Analysis on the growth of fungal population and the interaction between population

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Abstract. The carbon cycle is an important part of the geochemical cycle of the Earth. As the primary decomposers of organic material in terrestrial ecosystems, fungi are critical agents of the global carbon cycle. Based on the two characteristics of a fungus, the growth rate of the fungus and the fungus' tolerance to moisture, this article will establish a mathematical model to describe fungi and their role in the decomposition of ground litter (dead plant material) and woody fibers. First of all, the growth rate of a single fungus population was determined by modifying the Logistic population growth model. According to the hyphal extension rate of a fungus comprehensively measured by temperature and humidity changes, the effect of hyphal extension rate on wood decomposition rate is obtained. Next, this article uses the competitive ranking of fungi and the moisture niche width to determine the moisture tolerance of fungi and the influence of fungus moisture tolerance on wood decomposition rate. Integrating the hyphal extension rate and moisture tolerance of fungi, we established a wood decomposition rate model for a single fungus community. Based on this model and Lotka-Volterra model, this paper quantified and described the interaction between different types of fungi, and constructed a model of wood decomposition rate under the interaction of multiple fungal populations. Finally, the sensitivity and stability of the model are analyzed and tested. For the role and importance of biodiversity in the ecosystem, we conducted a more comprehensive study, and analyzed the role and principle of biodiversity in the process of environmental factors change.

Keywords: Modified logistic population growth model, Wood decomposition of fungal population.

1. Introduction

1.1. Background

It is commonly acknowledged that the carbon cycle is an important part of the geochemical cycle. In particular, various fungi play a vital role in the biological cycle of carbon as the main decomposers of litter and wood. Through the decomposition of fungi, organic substances such as plant materials and woody fibers are converted into carbon dioxide and enter the atmosphere.

However, there are huge differences in the speed at which different fungi decompose organic materials. In this way, researchers have done many related studies and discovered two interesting traits of fungi, the growth rate and the moisture tolerance. Under further research, they have learned the role of fungi in decomposing the plant materials and woody fibers on the ground. To be specific, fungi with larger hyphal extension rate are easier to decompose wood faster. The faster the fungi grow and the stronger the competitiveness, the faster they decompose wood.

Therefore, exploring the relationship between the growth rate and moisture tolerance of different fungi and the rate of decomposition of woody fibers is of great help to the study of the decomposition of compounds during the carbon cycle.

1.2. Model assumptions

1. The rate of wood decomposition by fungi is only related to the temperature and humidity of the environment, and the physical and chemical properties of other environments have little effect on it.

2. The effect of moisture and heat released during wood decomposition by fungi on wood decomposition rate and external environment is negligible, and the physical and chemical properties of the environment are directly determined by the external environment.

3. Assuming that the maximum and minimum humidity levels where half of a fungal community can maintain its fastest growth rate are width_max and width_min, respectively. When the environmental humidity is less than width_min or greater than width_max, the overall growth rate of a fungal community is half of the ideal growth rate.

4. When n species of fungi compete with each other, it is assumed that each species has a major competitor (one of the remaining n-1 species) who has the greatest impact on it, while the remaining n-2 species have negligible impact on it.

5. When n species of fungi compete with each other, it is assumed that each species is subject to logistic growth model.

2. Wood decomposition rate model of single fungal community

2.1. Wood decomposition rate model of single fungal community

Hyphae are the branch cells of fungi, which constitute the filaments and structures of most fungi. Different kinds of hyphae play different roles in the life cycle of fungi. Therefore, hyphal extension is an important characteristic of fungi.

Step 1: determine the relationship F

From the reference [7], it can be obtained that there should be a certain relationship F between wood decomposition rate D and the hyphal extension rate. Besides, wood decomposition rate is also related to temperature and humidity. In terms of temperature, when the extension of mycelium is equal, the decomposition rate of wood increases with the increase of temperature; When the ambient temperature is constant, the wood decomposition rate increases with the increase of mycelial extension. In terms of humidity, it is known that each species has its own corresponding water niche width. When at a certain environmental humidity that can maintain the fastest growth rate of at least half of the fungal community, because the essence of mycelium extension is the growth rate of fungi, at this time, the extension of mycelium will change with the change of environmental humidity, so that the wood decomposition rate will also change.

To sum up, we can get the relationship F:

$$D = F(V_x, T) \tag{1}$$

Where $\boldsymbol{V_x}$ means hyphal extension rate, and T represents the temperature of environment.

*** Growth rate of single fungus**

First, the growth rate of single strain was studied. According to the logistic growth model, the growth process of a single strain is described by the following differential equation:

$$\frac{dN(t)}{dt} = rN(t) \tag{2}$$

However, it is universally recognized that the natural resources are limited. Thus, the fungal community is unable to grow indefinitely. In this case, we add a correction factor $1 - \frac{N(x)}{K}$ to the above equation to make the model self-suppressive:

$$\frac{dN(t)}{dt} = rN(t)\left(1 - \frac{N(t)}{K}\right) \tag{3}$$

Assuming that the number of a fungal community in the initial state is $N(0) = N_0$

Then, the differential equation can be obtained:

$$\begin{cases} \frac{dN(t)}{dt} = rN(t)\left(1 - \frac{N(t)}{K}\right) \\ N(0) = N_0 \end{cases} \tag{4}$$

After that,

$$N(t) = \frac{K}{1 + (\frac{K}{N_0} - 1)\, e^{-rt}} \tag{5}$$

In a word, when the environment temperature is suitable, the humidity remains unchanged, and the interaction among the fungi is not considered (that is, the competition among the populations is ignored), it is noted that for a certain local fungus, the resources of its location are limited. At this time, only the internal competition of its population is considered, Then, we can get that n species of fungi obey the above population growth equation.

* **Hyphal extension rate of single fungus**

Secondly, because the essence of mycelial extension is the growth rate of fungi, we use the strain growth rate V_1 to measure the mycelial extension rate V_2:

$$V_2 = uV_1 \tag{6}$$

Where:

$$V_1 = \sum_{i=1}^{n} \lim_{\Delta t \to 0} \frac{\Delta N_i(t)}{\Delta t} \tag{7}$$

* **Solution $\boldsymbol{D = F(V_x, T)}$**

Finally, according to the graph C in the reference [7], we fitted the curve of mycelial elongation and wood decomposition rate at 10℃, 16℃ and 22℃, the results are as shown below:

When t = 10℃, the equation $f_{10}(x) = 3.06\ln(x) + 3.03$

Table 1. The relationship between hyphal extension rate and decomposition rate when t=10℃

When t = 16℃, the equation $f_{16}(x) = 4.36\ln(x) + 6.24$

Hyphal extension rate	**1.38**	**1.73**	**2.08**	**2.5**	**3.55**
Decomposition rate	4.68	5.37	2.87	6.37	7.46

Table 2. The relationship between hyphal extension rate and decomposition rate when t=16℃

When t = 16℃, the equation $f_{16}(x) = 4.36\ln(x) + 6.24$

Hyphal extension rate	**1.38**	**1.98**	**2.5**	**4.38**	**5**
Decomposition rate	7.86	9.05	10.05	12.74	13.33

Table 3. The relationship between hyphal extension rate and decomposition rate when t=22℃

When t = 22℃, the equation $f_{22}(x) = 6.61\ln(x) + 10.24$

Hyphal extension rate	**0.88**	**1.1**	**2.5**	**5**	**7.5**
Decomposition rate	9.85	10.94	15.42	20.5	24.28

In conclusion, we obtained the relationship between mycelial extension and wood decomposition rate at three temperatures.

Step 2: determine the relationship G

From the reference [7], it can be obtained that there is a certain relationship G between wood decomposition rate D and moisture tolerance of fungi. Specifically, the moisture tolerance is calculated as the difference between each isolate's competitive ranking and their moisture niche width, both scaled to [0,1]. Therefore, the moisture trade-off ranged from −1 to 1, with 1 representing high competitive dominance/low moisture tolerance and −1 the reverse. Hence, we obtained the relationship G between the fungal moisture tolerance and the wood decomposition rate by simple fitting. The formulas are as follows:

$$D = G(\theta_x) \tag{8}$$

According to the definition, we can know the humidity tolerance of fungi:

$$\theta_x = Rank_x - Width_x \tag{9}$$

For $Rank_x$, by sorting the growth rate of all fungi at each time, we get the competitive advantage ranking of fungi in current environment and current time. Considering the influence of environmental humidity on the growth of fungi, we can get the curve of the number of different fungi population changing with time, so as to obtain the competitive advantage ranking of the fungi in the current environment and at the current time.

And for $Width_x$, different fungi have their own water niche width, so we input it as a parameter of fungi.

Step 3: get wood decomposition degree Y

To sum up, the degree of wood decomposition is determined by the hyphal extension rate and the moisture tolerance of fungi so that we define Y as the degree of wood decomposition. The formula is as follows:

$$Y = w_1 F(V_x, T) + w_2 G(\theta_x) \tag{10}$$

Where: $w_1 =$ the constant coefficient of decomposing wood rate on hyphal extension rate

$w_2 =$ the constant coefficient of decomposing wood rate on moisture tolerance

Although the degree of wood decomposition Y does not directly represent the rate of wood decomposition, from the above analysis, it is easy to know that the rate of wood decomposition D and the degree of wood decomposition Y have a simple linear relationship:

$$D \propto Y \tag{11}$$

So far, we can get the situation that fungi decompose litter and lignocellulosic fiber on the ground without considering the interaction of strains.

3. Wood decomposition rate model under multi fungi interaction

When there are two kinds of fungi competing with each other in the environment, there will be competition for survival resources between the two populations. At this time, by further modifying the population growth model of single fungus, we can get that the growth of multiple fungi follows the following equation:

$$\begin{cases} \frac{dN_1(t)}{dt} = r_1 N_1(t)\left(1 - \frac{N_1(t)}{K_1} - S_{12}\frac{N_2(t)}{K_2}\right) \\ \frac{dN_2(t)}{dt} = r_2 N_2(t)\left(1 - \frac{N_2(t)}{K_2} - S_{21}\frac{N_1(t)}{K_1}\right) \end{cases} \tag{12}$$

Therefore, when n species compete with each other in the environment, the growth model of each species is as follows

$$\frac{dN_x(t)}{dt} = r_x N_x(t)\left(1 - \sum_{i=1}^{n} S_{xi}\frac{N_x(t)}{K_x}\right) \tag{13}$$

Thus, we can get that S has the following two characteristics:

$$S_{ij}S_{ji} = 1 \tag{14}$$

$$S_{ii} = 1 \tag{15}$$

The interaction process of bacteria under the condition of n = 10 is simulated as follows:

Step 1: Give the S table and the parameters of the moisture niche width of each fungal population. (Other parameters are not specifically listed)

Here we defined S as the retardation coefficient between fungal populations and the S table is set as follows:

Table 4. Retardation coefficient between fungal populations

i / j	1	2	3	4	5	6	7	8	9	10
1	1	2	0.4	3.2	0.5	0.8	0.6	0.2	0.3	0.2
2	0.5	1	3	6	0.2	0.1	0.7	0.2	0.4	2
3	2.5	0.33	1	0.4	0.5	0.2	4	0.9	0.2	0.1
4	0.31	3.33	2.5	1	3	0.5	0.1	0.2	0.7	4
5	2	5	2	0.33	1	0.3.	0.1	0.1	0.2	0.6
6	1.25	10	5	2	0.33	1	0.2	0.5	0.2	0.7
7	1.67	1.43	0.25	10	10	5	1	1.67	0.2	0.3
8	5	5	1.11	5	10	2	0.6	1	5	5
9	3.33	2.5	5	1.43	5	5	5	0.2	1	0.2
10	5	0.5	10	0.25	1.67	1.43	3.33	0.2	5	1

Then, the moisture niche width of each fungal population is set as follows:

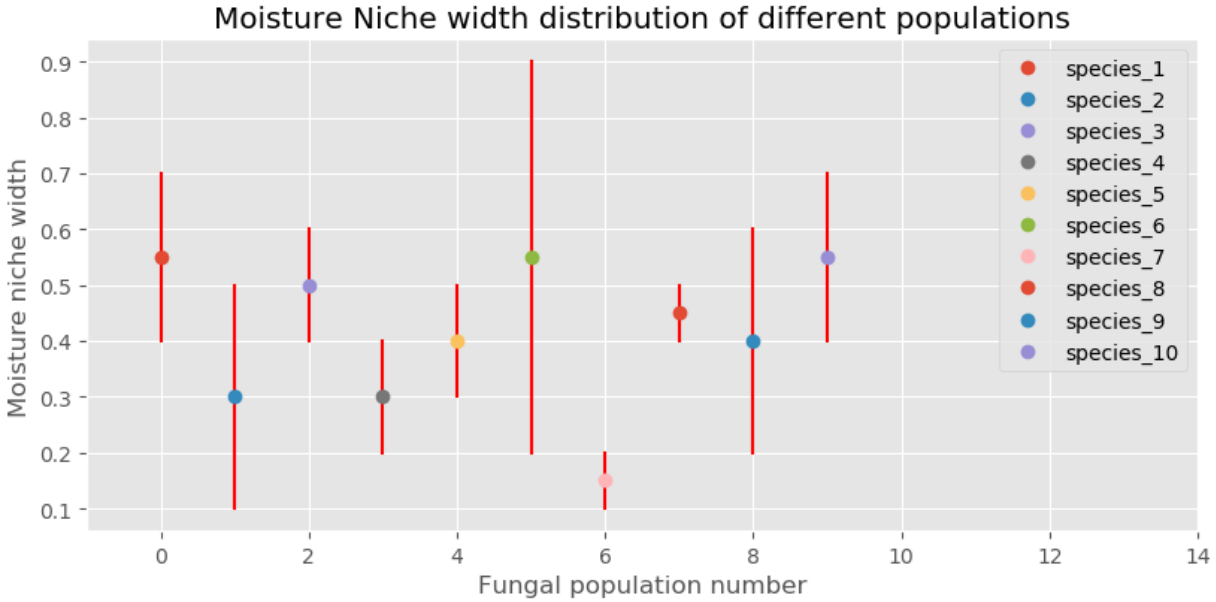


Figure 1. Moisture Niche Width Distribution of Different Populations

Step 2: Test whether S satisfies properties (14) and (15), and further simplify the model according to Hypothesis 4.

The simplified formula is as follows:

$$\begin{cases} \frac{dN_x(t)}{dt} = r_x N_x(t)\left(1 - \frac{N_x(t)}{K_x} - S_{xm}\frac{N_m(t)}{K_m}\right) \\ S_{xm} = Max(S_{xi}) \quad (i = 1,2,\ldots,n,\ i \neq x,\ S_{xi} > 1) \end{cases} \tag{16}$$

Step 3: Dimension reduction was performed on the block coefficient S table of each fungal population.

Table 5. New block coefficient of each fungal population

i	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
m	4	4	1	10	2	2	4	5	3	1
S_{im}	3.2	6	2.5	4	5	10	10	10	5	5

Step 4: Bring in calculation

Substitute the S and the moisture niche width of each fungal population into formula (16) to calculate the growth curve of the population number of each fungus over time.

Step 5: Calculate the degree of wood decomposition Y

So far, we have obtained the changes of wood decomposition degree with time under the interaction of a variety of fungi, as shown in the figure below.

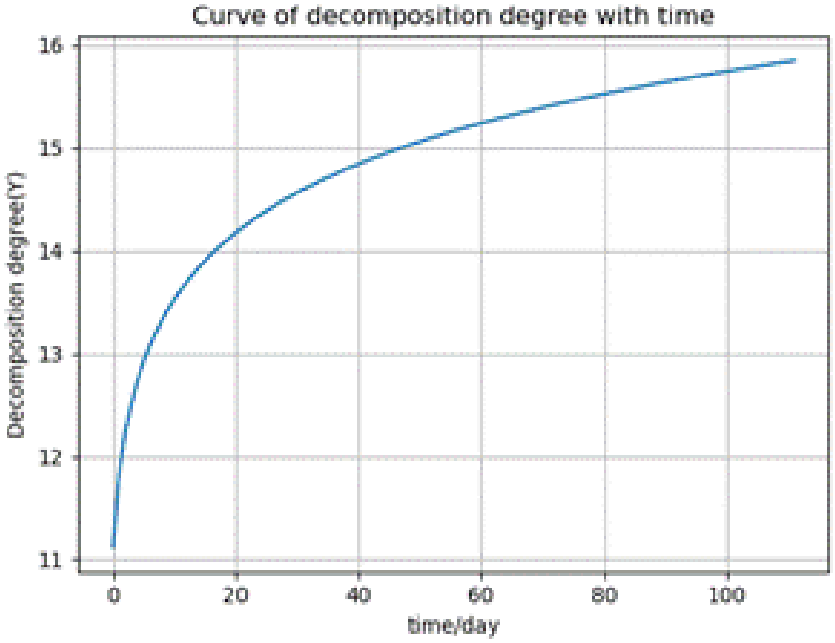


Figure 2. Curve decomposition degree with time

4. Conclusion

Firstly, by analyzing the growth of fungal population and the interaction between populations, we constructed a population growth model with multi-population interaction based on Lotka-Volterra model. Secondly, we also considered the influence of different environmental factors, including temperature and humidity, on the population, and combined with the information given by literatures, we obtained the quantitative model of the degree of wood decomposition by fungi in different environments. Based on the above models, we discussed in depth the dynamic characteristics of interactions between different fungal populations. Besides, the sensitivity of the model is analyzed, and the model is stable and reasonable.

References

[1] Ma Lin. analysis of "J" curve of population growth [J]. Biology teaching in middle school, 2000 (05): 21.

[2] Li Rui, Liu Fang, ran Guiping, Yang Xia. Climatic characteristics of relative humidity in Urumqi in recent 40 years [J]. Anhui Agricultural Sciences, 2010, 38 (27): 15121 - 15122 + 15133.

[3] Wu Anchi, Deng Xiangwen, Ren Xiaoli, Xiang Wenhua, Zhang Li, Ge Rong, Niu Zhongen, he Honglin, he Lijie. Spatial distribution pattern and influencing factors of species diversity in tree layer community of typical forest ecosystems in China. Acta ecologica Sinica, 2018, 38 (21): 7727 - 7738.

[4] Zhang Hua, Zheng Zhuo. Relationship between the distribution of airborne fungi and climate in Guangzhou [J]. Journal of Sun Yat sen University (Medical Science Edition), 2006, 27 (5): c2 - c2.

[5] Song Piao, Zhang Naili, Mak Ping, Guo Jixun. Effects of global warming on litter decomposition [J]. Acta ecologica Sinica, 2014, 34 (6): 1327 - 1339.

[6] Zhu Yanxia, Wang Ying. The great role of small and medium decomposers in ecosystem [J]. Excellent composition of junior high school students, 2015 (16).

[7] Nicky Lustenhouwer, Daniel S. Maynard, Mark A. Bradford, Daniel L. Lindner, Brad Oberle, Amy E. Zanne, Thomas W. Crowther. A trait-based understanding of wood decomposition by fungi [J]. Proceedings of the National Academy of Sciences, 2020 (republish).