

Linear Programming of The Los Angeles Olympics in 2028

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Abstract. As in 2028 in Los Angeles, the Games of XXXIV Olympia will be held, the sponsor of it is charging some companies to provide the food for all the athletes. In this essay, we assume that we are the employees in a company that is involved in this program. In order to get this opportunity, we, the authors, use the method of solution strategy in linear programming to analyze the whole program. By using it, we analyze the supply and nutrients and the profit we received. Finally we use our data analysis to make a conclusion for this Olympia Game.

Keywords: linear programming, Olympics, 2028, food supply, data analysis.

1. Introduction

1.1. Background

In 2028, the Games of the XXXIV Olympia will be held in Los Angeles, United States. This is the third time that the Olympics have been held in Los Angeles.

In previous Olympics Games in Japan, the committee found that there was an extremely huge problem in food waste. Lots of reports showed out that, "On the day of the opening ceremony at the Olympic Stadium, the large number of staff on duty meant that all of the volumes were large, and a lot of the food was not consumed," said by organizing committee spokesperson Masanori Takaya, "We apologize for the huge waste. "Food waste is always a major problem that we, all humans, are facing and need to be serious about. Around the world, when we are living in an environment where we have whatever we want, there are as many as an 811 million people who still go hungry. It is such a huge number, so when there are so many people who are still hungry, it is inexcusable that people are still wasting food inadvertently. Especially in the United States, America wastes roughly 40 percent of its food. Including not only waste by people, but also environmental waste. Therefore, knowing that the environment will inevitably waste a certain amount of food, we should reduce the waste made by humans and then consider optimizing the value of the food.

Now, the Olympics Committee is planning to find a company that could provide all the food supply during the Games of the XXXIV Olympia. To significantly reduce the food waste problem, there are planning to find a company that do well in distribute nutrition and food supply. Moreover, there are several companies planning to compete to get this food-providing at now. For us, we are the employees in a company which provides the food for the Games of the XXXIV Olympia. In this situation, we need to achieve two goals. One is the requirement that the Olympics Committee purpose, which is to providing more nutrition for all the athletes in the Olympics. Another is to maximize our profit because our company wants to gain the most revenue from this food-providing project. And these two constrains "which hold with a stated probability" (Dantzig, 2002, p.46) [1]. Based on these two limits, we are creating a project using several constraints in order to accomplish these two outcomes.

Our main job is to grow grain and sell it. We want to prepare for the Supply of grain for the Olympics, and we need to plant the grain in the right place in advance according to the demand to maximize profits. We want to maximize our benefits while ensuring adequate nutrition for our athletes

As a grain supplier, we have to consider the diversity of grains, and the nutritional content of each grain is different. Each source has a certain supply of units to distribute to the destinations, and each destination has a certain demand in form of nutrition for units to be received from the sources. Therefore, what we need to do is to reasonably distribute the supply of different grains in different producing areas, while maximize the total profit of suppliers under the condition of ensuring the nutrition of grains in the Olympic Village.

Therefore, the problem is essentially a transportation problem [2].

To describe the general model for the transportation problem, we need to use terms that are considerably less specific than those for the components of the prototype example. In particular, the general transportation problem is concerned (literally or figuratively) with distributing any commodity from any group of supply centers, called sources, to any group of receiving centers, called destinations, in such a way as to minimize the total distribution cost or maximize the profit. Each source has a certain supply of units to distribute to the destinations, and each destination has a certain demand for units to be received from the sources. The model for a transportation problem makes the following assumption about these supplies and demands.

The requirements assumption: Each source has a fixed supply of units, where the entire supply must be distributed to the destinations. Similarly, each destination has a fixed demand for units, where this entire demand must be received from the sources.

For the question studied in this paper, The Olympic Village belongs to destination, origins belong to Sources, the yield of grain is supply, and the demand for grain in the Olympic Village is demand. Therefore, we could develop the supply optimization model based on the mathematical modelling processes for transportation problem.

1.2. Literature review

In the article “Socio-technical innovations for total food chain safety during the 2008 Beijing Olympics and Paralympics and beyond” written by Xuli Wu et al., the authors analyzed how the food chain made the food athletes eat safer. However, they just find out it by opting out the data in charts but not making a deeper analysis. Similar to the previous essay, another four articles which have the same shortcomings: Towards fair ranking of Olympics achievements: the case of Sydney 2000 by L. Churilov and A. Filtman, Special Olympics Canada 2014 Summer Games-sustainable Food Materials & Waste Procedures by Ann Lee et al., Sustainable Food: An Inter-Organisational Approach for the London 2012 Olympics by Jessica Jane Spayde, and Temporary Commercial Network Programming For Beijing Olympic Games Based on Data Mining And Genetic Algorithms by Yan Chen. Besides, other researchers have also studied linear programming, such as the article: Management Models and Industrial Applications of Linear Programming by A. Charnes and W. W. Cooper, Linear Programming by Robert Fourer, and A Proposed Linear Programming Based Algorithm to Solve Arc Routing Problems by Hashnayne Ahmed. These are all professional analyses, but they only have theorems and derivations. They’re lacking applications in life. In articles Keeping the Olympics moving written by Tony James, Sports and politics: The Olympics and the Los Angeles games written by Bill Shaikin, and Yale Physics Olympics written by Karen Bouffard, the data is so old that they cannot be used in the recent or future Olympics games.

In this essay, we are going to use linear programming to apply it to a specific life event — 2028 Olympics in Los Angeles, United States. As we all know, the best way to solve problem in integer is linear programming [3]. In this way, we get rid of all the disadvantages the previous researches made. Therefore, we can have a deeper understanding of linear programming and food transportation, and distribution of food supply in the Olympics Games.

1.3. Information allocation

1.3.1 Identity matrix

First, we're going to set up a matrix A, like that:

$$\begin{bmatrix} A_{11} & \cdots & A_{1n} \\ \vdots & A_{ij} & \vdots \\ A_{m1} & \cdots & A_{mn} \end{bmatrix}$$

This is an identity matrix. m represents the total number of the grain types, and n represents the total number of the origins. Matrix A is 0-1 matrix, and $A_{ij}=1$ means the i th grain is produced in the j th origin, otherwise, $A_{ij}=0$.

There are 15 origins [Southern Seaboard, Prairie Gateway, Northern Great Plains, Northern Crescent, Mississippi Portal, Heartland, Eastern Upland, Southern Seaboard (VA, NC, SC), Southern Seaboard (AL, GA), Fruitful Rim, Mississippi River Delta, Gulf Coast, California, Arkansas Non-Delta, Basin and Range].

There are 8 grains total [Soybean, Peanut, Rice, Sorghum, Barley, Wheat, Oats, Corn]

1.3.2 Cost Matrix

Four cost matrixes are defined. Each of them represents one type of costs, which are Primary cost (C_p), Operation cost (C_o), Environmental cost (C_e), and Capital cost (C_c), respectively. The matrices of these four types of expenditure have the same dimension as matrix A .

The Primary Cost (C_p) matrix is as follows:

$$\begin{bmatrix} C_{p11} & \cdots & C_{p1n} \\ \vdots & C_{p_{ij}} & \vdots \\ C_{p_{m1}} & \cdots & C_{p_{mn}} \end{bmatrix}$$

$C_{p_{ij}}$ represents the primary cost of grain i produced from origin j in dollars per bushel.

The definition of operation cost (C_o), environmental cost (C_e), and capital cost (C_c) are the same with C_p . C_t represents the total cost of grain i produced from origin j in dollars per bushel.

$$C_{tij} = C_{p_{ij}} + C_{o_{ij}} + C_{e_{ij}} + C_{c_{ij}}$$

M_1 , M_2 , M_3 , M_4 represent the budget for the above four types of expenditure. M_1 represent the budget for primary cost. M_2 represent the budget for operation cost. M_3 represent the budget for environmental cost. M_4 represent the budget for capital cost.

1.3.3 Price

We use “ R ” represent the price of the grains, which is with the same dimension as matrix A :

$$\begin{bmatrix} R_{11} & \cdots & R_{1n} \\ \vdots & R_{ij} & \vdots \\ R_{m1} & \cdots & R_{mn} \end{bmatrix}$$

R_{ij} represent the price of grain i produced from origin j in dollars per bushel.

1.3.4 Nutrition

In nutrition part, the matrix N is defined:

$$\begin{bmatrix} N_{11} & \cdots & N_{1a} \\ \vdots & N_{ik} & \vdots \\ N_{m1} & \cdots & N_{ma} \end{bmatrix}$$

The “m” represents the total number of the grain types, and “a” means the number of nutrition types. N_{ik} represents the content of nutrition k contained in the i th type of grain.

There are totally three kinds of nutrition which are fats, protein and carbohydrate.

Q1, Q2 and Q3 represents the minimum amount of different nutrition required for athletes during the Olympic Games. Q1 represent the minimum requirement of fats. And Q2 represent the minimum requirement of protein. Q3 represent the minimum requirement of carbohydrate.

1.3.5 Yield

Matrix Y is defined to represent the production of different kinds of grain from different origins:

$$\begin{bmatrix} Y_{11} & \cdots & Y_{1n} \\ \vdots & Y_{ij} & \vdots \\ Y_{m1} & \cdots & Y_{mn} \end{bmatrix}$$

Y_{ij} represent the maximum amount of yield of grain i from origin j, in bushel.

2. Model assumptions

(1) Assuming that the cost of grain transportation between each producing area and the Olympic Village is constant (independent of grain type and transportation volume), the total transportation cost is a constant, which does not affect the optimization result of supply plan. Therefore, transportation cost is not considered in the model.

(3) The yield and quality of grain are constant. Our data is based on the last three years and then we take the average, which is the average of the last three years. We base our calculations on this data. Therefore, constant value is taken regardless of numerical fluctuation

(4) The types of cereals are the above, and there is no case of private customization. The data looked at the top eight grains in greatest demand. If too many grains were considered, which would increase costs significantly, eight major grains were used, leaving out smaller amounts of demand.

(5) The total budget is 1.3 million yuan. The proportion of M1, M2, M3, M4 in the budget can be flexible and changeable. The reason for this setting is to consider that the company budget cannot be infinitely large, so a reasonable value is set for simulation.

3. Mathematical Model

3.1. Decision variable

Our goal is to set up a reasonable supply plan that maximizes the overall benefits and meets the athletes' daily nutritional needs. This program is designed to maximize profitability and meet nutritional requirements by supplying specific types of grain and the amount of grain planned from appropriate origins. So, we set up a matrix x_{ij} as our decision variable. The decision variable is x_{ij} , which represents the amount of the grain i supplied from origin j in bushel.

3.2. Objective function

$$\text{Max } F = \sum_{i=1}^m \sum_{j=1}^n (R_{ij} - C_{t_{ij}})x_{ij} \quad (1)$$

F represents the total profit. The m represents the row of the matrix. And the n represents the column of the matrix. The objective is to maximize the value of F. Matrix R_{ij} represent price of grain i produced from origin j in dollars per bushel. And Matrix C_{ij} represent total cost of grain i produced from origin j in dollars per bushel. Therefore $(R_{ij} - C_{t_{ij}})$ represent the profit of grain i produced from origin j in dollars per bushel. Then use the profit times the decision variable x_{ij} , represent the total profit of grain I produced from origin j in amount that decision variable set in dollars per bushel. Lastly, $\sum_{i=1}^m \sum_{j=1}^n (R_{ij} - C_{t_{ij}})x_{ij}$ represent sum of all the total profit of grain i produced from origin j in amount that decision variable set in dollars per bushel. And that represent the total profit which we want to maximize.

3.3. Constraints

3.3.1 Cost Constraints

$$\sum_{i=1}^m \sum_{j=1}^n x_{ij}Cp_{ij} \leq M_1 \quad (2)$$

Equation (2) means the total primary cost should not exceeds the budget of expenditure M1.

$$\sum_{i=1}^m \sum_{j=1}^n x_{ij}Co_{ij} \leq M_2 \quad (3)$$

Equation (3) means the total operation cost should not exceeds the budget of expenditure M2.

$$\sum_{i=1}^m \sum_{j=1}^n x_{ij}Ce_{ij} \leq M_3 \quad (4)$$

Equation (4) means the total environmental cost should not exceeds the budget of expenditure M3.

$$\sum_{i=1}^m \sum_{j=1}^n x_{ij}Cc_{ij} \leq M_4 \quad (5)$$

Equation (5) means the total capital cost should not exceeds the budget of expenditure M4.

3.3.2 Nutrition constraint

$$\sum_{i=1}^m N_{ik} \sum_{j=1}^n x_{ij} \geq Q_k \quad (6)$$

Equation (6) mean the content of nutrition k of total grains should exceed the minimum nutrition requirement Q_k .

3.3.3 Production capacity

$$x_{ij} \leq Y_{ij} \quad (7)$$

Equation (7) means the maximum amount we supplied should not exceed the amount of yield.

3.3.4 Auxiliary restriction

$$0 \leq x_{ij} \leq 100,000 \bullet A_{ij} \quad (8)$$

Equation (8) means limitation the range of x_{ij} , which uses “big-M” method. When $A_{ij}=0$, x_{ij} is restricted to 0, otherwise, x_{ij} becomes a non-negative variable whose range is almost unlimited.

3.4. Short summary

We have mn decision variables. And this objective function is single and linear [4]. We have 4 types of constraints. And every constraint is linear. For summary, the proposed model belongs to

linear programming model [5]. The proposed model could be represented as the following general form:

GENREAL MAXIMUM PROBLEM

$$\max F = M^T x$$

s. t.

$$T_{eq}x = b_{eq} \text{ (EQUALITY CONSTRAINTS)}$$

$$Tx \leq b \text{ (INEQUALITY CONSTRAINTS)}$$

M^T represents the coefficient of decision variables in the objective function. The first line of constraints refers to the set of equality constraints in the model, and the second line refers to the set of inequality constraints [6].

We can solve it using the Simplex method. simplex method, standard technique in linear programming for solving an optimization problem, typically one involving a function and several constraints expressed as inequalities. The inequalities define a polygonal region, and the solution is typically at one of the vertices. The simplex method is a systematic procedure for testing the vertices as possible solutions.

We can use linear programming Solver and call (Dantzig) Simplex methods [7] to solve. Then the linear programming solver can be used to solve the problem.

4. Case study

4.1. Input information

NAME	ORIGIN	CP	CO	CE	CC	TOTAL CO	Price	PRODUCTION
Soybean	Southern seaboard	3.77	1.09	0.27	4.15	9.28	10.14	8987
Soybean	Prairie gateway	2.42	0.85	0.35	5.17	8.79	9.5	15933
Soybean	Northern great plains	2.66	0.95	0.29	6.01	9.91	9.25	24890
Soybean	Northern crescent	2.5	1.09	0.25	4.84	8.68	9.69	8164
Soybean	Mississippi portal	3.21	0.84	0.6	4.68	9.33	9.76	52745
Soybean	Heartland	2.26	0.76	0.19	5.34	8.55	9.72	16704
Soybean	Estern upland	3.16	0.6	0.21	3.78	7.75	9.76	17325
Peanut	Southern seaboard (VA,NC,SC)	2.04	0.58	0.23	1.49	4.34	4.84	38612.04549
Peanut	Southern seaboard (AL,GA)	1.67	0.48	0.19	1.72	4.06	4.4	51256.72732
Peanut	Prairie gateway	2.02	1.01	0.78	3.45	7.26	6.82	32671.77276
Peanut	Fruitful rim	2.59	0.51	0.29	2.17	5.56	4.62	52572.09096
Rice	Mississippi river delta	2.05	0.32	0.52	1.93	4.82	11.21	102854.01
Rice	Gulf coast	1.63	0.57	0.61	1.83	4.64	10.92	104377.1
Rice	California	1.78	1.11	0.54	2.66	6.09	18.1	100531.6
Rice	Arkansas non-delta	1.88	0.57	0.52	2.07	5.04	11.26	99191.4
Sorghum	Prairie gateway	1.1	0.57	0.11	2.29	4.07	4.43	23947
Sorghum	Northern great plains	1.18	0.69	0.13	2.85	4.85	4.28	16320
Sorghum	Heartland	1.42	0.85	0.25	2.56	5.08	4.58	9250
Sorghum	Fruitful rim	1.14	0.48	0.18	2.22	4.02	4.08	94024
Barley	Northern great plains	1.5	0.57	0.2	3.51	5.78	4.29	16038
Barley	Northern crescent	1.59	0.89	0.28	3.92	6.68	3.65	1456
Barley	Heartland	1.6	0.83	0.41	4.69	7.53	3.71	3306
Barley	Fruitful rim	1.23	0.67	0.38	3.03	5.31	4.82	30923
Barley	Basin and range	1.29	0.63	0.36	3.6	5.88	4.67	13432
Wheat	Prairie gateway	1.49	0.75	0.28	4.1	6.62	4.3	25848
Wheat	Northern great plains	1.46	0.48	0.12	3.03	5.09	4.53	44250
Wheat	Northern crescent	1.82	0.61	0.15	3.36	5.94	5.17	7268
Wheat	Heartland	1.93	0.5	0.14	3.59	6.16	5.02	6570
Wheat	Fruitful rim	1.43	0.59	0.41	3.31	5.74	5.14	51600
Wheat	Basin and range	1.57	0.47	0.18	2.62	4.84	5.01	59080
Oats	Prairie gateway	1.42	0.54	0.29	3.67	5.92	3.23	3024
Oats	Northern great plains	0.72	0.37	0.15	2.72	3.96	2.42	9545
Oats	Northern crescent	1.25	0.59	0.37	4.73	6.94	2.8	1180
Oats	Heartland	0.74	0.36	0.21	4.11	5.42	2.77	1920
Corn	Southern seaboard	2.24	0.51	0.37	1.9	5.02	4.08	21842
Corn	Prairie gateway	1.44	0.31	0.3	1.93	3.98	3.68	65816
Corn	Northern great plains	1.37	0.33	0.16	1.67	3.53	3.42	90706
Corn	Northern crescent	1.47	0.42	0.15	1.6	3.64	3.64	24566
Corn	Heartland	1.34	0.28	0.12	1.81	3.55	3.59	57682
Corn	Estern upland	1.51	0.25	0.1	0.2	2.06	3.79	17444

Figure 1. Cost, price and yield of grain i from origin j in dollars per bushel.

This graph represent cost, price, of grain i from origin j in dollars per bushel. In the following data. CP represents primary cost, CO represents operation cost, CE represents environmental cost, and CC represents capital cost of grain i from origin j in dollars per bushel. Total cost represents the sum of CP, CO, CE and CC. And the price which represent the revenue of grain i from origin j. The production represents the yield of the grain i from origin j in bushel [8].

Agriculture Prouct	Protein(g/Bushel)	Fat(g/Bushel)	Carbohydrate(g/Bushel)
Soybean	4517.7564	2449.386	2694.3246
Peanut	898.1127	1896.0157	598.7418
Rice	650.416	52.364	7909.72
Sorghum	2267.96	816.462	18778.626
Barley	2721.55	500.7652	16002.714
Wheat	3592.4328	680.385	19595.088
Oats	2453.035	1001.535	9623.445
Corn	762.036	254.012	4826.228

Figure 2. Nutrition data in grams per bushel.

This graph represents the protein, fat and carbohydrate content of each grain in grams per bushel.

At previous model assumptions, the company has a budget of 1.3 million. In this case, the value of M1 set to be 400,000, M2 to 200,000, and M3 to 100,000 and value of M4 is 600,000. This represents that our primary cost budget is 400,000 dollars, operation cost budget is 200,000 dollars, environmental cost budget is 100,000 dollars, capital cost budget is 600,000 dollars.

In terms of the number of people in the Olympics, we simulated 12,000 athletes to obtain Olympics and the average weight of the athlete is 72.8 kg. The protein amount that an athlete need is $0.8 \times 72.8 = 58.24\text{g}$, since the recommended dietary allowance to prevent deficiency for an average sedentary adult is 0.8g per kg of body weight. And carbohydrate that an athlete intake is 4g per kg of body. So, the amount of carbohydrate of an average weight athlete needs is $4 \times 72.8 = 291.2\text{g}$. And the other important nutrition that an athlete needs is fat, and the fat that an athlete needs per day is at least 70g. Thus, the value of Q1 is set to be $58.24 \times 12000 = 698880\text{g}$. The value of Q2 is set to be $70 \times 12000 = 840000\text{g}$. The value of Q3 is set to be $291.2 \times 12000 = 3494400\text{g}$. Since then, the description of case parameters is complete.

4.2. Result

	Southern	Prairie	Northern	Northern	Mississipp	Heartland	Eastern up	Southern	Southern	Fruitful	Mississipp	Gulf coas	Californi	Arkansas	Basin and
Soybean'	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Peanut'	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Rice'	0	0	0	0	0	0	0	0	0	0	84554.88	0	100531.6	0	0
Sorghum'	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Barley'	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Wheat'	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Oats'	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Corn'	0	0	0	0	0	0	17444	0	0	0	0	0	0	0	0

Figure 3. Supply plan of the grain

This is our supply plan. The company plan to grow rice at 84554.88 bushel in the Mississippi River Delta. And 100,531.6 bushel of rice is grown in California. In Eastern Upland, 17444 bushel was planted. These grains will form our staple food for the Games to maximize our profits and meet the daily nutritional needs of the athletes.

And in this case, our total profit is $1.7779\text{e}+06$ dollars per day in standard form which is 1777868.2995 dollars per day.

4.3. Discussion

As mentioned in the previous Model Assumption, the total budget is 1.3 million dollars, but the proportion of each cost is flexible. So, the following discussion is about the impact of changing the budget parameters of different costs on overall profit. Because the total amount of cost is constant, when the budget of one cost increases, the budget of other parts must decrease. The following is the total profit reflected by the different cost parameters, as shown in the figure.

Origin Case	When M1 (Primary Cost) increase			When M2 (Operation Cost) increase		
40, 20, 10, 60	50, 10, 10, 60	50, 20, 0, 60	50, 20, 10, 50	30, 30, 10, 60	40, 30, 0, 60	40, 30, 10, 50
1777900	1387800	512.4948	1777900	1673800	348.3794	1777900
	When M3 (Environmental Cost) increase			When M4 (Capital Cost) increase		
	30, 20, 20, 60	40, 10, 20, 60	40, 20, 20, 50	30, 20, 10, 70	40, 10, 10, 70	40, 20, 0, 70
	1673800	1395200	2006100	1673800	1387800	298.6347

Figure 4. Total profit reflected by the different cost parameters per dollar

After we compare the total profit obtained by changing the proportion of total budget with the total profit obtained by the origin case. Only group M1=40, M2=20, M3=20, M4=50 is expected to get a higher total profit than the previous setup budget. This group of budgets increase 100,000 dollars of the Environmental cost and cut down 100,000 dollars of capital cost. So, the total profit is 3,784,000 more than the total profit of the original group.

Because this group of budgets was the only one that increased total profit. So, we cut another 100,000 from Capital cost, and then transferred to environmental cost. But what we found was that the total profit did not increase, it was actually less than the total profit of the original group which is \$1.6806e+06.

Therefore, we believe that the budgets of 40, 20, 20 and 50 have at least verified that more profits can be obtained by changing the proportion of each budget in the total budget, which provides a direction for our subsequent optimization model.

5. Conclusion

In this paper, we refer to and make the data about grain cost and return, and build a linear programming model. And this model, set into the Olympic Games case and discussed. With this model, we get a set of optimal solutions, which is we called active optimal solutions [9]. Since “it is not possible to get in a position where the programming problem admits of no solution” (Dantzig, 1955, p. 197) [10]. Moreover, on this basis, we explore the impact of changing the budget ratio on the total profit.

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