

Research on Artificial Intelligence-driven Mathematical Knowledge Mapping Construction and Personalized Teaching Path Optimization

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Abstract. At present, the application of artificial intelligence (AI) in the field of education is constantly deepening. Mathematics, as an abstract and logical discipline, is difficult to teach and the differences between students are obvious, and the existing teaching methods often fail to meet the needs of personalized teaching. Based on this, this study will combine AI technology to construct a corresponding mathematical knowledge map to optimize the teaching path and accelerate the realization of personalized teaching in mathematics. The study systematically introduces the construction process of mathematical knowledge graph from the levels of mathematical content extraction, integration and processing, visualization and display, and puts forward an effective program to realize personalized teaching of mathematics based on knowledge graph, which includes providing accurate learning resources and personalized error correction services, and utilizing knowledge graph-driven learning resource recommendation system. In order to build a personalized teaching path in mathematics, and then significantly improve the learning effect of students in mathematics.

Keywords: Artificial Intelligence; Mathematical Knowledge Mapping; Personalized Instruction; Path Optimization; Intelligent Recommendation.

1. Introduction

With the profound changes in educational concepts and the rapid development of information technology, the use of information technology to improve the quality of mathematics education has become an important issue in educational practice. The traditional mathematics teaching mode faces the outstanding problems of large individual differences among students, uneven learning progress and comprehension ability, and it is difficult to realize tailor-made teaching in the classroom. Constructing knowledge graphs can provide new ideas for solving problems, and if artificial intelligence technology is integrated into the construction process, a series of functions such as autonomous query of knowledge graphs and personalized recommendation can be realized. Especially for the subject of mathematics, the systematic and structured representation of mathematical knowledge can help teachers understand students' learning status more deeply and provide personalized learning path recommendations based on their mastery. At present, there are relatively few systematic studies on optimizing personalized teaching paths for mathematical knowledge graphs, and this study aims to fill this gap and better construct mathematical knowledge graphs based on AI technology, in order to deeply explore its feasibility in personalized mathematics teaching practice.

2. Overview Related to Knowledge Graphs

2.1 Conceptualization of Knowledge Graph Definition

The so-called knowledge map is essentially a structured semantic knowledge base, which can symbolically describe related concepts and their interrelationships. Generally, the basic unit of knowledge graph is "triad", i.e. "individual-relationship-individual", and each triad can better reflect the semantic connection between different entities. Under the perspective of graph, knowledge graph is a conceptual network, in which each node represents an entity, and the edges represent the

interrelationships between different entities. Knowledge graph construction requires extracting relevant knowledge elements from raw data and storing them in the data layer, schema layer. This process will involve the application of automated or semi-automated technical means, such as information extraction, knowledge fusion, knowledge processing, etc., as shown in Figure 1 below. Commonly used knowledge graph construction methods mainly include Semantic Wiki, XTM, ontology construction and so on.

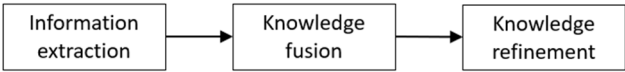


Fig 1. Steps of knowledge mapping construction

2.2 Knowledge Graph Ontology Construction

Ontology construction, as an important part of knowledge map construction, mainly describes various concepts and their interrelationships in the domain in a formalized way. The core goal of ontology construction is to better clarify the definition, hierarchy, and interrelationships of concepts in the domain, so as to lay a solid foundation for the construction of knowledge graph. Generally speaking, there are big differences in different domain ontology construction methods, and the common ones mainly involve IDEF5, AFM, skeleton method and so on. Under the application of modern technology, semi-automatic and automatic construction methods have also been gradually developed, which can effectively utilize machine learning, natural language processing and other advanced technologies to improve the efficiency of knowledge graph construction. Figure 2 below shows the basic process of ontology construction:

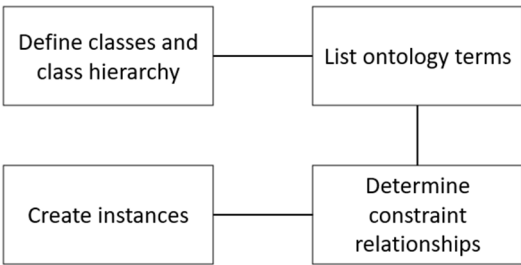


Fig 2. Basic flow structure of ontology construction

3. Research on Artificial Intelligence-driven Mathematical Knowledge Graph Construction

3.1 Math Content Information Extraction

Mathematical content information extraction is the first step in the construction of mathematical knowledge mapping, and the core task of content information extraction is to fully analyze the textbooks or learning materials of different grades and chapters, and to extract entities, attributes, and relevant connections between entities related to mathematical knowledge. In middle school mathematics, the information extraction phase needs to determine the specific concepts and theorems to be extracted. For example, in the chapter "Algebra", core concepts related to algebra should be extracted from the textbook, such as "variables", "constants", "equations", "solutions", "variables", "constants", "equations", "equations", "equations", and "solutions". ", "solution", and so on. On the basis of identifying these concepts, the interrelationships between the core concepts need to be clarified. For example, "equation" and "solution" for the "inclusion" relationship, the solution of the equation is the result of the equation, "variable" and "constant". "constants" should be opposing concepts.

Formulas are also an important part of information extraction in the construction of mathematical knowledge maps. For example, in the chapter "Geometry", the formula "Parallelogram Area

Formula" is very important, which needs to be extracted as a knowledge entity and associated with other related concepts, such as "Rectangle", "Rectangle" and "Parallelogram". Rectangles", "rectangles" and "parallelograms". When extracting the formula, it is necessary to focus not only on the formula itself, but also to fully understand the elements of the formula and their mathematical relationships with each other, for example, the "base" and "height" of the formula are indispensable variables, which are closely related to the geometric features of the actual figure.

In practice, information extraction can fully rely on natural language processing technology, based on word separation, named entity recognition, relationship extraction and other technical means to achieve. With specific reference to the problem of "solving algebraic equations", combining with text analysis, we can extract the entity of "algebraic equations" and the attribute of "solution", and identify the relationship between the two, and ultimately form a The triad (algebraic equation-solution-solution) in the knowledge graph is finally formed. Table 1 below shows an example of knowledge fusion in the construction of middle school mathematics knowledge mapping.

Table 1. Examples related to knowledge fusion in mathematical knowledge mapping construction

Concept	Relationship	Concept
Algebraic concept	Incorporate	Algebraic formula concept
Plane geometry	Incorporate	Polygon concepts, circle concepts, view concepts
Polygon concept	Incorporate	Rectangle Concept, Triangle Concept, Square Concept
Polygon concept	Incorporate	Calculate the perimeter of a polygon, calculate the area of a polygon
...	...	...
The concept of a function	Incorporate	Images of functions, function optimization, function solving

3.2 Integration of Math Content Knowledge

In the educational level of middle school mathematics, knowledge fusion involves the effective integration of different mathematical concepts, and also needs to focus on resolving ambiguities, redundancies, and relational contradictions among entities. Based on this, artificial intelligence technology, especially the semantic similarity calculation method in natural language processing (NLP), can be widely used in the optimization of mathematical knowledge graph construction, and the specific common language similarity calculation method is as follows:

$$\text{Sim} (A, B) = \frac{\log_p \text{Com} (A, B)}{\log_p \text{description} (A, B)} \quad (1)$$

In the above equation $\text{Sim} (A, B)$ represents the similarity between A and B, while $\text{Com} (A, B)$ represents the commonality between the knowledge points and $\text{description} (A, B)$ represents the personality.

In the textbook of junior high school mathematics, many concepts and terms are repeatedly mentioned in different chapters, but their meanings or specific scenarios of use are somewhat different. For example, the concept of "equation" is interpreted differently in different types of math problems. In junior high school mathematics, equations can be algebraic equations, linear equations, quadratic equations and other types of equations, and different types of equations involve different solutions, techniques, steps and so on in specific math problems. This requires the use of semantic similarity computation methods in natural language processing technology to recognize and align the entities of "equations" in different contexts.

In practical applications, the effective combination of AI technology, especially machine learning and natural language processing technology, can realize automated knowledge fusion. In the construction of middle school mathematics knowledge mapping, the AI system will automatically determine the actual relationship between entities with the help of pre-trained semantic models and a

large amount of mathematics textbook data, and merge entities based on semantic similarity. For example, for "parallelogram" and "rectangle", the AI system can fully combine their geometric characteristics, such as parallel sides and their application in mathematical formulas, to determine whether it is possible to merge them, thus further optimizing the efficiency of knowledge mapping storage and query.

3.3 Mathematical Content Knowledge Processing

Quality assessment focuses on determining whether new knowledge meets the knowledge base criteria. In junior high school mathematics, the concept of fraction has different definitions and applications in different chapters, for example, fractions can be used as ratios, proportions, or numerical magnitudes. If the AI system mistakenly treats the concept of "fraction" in different contexts as identical in the integration process, it will result in redundancy and errors in the knowledge base. Therefore, the quality assessment function needs to detect the semantic similarity and determine the match between the new knowledge and the entities in the existing knowledge base. The smaller the quality level J , the better the fit between the knowledge and the existing knowledge base.

If the new pedagogical knowledge is the concept of "fractional simplicity", and the existing knowledge base already contains "basic properties of fractions" and "addition and subtraction of fractions", the AI system can calculate the semantic similarity between "the approximation of fractions" and the existing knowledge, and determine whether it is related to the concept of fractions in the knowledge system. If the similarity is high and in line with the mathematical laws, it can be recognized as qualified based on the quality assessment function and added to the knowledge base. The specific quality assessment function is as follows:

$$J = \frac{1}{2} \sum_{i=1}^m (y_i - Y)^2 \quad (2)$$

In the above equation J is the quality level, y_i denotes the obtained knowledge and Y is the supervised corpus. In addition, knowledge base hierarchization is more important based on the degree of similarity and association between entities. This allows setting different levels to present mathematical knowledge in a structured and hierarchical form. For example, the knowledge of equations can be effectively categorized according to different types (linear equations, quadratic equations, etc.), and subdivided under each type to correspond to the solutions, application scenarios, etc. The AI technology will combine the similarity and classification relationship of mathematical knowledge to automatically categorize and sort them to form a clear, hierarchical knowledge structure system. As shown in Table 2 below, it is an example of knowledge processing for middle school math knowledge mapping.

Table 2. Example of knowledge processing for knowledge mapping construction

Concept	Relationship	Concept
Algebraic concept	Level 1 contains	Concept of Equation, Concept of Algebraic Equation
Algebraic concept	Level 2 contains	Concept of Equation, Concept of System of Equation
Probability and statistical concepts	Level 1 contains	Representation of data, representation of data, collection of data, processing of data
Plane geometry	Level 1 contains	Polygon concepts, circle concepts, view concepts
The Circular Concept	Level 1 contains	The concept of central symmetry, the concept of axial symmetry
...	...	...
Plane geometry	Level 3 contains	Isosceles triangle concept, equilateral triangle concept, right triangle concept

3.4 Visualization of Mathematical Knowledge Graphs

After the mathematical knowledge processing is completed, the definition of each knowledge point in the knowledge graph and its hierarchical structure division is realized accordingly. Based on the Neo4j engine, Java language can be implemented in Windows knowledge map visualization, see Figure 3 below.

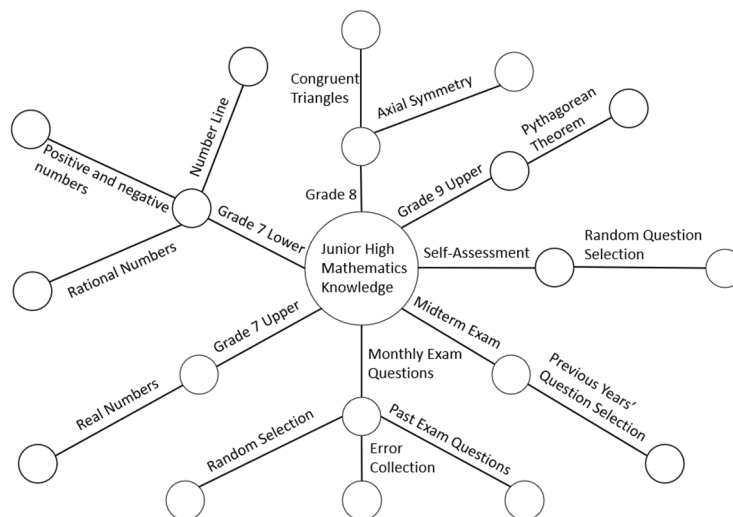


Fig 3. Visualization of knowledge mapping in middle school mathematics

As shown in the figure, after visualizing the mathematical knowledge graph, clicking on each entity node can not only display the content that has been set, but also crawl similar teaching resources on the Internet to realize the systematization of mathematical knowledge. And after adding new knowledge content, the knowledge graph can give automatic reasoning ability to assign the new knowledge to a certain branch, realizing the automatic updating and development of new knowledge.

4. Research on the Optimization of Personalized Teaching Paths Based on Mathematical Knowledge Mapping

4.1 Personalized Learning Needs Analysis and Path Planning

In junior high school mathematics teaching, due to the significant differences in students' mathematical foundation, thinking styles, and learning progress, it is particularly important to analyze students' personalized learning needs. Based on mathematical knowledge mapping, the analysis of personalized learning needs to fully assess the students' existing knowledge mastery, identify students' knowledge weaknesses, cognitive bias. For example, in the learning process of the chapter "Fractions", some students have difficulties in understanding "addition and subtraction of fractions", and some students do not have a thorough understanding of "simplification of fractions". Based on the in-depth analysis of students' pre-test data and learning feedback, the AI system can identify in real time which knowledge points students have learning barriers or knowledge blind spots in their learning.

After accurately identifying students' learning needs, the mathematical knowledge graph can give full play to its advantages and help the AI system plan personalized learning paths for students. Take the chapter "Equations" as an example, for students with a good foundation, the knowledge map can be used to directly plan the path of higher-order "equation solving skills" or "complex application problems"; for students with a weak foundation, the knowledge map can be used to directly plan the path of "linear equations" or "complex application problems". For students with a weak foundation, they can supplement and strengthen their theoretical knowledge based on the basic knowledge of

"linear equations" and "properties of equations" in the knowledge map, so as to gradually build up a complete system of solving equations.

In path planning, Math Knowledge Graph can help students better identify key learning nodes, and also provide flexible cross-chapter and cross-knowledge point learning suggestions. In the learning of "function" related content, if students lack knowledge of "proportion", the AI system will recommend students to review the knowledge of proportion first, and then further study function on this basis, to ensure that students progress step by step and avoid subsequent learning difficulties due to the lack of solid foundation. The AI system will recommend students to review the knowledge of proportion first.

4.2 Knowledge Graph-Driven Learning Resource Recommender System

In the process of personalized teaching based on mathematical knowledge mapping, learning resource recommendation system plays an important role. Take the middle school math module "Geometry" as an example, in the process of learning plane geometry, students often encounter concepts such as "the sum of the interior angles of triangles", "the nature of parallel lines" and "the calculation of area and volume". The concepts of "area and volume calculation" are often encountered by students in the process of learning plane geometry. With the help of knowledge mapping, the system will analyze students' actual mastery of these knowledge points in real time, and find out that students may have knowledge blind spots and understanding deviations. If a student's understanding of "the properties of angles and parallel lines" has deviations, the knowledge map can accurately identify the student's learning weaknesses based on his/her learning progress and feedback information.

Based on this analysis process, the Learning Resource Recommendation System will automatically recommend learning resources related to "Properties of Parallel Lines" for students. For example, the system can recommend a basic video course to explain in detail the connection between parallel lines and angles, helping students to understand and consolidate their knowledge. At the same time, targeted practice questions can also be pushed, so that students can deepen their understanding by doing the questions. In the process of learning resources selection, the recommendation system will accurately push the corresponding teaching videos and courseware according to the relevance of the knowledge graph nodes, and also recommend different forms of learning materials according to students' learning habits and preferences, which commonly include interactive question banks, graphical display tools, case studies, etc.

5. Conclusion

In summary, the construction of mathematical knowledge maps helps to optimize the implementation of personalized teaching paths. In the study, mathematical knowledge mapping is precisely constructed by extracting, fusing, processing and visualizing mathematical knowledge, which provides students with tailor-made learning paths. The study can provide new perspectives and specific methods for the effective implementation of intelligent teaching in the field of education, which can promote the efficient allocation of teaching resources and accelerate the process of personalized learning for students.

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