Time Trial Pacing Optimization Strategy Based on Muscle Fatigue and Aerobic and Anaerobic Respiration

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Abstract: In the time trial, the rider should organize his/her power wisely so that he/she can minimize the time for the entire contest. In order to design the optimal power output strategy for different riders in different road situations, we build a model to describe the rider based on the aerobic and anaerobic respiration and muscle fatigue and recovery process. Then we use the greedy algorithm to obtain the optimal pacing strategy for the rider on the plane road. Then we use the image processing to get the road data in UCI and Tokyo Olympics and use modify the model so that it can give the optimal pacing strategy on the roads with turns and slope.

Keywords: Aerobic and anaerobic respiration, Image processing, Muscle fatigue and recovery.

1. Introduction

Recently, it has been widely studied in sports science about the improvement of the grades for time trial. The optimal pacing strategy on flat straight course has been studied by De Koning et al\cite{1}, where they say the optimal strategy is to speed up to a velocity a little bit lower than the critical velocity and keep it to the end, which is in agreement with Jenny de Jong’s\cite{2} study. Andrea Zignoli\cite{3} established a detailed 3D model and system to study this problem, and got the optimal pacing strategy in sharp turns and cornering. However, the optimal pacing strategy in complex weather and for team time trial has not been well studied yet.

In this paper, we establish a model based on S. Alireza Fayazi’s\cite{4} muscle strength fatigue and recover model, and modify it with the aerobic and anaerobic respiration model put forward by Faraz Ashtiani\cite{7}. We analyze the optimal pacing strategies time trial specialist and climber for female and male rider in flat straight course. And we find that our model can get good pacing strategies for different rider.

In Section 2.5, we study the Tokyo Olympics and the 2021 UCI individual time trial with complex terrain such as sharp turning slopes. Finally, the optimization model calculates that the shortest time of the Tokyo individual time trial and the UCI individual time trial is 3304s and 3450s, which is close to the time of 3304s and 2820s used by the champions of the two events. The effects of gender and runner type on individual time trial performance were also investigated. Compared to climbers, specialist cyclists have less explosive power and lower early speed, but have more endurance, can maintain a higher speed in the second half, and complete the entire the whole track in less time. In addition, the output power and speed of female players are lower than that of male players, so their performance is generally lower than that of male players.

Then, under complex weather conditions (taking the wind direction as an example), the sensitivity of our model is analyzed, and the wind direction and wind speed will have a greater impact on the final results of the model. Then based on the model, the optimal time-trial physical output power arrangement measurement for team competition is given.

After that, based on the wind resistance attenuation received by different positions in the team competition team proposed by Bert Blocken et al.\cite{8}, we extend the above model to the team competition situation. The optimal strategy arrangement method and the computer solution method are discussed, and the assumption of the team member position arrangement method is given in combination with the actual situation.

2. Organization of the Text

2.1. Problem Background

The bicycle road timing race originates from European and is a race at which athlete starts alone at specified intervals, their rank is based on the timing of their final arrival to the finish. Because the athletes have limited physical strength, so reasonable plan to use physical strength to achieve the shortest time is needed. The complex weather and landforms also pose great challenges to the pacing strategy.

The left picture in Fig1 shows the track of the Tokyo Olympic time trial. On the way, the track is not always straight, most of which are arcs with a certain curvature. The right shows the elevation of the men’s time trial course, showing that most of the course is ups and downs.
2.2. Method

The question requires us to establish a strategy for describing optimal pacing strategy used in a single time trial with sharp turns and slopes in complex weather conditions. Our approach includes the following:

- Based on muscle fatigue and recovery, we establish the muscle maximum isometric force model to describe its decay and rest.
- We calculated the optimal pacing strategy for the men’s/women’s time trial at the Tokyo 2021 Olympics considering the sharp turns and elevation.
- Taking the wind direction and velocity as an example, we study the effect of complex weather conditions on the model.
- Considering a range of bias, we consider the effects of the derivation of the rider when performing the optimal strategy.
- Using the single time trial model, we initially solved the optimal pacing strategy strategy for the team time trial.

2.3. General Assumptions and Model

In order to simplify the model, we made assumptions and justifications as follows:

Assumption 1 The rider and the bicycle can be considered as a whole part and studied as point mass, ignoring the internal relationships

Justification Compared with the entire track, the size of the rider and the bicycle is negligible. Also, ignoring the interaction between them can greatly simplify the model while without too much loss of precision. [4,6]

Assumption 2 Air density is approximately constant at all altitudes

Justification Since the track altitude changes in the Tokyo Olympics and UCI are not very obvious, we approximate that air density is at about 1.29m/cm3

Assumption 3 The frictional force received by a bicycle is static friction

Justification The direction of force on the two wheels of the bicycle during exercise is relatively complex. When the bicycle is in motion, the front wheel receives the rolling friction force backwards, and the rear wheel receives forward rolling friction. When sliding, both wheels receive backward sliding friction, so we use \( \mu m \cos(\theta) \) to simplify the model by approximating the frictional resistance received by the bicycle.

Assumption 4 The anaerobic power level is sufficiently high to get to a velocity that can be maintained indefinitely at critical power.

Justification The rider for the time trial is well trained, usually he/she can finish the whole trial. In this model he/she just wants to optimize the pacing strategy to get a better score. So his/her anaerobic power level is qualified.

Assumption 5 Muscle fatigue does not influence the linearity of force-velocity relationship.

Justification The force-velocity relationship is determined by the Newton’s second law, the muscle fatigue will only affect the maximum force the rider can apply.

2.4. Human Body Mechanism Model

2.4.1. Model Establishment

For a bicycle rider power output curve, we need to build the relationship of its own energy changes with the forward driving force at first. According to the reference[4], we know that muscle is in a certain regular contraction and expansion stage, which needs ATP(adenosine triphosphate) as the stable supply. And ATP’s production rate has is related to oxygen supply. When the oxygen supply is in short at the state of high intensity exercise, the muscle needs a certain amount of anaerobic ATP when in hypoxia to ensure the high intensity of energy consumption. But what accompanies the anaerobic state is the production and accumulation of lactic acid, making the muscle fatigue.

In this paper, we temporarily ignore the complex dynamic process of fatigue accumulation, only consider the maximum power generated under the fatigue limit affected by dynamic process of muscle fatigue and recovery. Then we build the corresponding mathematical model.

The maximum force muscle can exert when rest is called MVC(Maximum Voluntary Contraction). When the \( F_{max,iso} \) (maximum isometric force) is applied continuously, the \( F_{max,iso} \) will decline exponentially over time. More generally, \( F_{max,iso} \) can be described with the following relationship[5] when the \( F_{iso} \) (isometric force) is applied.

\[
\frac{dF_{max,iso}(t)}{dt} = -kF_{max,iso}(t) \frac{F_{iso}(t)}{MVC}
\]

Where \( k \) is a constant varying with each individual. When the continuous force is applied, \( F_{max,iso}(t) \) is smaller than the MVC, \( F_{max,iso}(t) \) is the currently applied isometric force. However when \( F_{max,iso}(t) = 0 \), as described in formula (1), \( F_{max,iso}(t) \) should be the previous value without any change. In our common sense, this is unreasonable because when no external force is applied, the muscles are resting, \( F_{max,iso} \) should gradually recover to the MVC.
[6] points out that the recovery of $F_{\text{max,iso}}$ should also approximately present the exponential relation with time:

$$\frac{dF_{\text{max,iso}}(t)}{dt} = R \left( MV C - F_{\text{max,iso}}(t) \right)$$  \hspace{1cm} (2)

Where R is the recovery coefficient, a constant. It is also noted in [7] that when there is a set of muscle fibers in the activation mode, some are tired and some are recovering. In other words, fatigue and recovery occur simultaneously. So, in order to describe the relationship between fatigue and recovery, we use a linear combination of (1) and (2), as follows:

$$\frac{dF_{\text{max,iso}}(t)}{dt} = -k F_{\text{max,iso}}(t) \frac{F_{\text{max,iso}}(t)}{MV C} + R \left( MV C - F_{\text{max,iso}}(t) \right) \hspace{1cm} (3)$$

When a person continuously exerts the maximum isometric force, that is, $F_{\text{iso}} = F_{\text{max,iso}}$ at this time we can obtain the applied isometric force from (1). And we name it as $F_{\text{th,iso}}$. So: $F_{\text{th,iso}} = MV C \cdot R \left( 1 + \sqrt{1 + 4k/R} \right)$  \hspace{1cm} (4)

(4) We can assume that when the applied force is $F_{\text{th,iso}}$, there will be no physical loss, which means the fatigue counteracts the recovery. Also, to describe the rider's fatigue, it is known that when the $F_{\text{max,iso}} = F_{\text{th,iso}}$ the maximum applied isometric force is the critical value, the rider is the most exhausted. When $F_{\text{max,iso}} = MV C$, the rider is resting and completely untired. Thus, we define the fatigue degree as:

$$\text{SoF}(t) = \frac{MV C - F_{\text{max,iso}}(t)}{MV C - F_{\text{th,iso}}}$$  \hspace{1cm} (5)

$0 \leq \text{SoF} \leq 1$ We consider that the force applied by the rider's is equal to the force received by the pedal $F_{\text{rider}} = F_{\text{iso}}$. By Newton's second law, the kinematic equations for cycling can be established as:

$$m_t \frac{dv_c}{dt} = \eta_g \frac{C_p}{r_g} F_{\text{rider}} - \frac{1}{2} C_p A v^2 - m_t g (\mu \cos(\theta) + \sin(\theta)) - F_b$$ \hspace{1cm} (6)

$m_t = m_b + m_r$ is the total mass, $m_b$ is the mass for bicycle and rider, $\mu$ is Rolling Resistance Coeff, $A$ is Frontal Area, $r_g$ is Wheel radius, $l_c$ is Crank arm length, $\eta_g$ is Gearbox efficiency, $\theta$ is the slope of the road, $F_b$ is wheel friction braking force, and we set it as 0 in this model. Then the current output of the power can be calculated as:

$$P_{\text{rider}} = \frac{F_{\text{rider}} \eta_g l_c v}{r_g s_w}$$ \hspace{1cm} (7)

However, the Faraz Ashtian[7] points out that this modeling assumes that the power output is constant, which is trivially not true. The maximum value of the rider’s output power should be affected by its aerobic and anaerobic respiration. They point out that there is a CP (critical point) for breathing, when the output power below the point the rider recovers and stores energy, and consumes energy the stored energy when above the. The meaning of the CP is shown in the Figure 2. Its storage energy is can be described by the following relationship[7]:

$$\frac{dW_{\text{rem}}}{dt} = CP - P, P > CP$$ \hspace{1cm} (8)

, where $W_{\text{rem}}$ is the remaining(stored) energy. And the maximum value of it is AWC, which is related to the rider’s physical fitness. $T_{\text{rec}}$ is the delay for the restoring the remaining energy.

And they also indicate that the maximum output power $P_{\max}$ has such relation with the residual energy:

$$P_{\max}(t) = aW_{\text{rem}}^2(t) + bW_{\text{rem}}(t) + CP$$ \hspace{1cm} (9)

where a,b are constant. According to the timing race knowledge, both the speed and the output power have upper limits. At this point, the purpose becomes getting the shortest time for a given course. Combined with the above conditions, then the above model can be converted to the following planning problem:

$$\min \left\{ \int_{t_0}^{t_f} dt = \int_{x_0}^{x_f} \frac{dx}{v(t)} \right\} \hspace{1cm} (10)$$

$0 \leq P_{\text{rider}}(t) \leq P_{\max}$

$0 \leq F_{\text{rider}} \leq F_{\text{max,iso}}$

$0 \leq \text{SoF}(t) \leq 1$

$0 \leq v(t) \leq v_{\max}$

$0 \leq W_{\text{rem}}(t) \leq AWC$

Figure 2. The meaning of the CP

2.4.2. Model Solving

The optimization problem is an optimal control problem, and the common methods include the gradient method, the dynamic planning method, and the greedy algorithm. Here we adopt the method of the greedy algorithm to solve the problem. For the selection of the constants, we referenced [6][7]. Figure 3 shows the solution flow diagram. We use the greedy algorithm to search for the $F_{\text{iso}}$, and its pseudocode is as follows:
2.4.3. Result and Discussion

According to the mathematical model and solution algorithm described above, we can show the basic qualities of a rider with parameters and make his/her power profile. According to the Cyklopedia’s definition for the profile, we used four common parameters: sprint abilities (5s), anaerobic capacity described by 1 minute maximum power, 5 minutes to tell us about VO2 max capability and 20 min to describe FTP. All four above parameters can be obtained from the maximum power output curve calculated on the straight plane course.

In addition, we included two parameters, AWC and CP, as competence properties of an individual’s power profile. We have developed the power profile for time trail specialist and climber for both female and male, which is shown in the appendix. First of all, we will make some analysis of the output on the straight plane course.

2.5. Real Track Model

2.5.1. Plane and Straight Lap Model

To make it simple, we studied the model in a flat straight track. Here we set the length of the straight track to 4km, CP = 400W, referring the parameter setting in [6][7] and using the greedy algorithm method in the previous section, the simulation results are in the Figure4:

From the results from Figure 4, we can see that the speed rises rapidly and maintains at a relatively large level, and the output power also rises rapidly, then it quickly drops and maintains at a relatively large level. Both maximum and current applied force leveled off a value after attenuation. However, the current applied force decays rapidly with partial fluctuations, which is caused by the delayed replenishment of residual energy by aerobic respiration. This is because at the beginning of the trial, rider needs output large force in order to quickly improve the speed, when the speed is stable, the force needed is not that much big, so the power will then fall quickly and then level off.

So in the flat straight track trial, the optimal pacing strategy is to quickly accelerate to a large speed and then try to keep this speed till the end of this trial. This strategy is also called the full sprint method, which corresponds the previous research. What makes it different is that, We introduce an aerobic and anaerobic breathing model to limit the output power. As is can be seen from Figure4, in the full sprint strategy, the applied force is basically not bounded by the maximum applied force, speed is also not limited by the maximum speed. The main limit on speed (that is, shortest
time requirement) is reflected in the CP. Which means, the current power should be kept at the CP level as far as possible to maintain an efficient physical output. Also the simultaneous storage energy should be kept at a relatively large value to support the maximum available power slightly higher than the CP to maintain the optimal remaining energy stored using aerobic respiration during anaerobic respiration.

Therefore, for time trial rider on straight flat roads, having a good CP is crucial to improve their grades.

2.5.2. Practical Course Model

In order to optimize the optimal physical output strategy in different tracks, the model needs to be applied according to the characteristics of the course, making the output results have practical significance. From the route map and altitude plane in Figure 5, it can be seen that the 2021 UCI World Championship time trial track is relatively flat as a whole, but there are certain sharp turns, while the track of the men’s time trial at the 2021 Tokyo Olympics has many sharp turns and slopes.

![Figure 5. The map of Tokyo Olympics course and UCI course](image)

For the consideration of the slope, according to the altitude information of 2021UCI found on the official website, the altitude is maintained within the range of 2~8m and the change is small, so the influence of the slope can be ignored. However, in the track of the 2021 Tokyo Olympics, it can be found that the slope changes more obviously. Due to the lack of altitude and distance data, we approximate the slope of the track by image processing.

After processing the map in the way of color feature extraction, grayscale, hole filling and binarization with the help of OpenCV lib, the findcontours function can be used to get a clear track outline, and the pixels of interest can be selected after screening, which can estimate the approximate slope distribution of the track, shown in the figure 6. The modeling of the slope has been discussed in the formula 4.1.

![Figure 6. Result of image processing](image)

2.5.3. Sharp Turn Model and External Wind Model

For the consideration of sharp turns, compared with straight roads, the friction between the wheels and the ground increases due to the centripetal force when the rider passes through the corners. Slippage will affect the performance of the race and even cause accidents, so the rider needs to actively reduce the speed when turning, and reasonably distribute physical energy to keep himself at a suitable speed.

As what is shown in 7.

![Figure 7. Result of image processing](image)

The rider is mainly subject to frictional resistance in the horizontal direction when cornering, so the power is generated by the rider himself, and the combined force of the two needs to be able to provide enough centripetal force to achieve the turn. \( F_{in} \) represents the centripetal force of the rider, the radius of the curve, \( F_{f} \) is the frictional force between the rider’s wheel and the ground. The speed of the rider is \( v \). The centripetal force and the rider’s own power should be less than the frictional resistance between the
ground and the wheel, so we can get:

$$\sqrt{F_n^2 + \left(\frac{m\omega}{r}\right)^2} \leq \mu mg \quad (11)$$

$$\sqrt{\left(\frac{dv}{dt}\right)^2 + \left(\frac{v}{r}\right)^2} \leq \mu g \quad (12)$$

It can be seen from the above formula that the larger the radius of the curve, the greater the maximum speed allowed for turning, and the friction coefficient is also an important factor. In different weather conditions, it is necessary to pay attention to the arrangement of the cornering speed.

2.5.4. External Wind Resistance Model

A large part of the cyclist’s energy is used to overcome wind resistance to do work during the riding process, so the wind direction and wind strength will have a certain impact on the cyclist’s performance, so the above model is modified to add the influence of wind, as shown in Figure 15.

![Figure 8. Result of image processing](image)

We first establish a Cartesian coordinate system. The east direction is the initial axis, and the counterclockwise direction is the positive direction. Let \( \beta \) be the direction of the wind, \( u_0 \) is the wind strength, \( \alpha(s) \) represents the direction of the rider at the \( s \) position, then the relative wind speed of the rider can be expressed as:

$$u = u_0 \cdot \cos(\alpha(s) - \beta) \quad (13)$$

After modifying the original balance equation, due to the influence of wind speed, the wind resistance of the rider will change accordingly, just modifying the partial equation of wind resistance, and then we get:

$$m\frac{dx}{dt} = \frac{\rho A}{2} F_{wind} - \frac{1}{2} \rho A (v + u)^2 - m\gamma(\mu \cos(\theta) + \sin(\theta)) - F_e \quad (14)$$

Since the road conditions of the actual track are too complex to obtain the actual orientation angle of each section of the track, the track is divided into corresponding sections according to the turning situation in Figure 7, and the orientation angles of these sections of the track are approximately calculated.

Figure 9 and Figure 10 approximately reflect the direction of the track. There are obvious differences between the tracks of the 2021 Tokyo race and the 2021 UCI race. The Tokyo track has many curves, and there are four turns with a large radius, the 2021 UCI track has fewer corners, and the turning radius of the corners is small, mainly sharp turns. In order to be closer to the actual situation of the competition that day, we checked the weather conditions on the day of the competition. The day of the 2021 Tokyo competition was a Grade 2 southwesterly wind. It can be determined that the wind speed \( u_0 \) is 2.1 m/s, and the wind direction is 73°. In 2021 UCI Brussels, Belgium, the southeast wind was level 2, so the wind speed on the day can be set to 1.9 m/s, and the wind direction is 134°, and the individual competition strategy is re-solved according to the modified model.

2.5.5. Corrected Model Results

After considering the effects of wind resistance and cornering, the revised model is used to solve the optimal strategy for the 2021 Tokyo individual race and the 2021 UCI track. Figures 12 and 13 respectively show the riding states of the time trial professional riders at different positions on the Tokyo track and the UCI track under the optimal strategy.

It can be seen from the figure 11 that the speed change of the Tokyo race in 2021 is more obvious, while the speed fluctuation of the 2021 UCI track is not large. This is because the terrain of the Tokyo track changes greatly in altitude and there are many curves, so the rider’s speed is difficult to stabilize in the sharp turning section. It is necessary to actively reduce the driving force and output power of the rider.
to reduce the speed, so that the rider can smoothly pass the
curve. On the way up the mountain, due to the effect of gravity,
the rider needs to increase the driving force and output power
to maintain the speed at a high level. During the downhill
process, the rider needs to reduce the driving force and
output power to save physical energy, and save physical
energy for subsequent acceleration to achieve optimal time. It
can be seen from the figure that the Climber has stronger
explosive power than the Specialist but lacks endurance. The
Specialist can reasonably plan its own physical strength and
is suitable for long-distance riding, so the Specialist’s
performance on different tracks is better.

Figure 11. The optimal solution of Tokyo course

Figure 12. The optimal solution of 2021 UCI course
Table 1. Optimal results for different riders

<table>
<thead>
<tr>
<th>The type of Cyclist</th>
<th>The optimal time in Tokyo course(s)</th>
<th>The optimal time in UCI course(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specialist(man)</td>
<td>3450</td>
<td>3630</td>
</tr>
<tr>
<td>Climber(man)</td>
<td>3772</td>
<td>3818</td>
</tr>
<tr>
<td>Specialist(woman)</td>
<td>3998</td>
<td>4040</td>
</tr>
<tr>
<td>Climber(woman)</td>
<td>4198</td>
<td>4252</td>
</tr>
</tbody>
</table>

The results of the query show that the 2021 Tokyo Men’s Individual Time Trial Champion takes 3304s, which is close to the optimal time of 3450s planned by the model, and the 2021 UCI Men’s Individual Time Trial Champion is 2820s, which is close to the optimal time of 3450s planned by the model, so the model has better fit. It can also be seen from the table that Climber’s performance is generally lower than that of Specialist, and the best performance of male riders is generally higher than that of female riders. The overall quality is generally higher than that of female riders, with stronger explosive power and longer lasting endurance, so the riding speed and maximum driving force of male riders are higher than that of female riders, so it takes less time for male riders to complete the full track.

3. Sensitivity Analysis

In practice, the rider is not able to pace according to the optimal strategy completely. So here we analyze the effect of the rider on the final result due to its deviations from the target power distribution. For simplicity, we analyze the cyclist specialist’s pacing strategy on a 1km flat straight road. For the road with complex conditions, it is similar with this.

Then we assume that each time the rider performs the optimal $F_{iso}(t)$ there is a deviation of the maximum percentage $\Delta$, which means that each time $F_{iso,\text{optimal}}(t)$ is obtained, to get $F_{iso}(t)$, the rider will randomly choose a number in $F_{iso,\text{optimal}}(t) \times [1 - \Delta, 1 + \Delta]$. And our strategy is a real-time strategy. That is to say, if the rider deviates from the optimal strategy during the ride, our strategy will be adjusted for the bias according to the previous step. The results obtained are showed in Figure 15.

![Figure 15. Optimal simulation results for derivation of 0%, 5%, 10%, 15%](image)

As is depicted from the Figure 15, when the deviation is not small, our model can correct the deviation of the rider so that it still approximately satisfies the optimal strategy. We also note that, if the starting derivation of $F_{iso}$ is large, our model will fail. The rider needs to meet our optimal strategy in the initial right sprint stage as he/she could. Generally speaking, our model has good fault tolerance for deviation.

4. Conclusion

In summary, we build a model based on the muscle fatigue and aerobic and anaerobic respiration to describe the relationship between the rider’s force and power. Then we use the greedy algorithm to find the optimal power output strategy for the rider. Also, we analyze the effect of the slope and turn which has not been studied yet. Based on our model, we can design different power output strategy for different riders based on different situations.

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References


