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Multi-Attribute Decision-Making Method Based on Dispersion Maximization and its Practical Application

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Abstract: Multi-attribute decision making is an important part of modern decision science, which has wide and practical applications in engineering design, economy, management, military and other fields. This paper first introduces the theory and sequencing steps of multi-attribute decision making method based on deviation maximization, and then makes specific application to the example of civil servant recruitment in government departments by using MATLAB programming. It makes the operation process of recruitment work clear and clear, and realizes scientific, normative and fair.

Keywords: Multi-attribute Decision Making; Maximization of Deviation; Objective Empowerment.

1. Introduction

In recent years, with the rapid development of economy and science and technology, both enterprises and individuals are often faced with a variety of choices, and objective things have complexity, uncertainty and the diversity of human thinking, we are always inclined to a variety of selection criteria to compare and sort objects. Under this background, multi-attribute decision making method has become an effective analysis method to solve many complicated problems in the real society. Classical multi-criteria decision making can generally be divided into two main areas, namely multi-objective decision making [1] and multi-attribute decision making [2]. The essence of multi-attribute decision making method is to make use of the decision information that has been mastered, and establish a mathematical model to make the optimal selection or ranking of the comprehensive evaluation values in the alternatives. Multi-attribute decision making mainly includes three aspects: the normalization of decision matrix, the determination of the weight of each attribute and the comprehensive ranking of the scheme. Among them, the attribute weights of alternative schemes are mainly determined by subjective weighting method, objective weighting method and combination weighting method. There is no uniform requirement for selecting the method of empowerment, and there is no absolute superiority or inferiority, which can be judged and chosen according to the actual situation of each decision maker. This paper intends to use the deviation maximization objective weighting method to make decision-making more scientific, standardized and fair.

2. Multi-attribute Decision Making Method based on Deviation Maximization

For a certain multi-attribute decision problem, suppose that the attribute weight information is completely unknown and the known decision matrix is A, and the normalized matrix B can be obtained after normalization. Suppose the weight vector of the attribute is $\omega = (\omega_1, \omega_2 \ldots, \omega_n), \omega_j \geq 0, j \in n$,and the constraint conditions are also satisfied: $\sum_{j=1}^{m} \omega_j^2 = 1$,

Then the comprehensive attribute value of each alternative can be defined as:

$$z_i(\omega) = \sum_{j=1}^{m} b_{ij}\omega_j \tag{1}$$

Multi-attribute decision making is to sort and prioritize the comprehensive attribute values of multiple alternatives. When solving a multi-attribute decision problem, if there is no significant difference in the attribute values of alternative solutions under the j attribute s_j, the attribute s_j has no influence on the decision making and ranking of the solutions. Conversely, if there is a large difference in the attribute values under the j attribute s_j of the alternative scheme, it indicates that the attribute s_j plays an important role in the decision and ordering of the scheme. In order to rank schemes, attributes with greater deviation in scheme attribute values (regardless of how important they are) should be given greater weight. If there is no difference in the attribute values of the alternative scheme under the attribute s_j, it means that the attribute s_j will have little or no influence on the decision and ordering of the scheme. In this case, its weight can be set to 0. For the attribute s_j, $V_{ij}(\omega)$ represents the deviation between alternative r_i and all other alternatives,and we have the following definition:

$$V_{ij}(\omega) = \sum_{k=1}^{n} \left| b_{ij}\omega_j - b_{kj}\omega_j \right|, \quad i \in m, j \in n.$$

And we assumption that:

$$V_j(\omega) = \sum_{i=1}^{n} V_{ij}(\omega) = \sum_{i=1}^{n}\sum_{k=1}^{n} \left| b_{ij} - b_{kj} \right| \omega_j, j \in n$$

For the attribute s_j,$V_j(\omega)$ represents the total deviation of all alternatives from the others. Based on the above analysis, it can be seen that the selection of ω of the weighted vector can maximize the total deviation of all attributes to all alternatives [3]. Therefore, the objective function is constructed as:

$$\max V(\omega) = \sum_{j=1}^{m} V_j(\omega) = \sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{k=1}^{n} \left| b_{ij} - b_{kj} \right| \omega_j$$

Thus, solving the weight vector ω is equivalent to solving the following optimization model (M-1.1):

$$\begin{cases} \max V(\omega) = \sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{k=1}^{n}\left|b_{ij}-b_{kj}\right|\omega_j \\ s.t \quad \omega_j \geq 0, j \in J, \sum_{j=1}^{m}\omega_j^2 = 1. \end{cases}$$

To obtain the optimal solution, the Lagrange function is constructed:

$$L(\omega, \varepsilon) = \sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{k=1}^{n}\left|b_{ij}-b_{kj}\right|\omega_j + \frac{1}{2}\varepsilon\left(\sum_{j=1}^{m}\omega_j^2 - 1\right)$$

Find its partial derivative, and let the two partial derivatives value 0, that is, to obtain the optimal solution:

$$\omega_j^* = \frac{\sum_{i=1}^{n}\sum_{k=1}^{n}\left|b_{ij}-b_{kj}\right|}{\sqrt{\sum_{j=1}^{m}\left[\sum_{i=1}^{n}\sum_{k=1}^{n}\left|b_{ij}-b_{kj}\right|\right]^2}}, \qquad j \in n.$$

Since the above formula obtains the unitized weight vector ω^*, while the traditional weight vector satisfies the normalization constraint conditions rather than the unitized constraint conditions, in order to achieve the consistency with people's usage, the normalization of ω^* is carried out, then assuming the following:

$$\omega_j = \frac{\omega_j^*}{\sum_{j=1}^{m}\omega_j^*}, j \in n.$$

That we get:

$$\omega_j = \frac{\sum_{i=1}^{n}\sum_{k=1}^{n}\left|b_{ij}-b_{kj}\right|}{\sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{k=1}^{n}\left|b_{ij}-b_{kj}\right|}, \ j \in n.$$

In the multi-index evaluation model, it is necessary to normalize the attribute values of the scheme to eliminate the incommensurability caused by different dimensions, orders of magnitude and units. The averaging method can not only eliminate the dimensions, but also retain the variation degree information of each indicator data, to reflect the original indicator characteristics more accurately and finally obtain more scientific and accurate evaluation results. In addition, if all the collected data are objective data, it is more accurate to adopt the mean-value method to standardize the data [4].

3. The Application of the Method based on Deviation Maximization

Based on the data given in the 2004 National Mathematical Modeling Competition for College Students (hereinafter referred to as CUMCM2004-D), this paper uses MATLAB programming to provide scientific decision-making for the fair and impartial selection of civil servants by government departments [5]. The specific analysis is as follows:

Due to the needs of work, a unit directly under the company plans to recruit 8 civil servants to the public. The specific recruitment methods and procedures are as follows:

(1) Public examination: anyone under the age of 30, college degree or above, healthy people can sign up for the examination, the examination subjects include comprehensive basic knowledge, professional knowledge and administrative vocational ability test three parts, each subject full score of 100 points. According to the high and low scores of the exam, 16 people are selected to enter the second stage of interview.

(2) Interview assessment: The interview assessment mainly evaluates the comprehensive quality of the applicant's knowledge, understanding of the problem, adaptability, expression ability, etc. According to certain standards, the interview expert group will give A grade for each candidate in all aspects, from high to A/B/C/D four grades, the specific results are shown in the table below:

Table 1. Scores of written test and expert interview for civil servant recruitment

Applicant	Written test score	Scope of knowledge	Understanding ability	Strain capacity	Expressive ability
personnel 1	290	A	A	B	B
personnel 2	288	A	B	A	C
personnel 3	288	B	A	D	C
personnel 4	285	A	B	B	B
personnel 5	283	B	A	B	C
personnel 6	283	B	D	A	B
personnel 7	280	A	B	C	B
personnel 8	280	B	A	A	C
personnel 9	280	B	B	A	B
personnel 10	280	D	B	A	C
personnel 11	278	D	C	B	A
personnel 12	277	A	B	C	A
personnel 13	275	B	C	D	A
personnel 14	275	D	B	A	B
personnel 15	274	A	B	C	B
personnel 16	273	B	A	B	C

First of all, the four interview levels of the candidate's specialty are quantified and assigned as follows: A=0.5,

B=0.3, C=0.15, D=0.05. The decision matrix A_{ij} is established as follows, where x_i(i=1,2... 16) Denotes the i candidate, s_1-Written test result, s_2-Knowledge range, s_3-understanding ability, s_4-Adaptability, s_5- Expression ability. The decision matrix A is obtained by quantifying each level assignment.

Table 2. Decision Matrix A

	s_1	s_2	s_3	s_4	s_5
x_1	290	0.5	0.5	0.3	0.3
x_2	288	0.5	0.3	0.5	0.15
x_3	288	0.3	0.5	0.05	0.15
x_4	285	0.5	0.3	0.3	0.3
x_5	283	0.3	0.5	0.3	0.15
x_6	283	0.3	0.05	0.5	0.3
x_7	280	0.5	0.3	0.15	0.3
x_8	280	0.3	0.5	0.5	0.15
x_9	280	0.3	0.3	0.5	0.3
x_{10}	280	0.05	0.3	0.5	0.15
x_{11}	278	0.05	0.15	0.3	0.5
x_{12}	277	0.5	0.3	0.15	0.5
x_{13}	275	0.3	0.15	0.05	0.5
x_{14}	275	0.05	0.3	0.5	0.3
x_{15}	274	0.5	0.3	0.15	0.3
x_{16}	273	0.3	0.5	0.3	0.15

The deviation maximization decision method is adopted to make the decision, and the steps are as follows:

step1: The decision matrix is normalized by MATLAB programming, and the normalized matrix B is obtained by means method in this paper, as shown in Table 3.

Table 3. Normalized Matrix B

	s_1	s_2	s_3	s_4	s_5
x_1	1.0000	1.0000	1.0000	0.5556	0.4286
x_2	0.8824	1.0000	0.5556	1.0000	0
x_3	0.8824	0.5556	1.0000	0	0
x_4	0.7059	1.0000	0.5556	0.5556	0.4286
x_5	0.5882	0.5556	1.0000	0.5556	0
x_6	0.5882	0.5556	0	1.0000	0.4286
x_7	0.4118	1.0000	0.5556	0.2222	0.4286
x_8	0.4118	0.5556	1.0000	1.0000	0
x_9	0.4118	0.5556	0.5556	1.0000	0.4286
x_{10}	0.4118	0	0.5556	1.0000	0
x_{11}	0.2941	0	0.2222	0.5556	1.0000
x_{12}	0.2353	1.0000	0.5556	0.2222	1.0000
x_{13}	0.1176	0.5556	0.2222	0	1.0000
x_{14}	0.1176	0	0.5556	1.0000	0.4286
x_{15}	0.0588	1.0000	0.5556	0.2222	0.4286
x_{16}	0	0.5556	1.0000	0.5556	0

step2: Use formula (2.2) to calculate the optimal weight vector:

ω = (0.1871, 0.2086, 0.17600, 0.2211, 0.2072)

From the value of the optimal weight vector, it can be seen that the calculation based on the deviation maximization method considers that s_4- strain ability is the most important in the recruitment assessment, followed by s_2- knowledge area and s_5- expression ability, which is more consistent with the objective reality.

step3: Use formula (2.1) to calculate the comprehensive attribute value of the candidate x_i:

$$z_1(\omega) = 0.7833;\ z_2(\omega) = 0.6926;\ z_3(\omega) = 0.4570;$$
$$z_4(\omega) = 0.6501;\ z_5(\omega) = 0.5248;\ z_6(\omega) = 0.5359;$$
$$z_7(\omega) = 0.5214;\ z_8(\omega) = 0.5900;\ z_9(\omega) = 0.6006;$$
$$z_{10}(\omega) = 0.3959;\ z_{11}(\omega) = 0.4242;\ z_{12}(\omega) = 0.6067;$$
$$z_{13}(\omega) = 0.3842;\ z_{14}(\omega) = 0.4297;\ z_{15}(\omega) = 0.4553;$$
$$z_{16}(\omega) = 0.4147;$$

step4: Sort and prioritize the schemes:

$$x_1 > x_2 > x_4 > x_{12} > x_9 > x_8 > x_6 > x_5 > x_7 > x_3 > x_{15} > x_{14} > x_{11} > x_{16} > x_{10} > x_{13}$$

Therefore, the optimal decision scheme is $x_1, x_2, x_4, x_{12}, x_9, x_8, x_6$ and x_5. Personnel selection is carried out according to the above five attributes, and the target selection is personnel 1, personnel 2, personnel 4, personnel 12, personnel 9, personnel 8, personnel 6 and personnel 5.

4. Conclusion and Prospect

Through the brief introduction and practical application of the multi-attribute decision method based on deviation maximization, it can be seen that this decision method is an objective weighting method, avoiding the subjectivity of subjective weighting method, and can obtain scientific, standardized and fair selection results. However, the paper also ignores the advantages of subjective and combined weighting methods, and only makes decisions based on

objective weighting, which is too simple and cannot get a better scheme. Secondly, the paper does not analyze the specific cases of subjective empowerment and combination empowerment, and it is not sure whether the subjective empowerment is closer to the facts and more humane. Finally, in the decision method based on maximization of deviation, the absolute value of deviation between attributes is calculated, which brings inconvenience in calculation, and can be improved on this basis.

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