate of the species1, m_1 denotes relative mortality, $N_1 m_1$ denotes the mortality rate of the species1, R denotes the available resources, h denotes the amount of precipitation at moment t;

Where S denotes the resource supply rate in (5), $N_1 g_1(R, N_1, t) u_1$ denotes the resource consumption of the species1, u_1 denotes the amount of resources consumed to reproduce a unit number of species.

Extending equations (4) and (5) to the multi-species case yields the following expressions.

$$g_i(R, N_i, h) = \emptyset_i \times \frac{\delta_{imax} N_{ih}}{N_i + N_{ih}} \times \frac{\mu_{imax} R}{R + R_{ih}} \times \frac{\lambda_{imax} h}{h + h_{max}} \tag{7}$$

$$\frac{dN_i}{dt} = N_i g_i(R, N_i, h) - N_i m_i \,, i = 1,2,3, \ldots., n \tag{8}$$

$$\frac{dR}{dt} = S - LR - \sum_{i=1}^{n} N_i g_i(R, N_i, h) u_i \,, i = 1,2,3, \ldots., n \tag{9}$$

2.2. Principles of plant community biomass at equilibrium

The biomass of the plant community in equilibrium [10]is constructed as follows.

$$B_{(n)} = aQ\left(S - \left(R^*_{min} + \frac{R^*_{max} - R^*_{min}}{n+1}\right)\right) \tag{10}$$

Where n is the number of randomly selected species in a community, $B_{(n)}$is the community biomass when a community reaches equilibrium, R^* is the critically available resource for each species, $R^*_{min}$is the minimum in R^*, R^*_{max} is the maximum in R^*, Q, a are factors. S is the balanced resource concentration.

A qualitative analysis of (10) leads to the conclusion that:

(1) Total plant biomass increases with species diversity;

(2) The total plant biomass is proportional to the maximum and minimum critical available resources R^*_{min} and R^*_{max}, and $\lim_{n\to+\infty} B(n) = S - R^*_{min}$, so the trend of $B_{(n)}$is to increase first and then stabilize.

3. Results

3.1. Modeling process

In this paper, the data based on the website www.nesdc.org.cn. In addition, this paper select the five species with the largest 3-year average number of plants in the five families and collect the local monthly average precipitation over a four-year period.

The maximum precipitation in the area $h_{max} = 68.7mm$, and for $\delta_{imax} = \mu_{imax} = 1, \emptyset_i = \delta_{imax}\mu_{imax}\lambda_{imax}N_{ih}$. R_{ih}, N_{ih} and m_i of N_i for each species when the maximum water uptake rate λ_{imax} and growth rate of each species reach $0.5\delta_{imax}$.

The initial value results are shown in the following Table 1.

Table 1. Initial values of each parameter for each species

Name	λ_{imax} (%)	N_{ih}	m_i	R_{ih}	Family
Achnatherum sibiricum(L.) Keng (Species 1)	15.43	16.95	1.20	53.71	Gramineae
Caragana microphylla Lam. (Species 2)	0.83	0.23	0.16	7.27	Leguminosae
A. frigida Willd. (Species 3)	5.90	1.33	1.45	5.44	Asteraceae
Kochia prostrate (L) Schrad. (Species 4)	5.83	0.24	1.75	0.02	Quinoa
A. ramosum L. (Species 5)	1.40	0.50	0.16	0.18	Liliaceae

In (6), the resource supply rate S is assigned the value 25,and the relative resource self-consumption rate L is assigned the value 0.1, the total resources consumed by all species $\sum_{i=1}^{n} N_i G_i(R, N_i, h) u_i = 9.85$, Substitute into (6) obtain :$\frac{dR}{dt} = 15.15 - 0.1R$, and R(0)=50, the expression of $R(t)$ is as follows.

$$R(t) = 151.5 - 10e^{-0.1t+2.3} \tag{11}$$

Obtaining $R(t) < 151.5$, Substituting (8) and other known conditions into (6) yields the following equation set:

$$\begin{cases} \frac{dN_1}{dt} = \frac{261.54N_1}{N_1 + 16.95} \times \frac{R}{R + 53.71} \times \frac{h}{h + 68.7} - 1.20N_1 \\ \frac{dN_1}{dt} = \frac{261.54N_1}{N_1 + 16.95} \times \frac{R}{R + 53.71} \times \frac{h}{h + 68.7} - 1.20N_1 \\ \frac{dN_3}{dt} = \frac{7.85N_3}{N_3 + 1.33} \times \frac{R}{R + 5.44} \times \frac{h}{h + 68.7} - 1.45N_3 \\ \frac{dN_4}{dt} = \frac{1.40N_4}{N_4 + 0.24} \times \frac{R}{R + 0.02} \times \frac{h}{h + 68.7} - 1.75N_4 \\ \frac{dN_5}{dt} = \frac{0.7N_5}{N_5 + 0.50} \times \frac{R}{R + 0.18} \times \frac{h}{h + 68.7} - 0.16N_5 \end{cases} \tag{12}$$

For $N = [N_1, N_2, N_3, N_4, N_5]$, $F(N, h, R) = \begin{bmatrix} \frac{261.54N_1}{N_1+16.95} \times \frac{R}{R+53.71} \times \frac{h}{h+68.7} - 1.20N_1 \\ \vdots \\ \frac{dN_5}{dt} = \frac{0.7N_5}{N_5+0.50} \times \frac{R}{R+0.18} \times \frac{h}{h+68.7} - 0.16N_5 \end{bmatrix}$, we know that:

$$\frac{dN}{dt} = F(N, h, R) \tag{13}$$

The existence of a constant solution to the above equation, i.e., the equilibrium point, is analyzed, in (13), for $\frac{dN}{dt} = F(N, h, R) = 0$, h = $\frac{h_{max}}{2}$.The left part of above equations is treated as a function $y(N_i)$ about N, the right part is treated as a straight line $L(R)$, the image is plotted as shown below Figure 2:

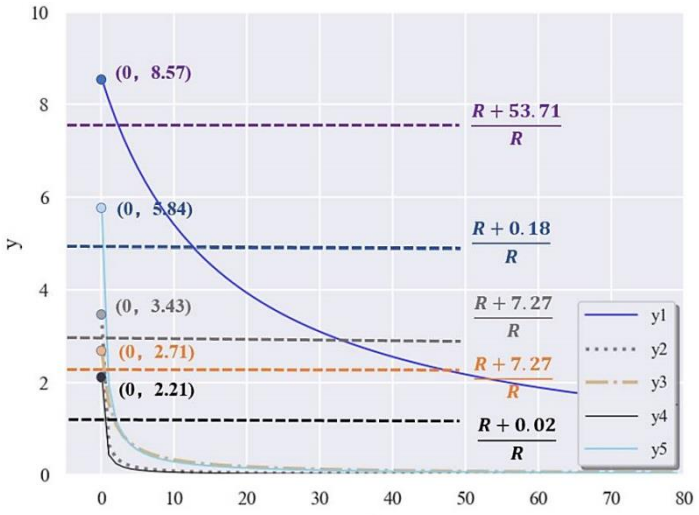


Figure 2. Graph of the function between $y(N_i)$ and $L(R)$

From the graph, when $L(R) \leq y_i(0)$, there is an intersection of $y(N_i)$ and $L(R)$, substitute into the equations:

$$\begin{cases} y_1(0) = \dfrac{R_1^* + 53.71}{R_1^*} = 8.57 \\ y_2(0) = \dfrac{R_2^* + 7.27}{R_2^*} = 3.43 \\ y_3(0) = \dfrac{R_3^* + 5.44}{R_3^*} = 2.71 \\ y_4(0) = \dfrac{R_4^* + 0.02}{R_4^*} = 2.21 \\ y_5(0) = \dfrac{R_5^* + 0.18}{R_5^*} = 5.84 \end{cases} \tag{14}$$

Where R_1^*, R_2^*, R_3^*, R_4^*, R_5^* are the available resources on the street respectively, thus solving for:

$$R_i^* = [R_1^*, R_2^*, R_3^*, R_4^*, R_5^*] = [7.10, 2.99, 3.18, 0.02, 0.04] \tag{15}$$

In case of weather conditions with no changes, the critical available resource of various Species R_i^* are constant, when $R_i \geq R_i^*$, the population density of equilibrium species $N_i \neq 0$ has a solution, there is a stable point.

Combine (13) and (9) to obtain the equation set, and for $\varphi(R) = S - LR - \sum_{i=1}^{n} N_i g_i(R, N_i, h) u_i$, we could know that:

Then the Jacobi matrix for this equation set can be listed as: $J = \begin{bmatrix} \frac{\partial F}{\partial R} & \frac{\partial F}{\partial h} \\ \frac{\partial \varphi}{\partial R} & \frac{\partial \varphi}{\partial h} \end{bmatrix}$, so it can be solved for its characteristic root, the Imaginary part of characteristic root is negative for Species1, Species3, Species4, the Real part of characteristic root is positive for Species2, Species 5. By the nature of the equilibrium point it is known that: only Species1, Species3 and Species 4 have a stable equilibrium. The $\frac{dN}{dt} - N$ graph for the five species is drawn as follows Figure 3.

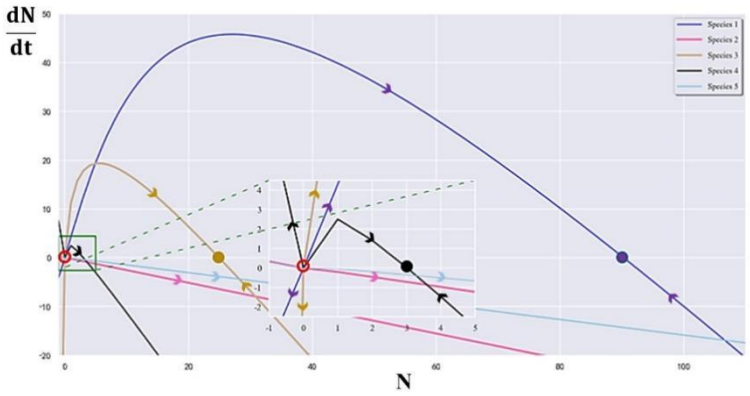


Figure 3. $\frac{dN}{dt} - N$ graph

From the above figure, it can be seen that Species 1, Species3 and Species4 all have two intersections with the x-axis, i.e. there are two equilibrium points, thus proving the above analysis. And since $F'(N,h,R)|_{N=0} > 0$, So N=0 is an unstable equilibrium point for $F(N,h,R)$, N_1^*, N_3^*, N_4^* which is other intersection with the x-axis, is the equilibrium point when the population reaches stability.

3.2. Analysis of experimental results

Bring (8) into (11), and $h = \frac{h_{max}}{2}$, the initial number of each species is [567, 8, 11, 3, 15], we use the recursive algorithm to predict changes in the population of each species over the following years, and the number of N_i is plotted against time as follows Figure 4:

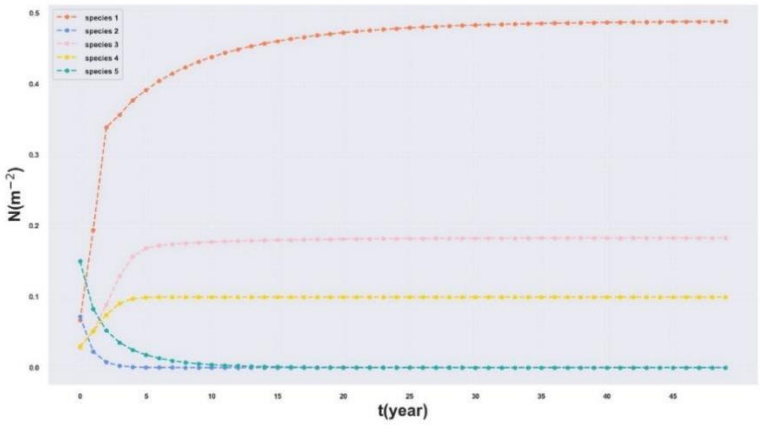


Figure 4. Variation curve of N_i with time

Through the above analysis, the following conclusions can be drawned.

- Species 1, Species 3 and Species 4 are increasing in number over time and eventually tend to stabilize. Species 2 and Species 5 are decreasing in number over time and will eventually die out, i.e., the number of species tends to 0.
- The number of Species 1 grows faster at first and then slows down, partly due to the limitation of individual size, and partly due to interspecific competition with Species 2, which means that Species 1 is stronger than Species 2 in the competition for resources to feed Species 2, probably because of its ability to compete for sunlight and water, which will eventually lead Species 2 to die out and Species 1 to reach its maximum capacity, i.e., the equilibrium point.
- It is highly likely that the eventual extinction of Species 5 is also due to interspecific competition. The principle of competitive exclusion in ecology suggests that if individual members of two populations consume about the same amount of resources, and the environment can sustain a larger maximum capacity for Species 1 than for Species 5, then Species 5 will eventually die out.

- Species 3 and Species 4 exhibit another, much rarer, situation between population competition, where Species 4 is less capable of competing for the resources required by Species 3 and Species 3 is less capable of competing for the resources of Species 4, so that both sides can eventually converge to an equilibrium to reach their respective maximum numbers.

To explore more directly the effect of dry weather, i.e., precipitation factor, on community development, the relationship between the relative growth rate of species and precipitation is further analyzed , and its results are shown in the figure below Figure 5.

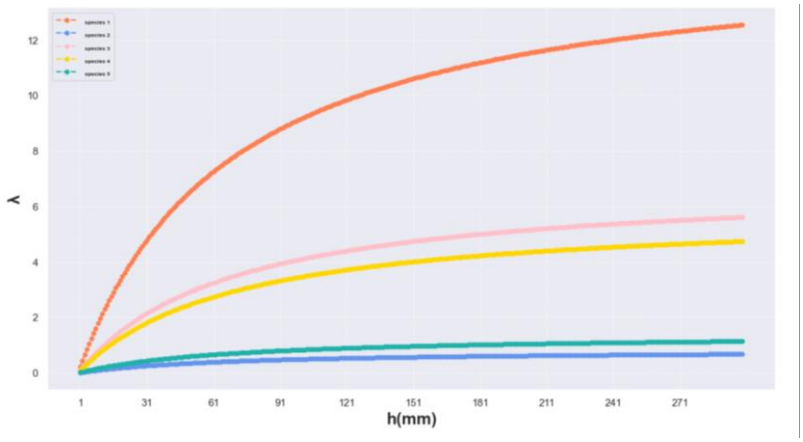


Figure 5. Relationship curves between species relative growth rate and precipitation

The relative growth rate of a species is positively correlated with precipitation, but precipitation does not always increase, and when precipitation reaches the λ_{max} of the species to absorb and utilize, the relative growth rate of the species does not increase but tends to stabilize.

Meanwhile, the λ_{max} values of different species can reflect the adaptability of different species to drought. The faster the relative growth rate stabilizes means that the species needs only a small amount of water to sustain itself, but its relative growth rate will be limited as a result. According to the above information, Species 1 changes more slowly but has the largest absorption and utilization capacity, i.e., λ_{imax}, resulting in the largest number of species among the five species. Species 2 and Species 3 reach their relative growth rates faster, but their λ_{max} values are smaller, resulting in their species numbers being relatively small among the five species. This conclusion is supported by the data on the number of plants of the five species that we collected earlier, i.e., the number of Species 1 > Species 3 >Species 4 > Species 2 ≈ Species 5.

From the previous inference it is clear that, for h = $\frac{h_{max}}{2}$, we can obtain:

$$\begin{cases} \dfrac{0.67N_{ih}\lambda_{ih}N_i}{N_i+N_{ih}} = \dfrac{R_i^*+R_{ih}}{R_i^*} \\ R_{min}^* = \min\{R_1^*, R_2^*, \ldots., R_n^*\} \\ R_{max}^* = \max\{R_1^*, R_2^*, \ldots., R_n^*\} \end{cases} \tag{16}$$

According to the survey data to obtain $B_{min} \approx 22$, it can be obtained:

$$B_{(n)} > B_{min} \Rightarrow n > \frac{R_{max}^* - 3}{3 - R_{min}^*} \tag{17}$$

The final conclusion can be drawn that, when n = $[\frac{R_{max}^*-3}{3-R_{min}^*}]$+1, The plant community may benefit.

In the following, we discuss the turnover of dominant species within the community in a categorical manner. Through the previous analyses, three species are selected, Species 1, Species 2, and Species 3, whose numbers did not decay for analysis.

- For $h = \frac{h_{max}}{2}$, i.e., without considering the effect of h with time, study the relationship between population density and R for three species. Then the equation set can be obtained as follows.

$$\begin{cases} \frac{dN_1}{dt} = \frac{174.36N_1}{N_1 + 16.95} \times \frac{R}{R + 53.71} - 1.20N_1 \\ \frac{dN_3}{dt} = \frac{5.23N_3}{N_3 + 1.33} \times \frac{R}{R + 5.44} - 1.45N_3 \\ \frac{dN_4}{dt} = \frac{0.93N_4}{N_4 + 0.24} \times \frac{R}{R + 0.02} - 1.75N_4 \end{cases} \tag{18}$$

The variation of population density N with R for the three species is plotted as shown below Figure 6.

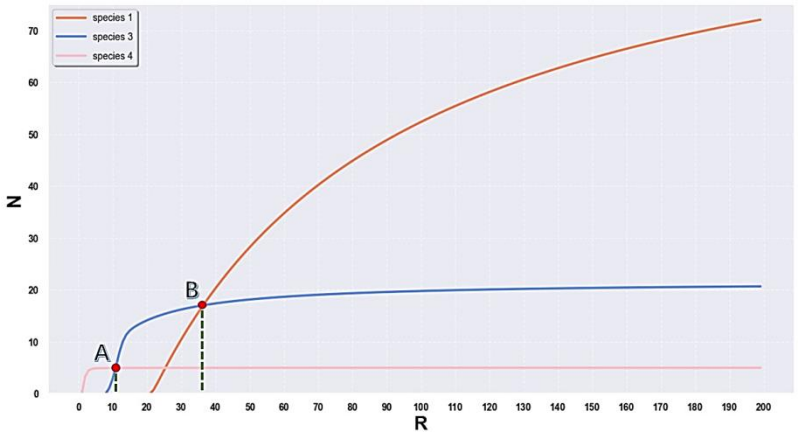


Figure 6. N-R graph

If the values of N_1, N_3, N_4 that eventually stabilize are $N_{1max}, N_{3max}, N_{4max}$, the following conclusions can be drawn from the analysis: the relationship between the three species satisfying $N_{4max} < N_{3max} < N_{1max}$ and $R_4^* < R_3^* < R_1^*$ results in the dominant species turnover in the figure above. Species 4 is the dominant species before point A, Species 3 is the dominant species between points A and B, and Species 1 is the dominant species after point B.

Similarly, the extension to n species still leads to similar conclusions: When the magnitude of $N_{max} = [N_{jmax}, N_{kmax}, \ldots\ldots, N_{zmax}]$ and $R^* = [R_j^*, R_k^*, \ldots\ldots, R_z^*]$ correspond to each other, species turnover occur, at this point in the community, the addition of a species or the deletion of a species did not affect the N-R curves of the extant and remaining species, i.e. the community can regulate itself so that it is not destroyed.

- Consider h as a variable and discuss the effect of h on R^*, in (13), since $\frac{dN_i}{dt} = 0$, for $K_i = 1 - \frac{N_{ih}\lambda_{ih}}{m_i(N_i + N_{ih})}$, we know that $R^* = \frac{R_{ih}(h + h_{max})}{K_i h + h_{max}}$, draw the $R^* - h$ graph as follows Figure 7:

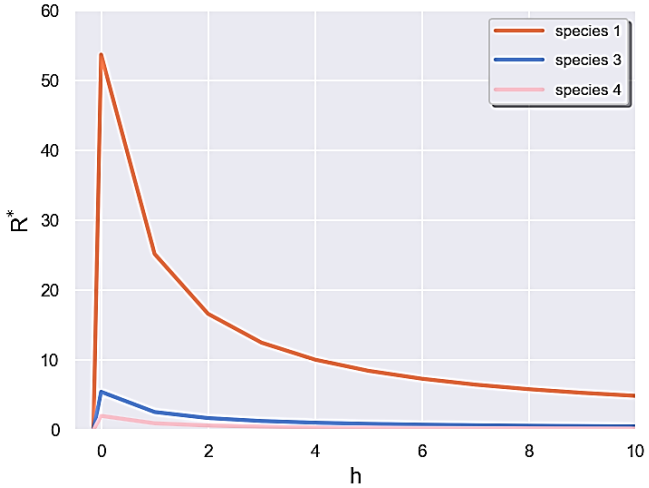


Figure 7. $R^* - h$ graph

It can be seen that the effect of weather influence h on R* is decreasing. Although the initial $R_4^* < R_3^* < R_1^*$, as the weather changes, there will appear $\triangle R_4^* < \triangle R_3^* < \triangle R_1^*$, the succession of the population will also change.

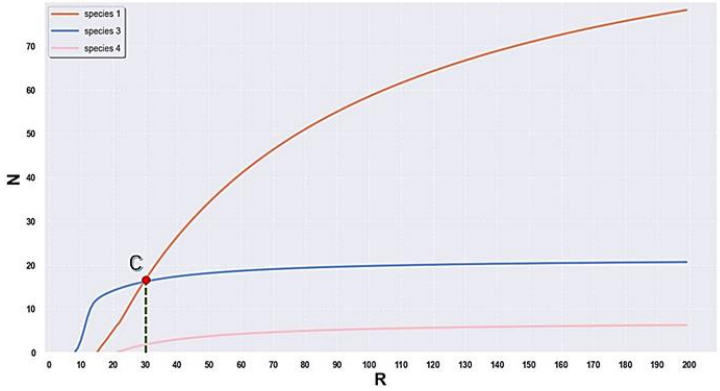


Figure 8. N- R* graph

From the figure 8, it can be seen that species 4 is the dominant species before point C and species 1 is the dominant species after point C. The successional structure of the dominant species becomes simpler, the diversity of species decreases, and the population size of species 3 cannot tend to stabilize or even tends to decay, at this time$B_{(n)}$ mentioned in the previous analysis will decline, which is detrimental to the development of the community.

However, if the influence of weather makes the stabilization value $N_{max}$to which each species tends to correspond to the ranking of the critical available resources R^*, the sequential succession of the dominant community will be reached, and eventually the number of each species tends to stabilize and the biomass $B_{(n)}$ at population equilibrium is maximum, which is most beneficial to the stability of the community.

4. Conclusions

The results of the change in the number of species with precipitation are obtained according to the grassland dynamical system model established in this paper, and the results show that among the five species selected in this paper, the number of species 1, 3 and 4 tends to increase and then stabilize over time, while the number of species II and V tends to decrease constantly over time. This conclusion is consistent with the fact that two plants are endangered in recent years in the actual data obtained in this paper.

Through introducing biomass to evaluate the stability of plant communities and species diversity, the results show that the number of species at plant community equilibrium is $N = [\frac{R^*_{max}-3}{3-R^*_{min}} + 1]$. The optimal number of species is obtained based on the calculated critical available resources, and the relative average error obtained is less than 5% when comparing this number with the number of species when the community is stable in the actual data of all years.

By analyzing the effect of dominant population succession on the number of various groups in the community as well as on the stability of the community, it can be concluded that: when the magnitude of $N_{max} = [N_{jmax}, N_{kmax}, \ldots\ldots, N_{zmax}]$ and $R^* = [R_j^*, R_k^*, \ldots\ldots, R_z^*]$ correspond to each other, the addition or subtraction of a species to the community does not affect the stability of the plant community, i.e., the community is within the range of self-regulation.

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