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Motion Path Study Based on Isometric Solenoids and Discrete Simulation

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Abstract: This paper focuses on the position, speed change and collision detection problems during the traveling process of the bench dragon from both physical and mathematical perspectives. Firstly, based on the equidistance solenoid equation and the mechanical arm model, this paper iteratively recursively starts from the dragon head and sequentially solves the position of each section of the bench in the traveling process. Secondly, this paper combines the velocity analysis of the rigid rod and the nature of isometric solenoid, and also solves the position of each section of the bench in the traveling process sequentially in the form of iterative recursion. Finally, this paper discretizes the disk-in trajectory and carries out collision detection, so as to obtain the time, position, velocity and other information at the termination moment. The study shows that the model can effectively solve the complex optimization problem in the marching process of the dragon dance team.

Keywords: Isometric Solenoids, Iterative Recursion, Rigid Body Rods, Discrete Simulation.

1. Introduction

The purpose of this paper is to study in depth the dynamics of the bench dragon during its traveling process from the physical and mathematical perspectives, so as to analyze the key factors such as the position, velocity change and collision detection during the traveling process of the bench dragon [1,2,3]. In this paper, firstly, based on the equidistant solenoidal equation and robotic arm model, the position of each section of the bench during the traveling process is solved by iterative recursive method [4,5]. Secondly, this paper combines the velocity analysis of rigid-body rod and the nature of isometric solenoid to further solve the position of each section of the bench during the traveling process. The introduction of the rigid-body rod model allows us to analyze the dynamic characteristics of the bench dragon during the traveling process more accurately, including the changes of velocity and acceleration [6,7]. Finally, in this paper, the disk-in trajectory is discretized and collision detection is carried out to obtain the key information such as time, position and velocity at the termination moment. Through the discrete simulation technique, we can simulate various situations that may be encountered by the bench dragon in the actual traveling process, so as to optimize the performance scheme and avoid the occurrence of collision and other unforeseen situations [8,9].

2. Modeling of Bench Dragon Travel Process Based on Isometric Solenoidal Equation and Recursive Method

Through field research, we collected data on a bench dragon, which consists of 223 sections of benches, of which the first section is the dragon's head, the next 221 sections are the dragon's body, and the last section is the dragon's tail. The length of the dragon's head is 341 cm, the length of the dragon's body and tail is 220 cm, and the width of all the benches is 30 cm. each bench has two holes, the diameter of which is 5.5 cm, and the centre of the hole is 27.5 cm away from the head of the nearest bench. the two adjacent benches are connected by a handle, and the two benches are connected by a handle, and the two benches are connected by a handle.

According to the research, the dragon dance team coils in clockwise along an isometric solenoid with a pitch of 55cm, and the center of each handle is located on the solenoid. The traveling speed of the handles in front of the dragon head is always 1 m/s. After consulting the information, the thread belongs to the isometric thread and the general expression can be given by the following equation:

$$\rho = \rho_0 + b\theta \tag{1}$$

where, ρ_0, θ are the parameters to be solved. After that, firstly, from the known parameter conditions, the pitch is 55 cm, θ >0, the isometric solenoid with counterclockwise positive direction centered at point O is established, and the parametric equation of the solenoid is:

$$\rho = \frac{11}{40\pi}\theta \tag{2}$$

Since the subsequent modeling process requires the use of the arc length formula in isometric solenoids, the arc length formula is derived by line integration, and the derivation process is shown below, which takes a combination of differential and integral methods for the derivation:

$$ds = \sqrt{(a\theta d\theta)^2 + (ad\theta)^2} = \sqrt{(a\theta)^2 + a^2}d\theta \tag{3}$$

$$\int \sqrt{A + Bx + Cx^2}dx = \frac{(2Cx+B)\sqrt{A+Bx+Cx^2}}{4C} + \frac{4AC-B^2}{8C}\int \frac{dx}{\sqrt{A+Bx+Cx^2}} \tag{4}$$

Integrating both sides simultaneously and applying the integral formula yields:

$$\frac{dx}{\sqrt{A+Bx+Cx^2}} = \frac{1}{\sqrt{C}}\operatorname{arcsinh}\left(\frac{2Cx+B}{\sqrt{4AC-B^2}}\right) \tag{5}$$

The final derived equation for the arc length of an isometric solenoid is as follows:

$$L(\theta) = \frac{b^2 \cdot \theta + \rho_0 \cdot b}{2 \cdot b^2} \cdot \sqrt{\rho_0^2 + b^2 + 2 \cdot \rho_0 \cdot b \cdot \theta + b^2 \cdot \theta} + \frac{|b|}{2} \cdot \operatorname{arcsinh}\left(\frac{\rho_0 + b \cdot \theta}{b}\right) \quad (6)$$

In order to obtain the speed as well as the position information of each node of the bench dragon under each moment, we regard each boarding as a robotic arm, and regard the handle as a joint, to construct a more intuitive robotic arm model, and the simulation model is shown in Figure 1 below.

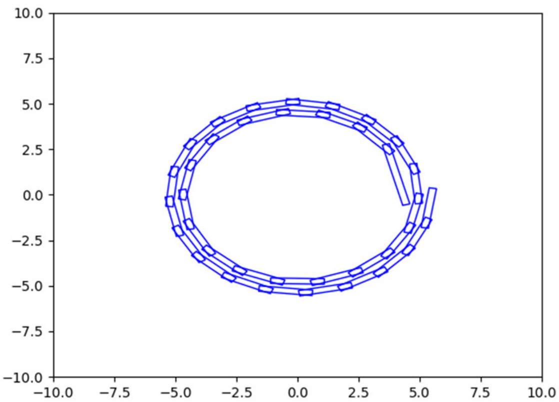

Figure 1. Simulation of robotic arm model

For this model, each plateen is considered as a linkage, and each linkage has a length of 220 cm (dragon body) or 341 cm (dragon head) and a width of 30 cm. Each handle is represented as a joint, and all joints are confined to isometric solenoids in the two-dimensional plane, so the problem of solving the motion of the handles before and after all the boards is converted into a multi-joint robot arm problem. For such a robotic arm model, it is only necessary to solve the position and speed of the faucet joint, and then through the recursive formula of speed and position between the joints, the position and speed of each joint can be solved sequentially.

Let the angle in the isometric solenoid corresponding to the faucet when it reaches a certain position be θ, the arc length as a function of θ be $L(\theta)$, and the total arc length be L_0. For the faucet, its velocity is along the tangential direction of the solenoid, with a magnitude of $v_0 = 1\mathrm{m/s}$. Therefore, the arc length traveled by the faucet is equal to the total arc length minus the tangential velocity times time, so the parametric equation for arc length versus time can be derived as:

$$L(\theta) = L_0 + v_0 t \quad (7)$$

$$\theta = \theta(t) \quad (8)$$

Since arc length, radian are functions of time, it is further possible to introduce $x(t)$ and $y(t)$, which are obtained according to the conversion relationship between polar and Cartesian coordinates:

$$\begin{cases} x(t) = \rho(t)\cos(\theta(t)) \\ y(t) = \rho(t)\sin(\theta(t)) \end{cases} \quad (9)$$

Let the position of the dragon head be $(x_0(t), y_0(t))$, the boarding length of the dragon head be $L_0 = 286$ cm, the position of the dragon body of the i-th section ($i = 1,2, \ldots 222$) be $(x_i(t), y_i(t))$, and the boarding length of the dragon body be $L_1 = 165$ cm, so that the relationship of the adjacent joints can be obtained according to the definition of Euclidean distance:

$$\begin{cases} \sqrt{(x_0(t) - x_1(t))^2 + (y_0(t) - y_1(t))^2} = L_0 \\ \sqrt{(x_i(t) - x_{i+1}(t))^2 + (y_i(t) - y_{i+1}(t))^2} = L_1 (i = 1,2, \ldots 222) \end{cases} \quad (10)$$

The position of each joint as a function of time can be obtained by combining the equations of the circle centered on the joint, the equations of the parameters of the solenoid, and the equations of the polar coordinates, i.e., equations (2), (8), and (9), being careful to limit the length of the arcs or else more than one solution may be found. Initially, the faucet is located at point A on the 16-th turn of the spiral, and this condition is used to find the required position information.

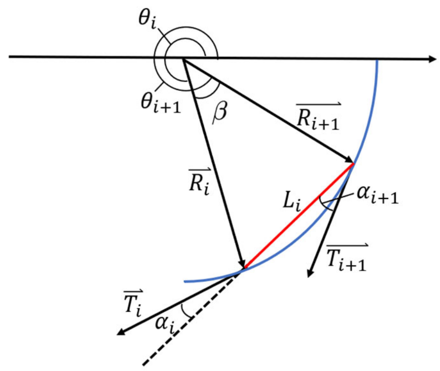

Figure 2. Joint Velocity Schematic

Figure 2 shows the two joints before and after boarding, the back handle of the i-th boarding is also the front handle of the $(i+1)$-th boarding, θ_i represents the polar angle of the back handle of the i-th boarding, θ_{i+1} represents the polar angle of the back handle of the $(i+1)$st boarding, β represents the difference of the polar angles, T_i, R_i represent the tangential and radial vectors, and the direction of the velocity on the spiral $\vec{v}_i$ is in the same direction as $\vec{T}_i$.

In this paper, the angle between each velocity and the rod is calculated, and the velocities at each joint are solved recursively. Checking the literature, it can be obtained that for any point P on the solenoid, its tangential vector consists of the parameters of the solenoid equation, and let the tangential vector at the point be $\vec{T}_i = (T_x, T_y)$T ⃗_i=(T_x, T_y), then the tangential vector is given by the following equation:

$$\begin{cases} T_x = \frac{dx}{d\theta} = b \cdot \cos\theta + (\rho_0 + b \cdot \theta)\sin\theta \\ T_y = \frac{dy}{d\theta} = b \cdot \sin\theta + (\rho_0 + b \cdot \theta)\cos\theta \end{cases} \quad (11)$$

For the i-th joint, the vector pointing to the next joint is $\vec{R}_i = (x_{i+1} - x_i, y_{i+1} - y_i)$, so for finding the angle α_i, it is simpler to find the angle of the two vectors i.e.

$$\alpha_i = \arccos\left(\frac{\vec{T}_i \cdot \vec{R}_i}{|\vec{T}_i||\vec{R}_i|}\right) \quad (12)$$

The link between the forward and backward velocities is given by the recursive modeling of the velocities of this joint:

$$v_i \cdot \alpha_i = v_{i+1} \cdot \alpha_{i+1} \tag{13}$$

Therefore, the 1m/s of the dragon's head can be used to solve the speed of the back part of the handle by iterative recursion.

The horizontal and vertical coordinate positions of the dragon dance team at each moment are given in Table 1 below, and the speed of the dragon dance team at each moment is given in Table 2 below.

Table 1. Horizontal and vertical coordinates of the position of the dragon dance team at each moment in time

	0 s	60 s	120 s	180 s	240 s	300 s
Mixer x (m)	8.800000	5.662178	4.453844	2.327034	2.351795	4.699969
Mixer y (m)	0	-5.902779	6.042917	6.355608	5.463400	1.644806
Section 1 of the Dragon's Body x (m)		7.372824	1.886354	4.763584	4.655571	3.063900
Section 1 of the Dragon's Body y (m)		3.611935	7.300518	4.859957	3.769426	3.989175
Section 51 of the Dragon's Body x (m)			5.169406	3.536561	5.813751	-6.189657
Section 51 of the Dragon's Body y (m)			6.677874	6.947749	4.069224	1.215279
Section 101 of the Dragon's Body x (m)				1.170586	5.162728	-5.776372
Section 101 of the Dragon's Body y (m)				8.594989	6.178448	4.571165
Section 151 of the Dragon's Body x (m)						7.427815
Section 151 of the Dragon's Body y (m)						3.681309
Section 201 of the Dragon's Body x (m)						
Section 201 of the Dragon's Body y (m)						
Dragon's Tail (back) x (m)						
Dragon's Tail (back) y (m)	\	\	\	\	λ	

Table 2. The speed of the dragon dance team at each moment

	0 s	60 s	120 s	180 s	240 s	300 s
Mixer (m/s)	1	1	1	1	1	1
Section 1 of the Dragon's Body (m/s)		0.999950	0.999934	0.999893	0.999827	0.999675
Section 51 of the Dragon's Body (m/s)			0.999506	0.999296	0.998910	0.998014
Section 101 of the Dragon's Body (m/s)				0.998961	0.998421	0.997258
Section 151 of the Dragon's Body (m/s)						0.996799
Section 20 of the Dragon's Body (m/s)						
Dragon's Tail (back) (m/s)						

3. A Discrete Simulation-based Collision Conflict Detection Model for Bench Dragons

During the disking-in process along the isometric solenoid, the boarden dragon may collide with different sections of the boarden due to the small radius of the dragon's head path. In order to avoid the collision, it can be determined first that no collision occurs in the first 300s of the benched dragon, so this paper starts to find the collision point after 300 s. The collision detection model is developed to determine the boundary collision condition. This paper establishes a collision detection model to determine the boundary collision conditions, and adopts the method of gradually narrowing the time and node search range in the process of solving the collision moments, which reduces the time complexity of the program and improves the solving accuracy.

First of all, it needs to be clear that the collision in the question is the collision between the dragon head and the dragon body, and it occurs in the inner circle. Because the radius is smaller in the inner circle, the arc of the dragon head under the same length (chord length) is larger, the angle across is larger, and it is more likely to collide with the dragon body segments, and the schematic diagram of the collision nodes is given in Figure 3 below.

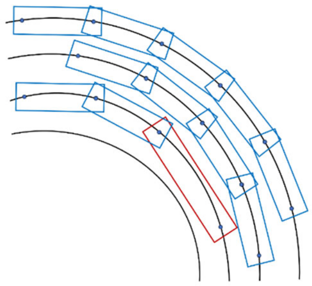

Figure 3. Schematic diagram of collision nodes

As can be seen from the figure, on the inside of the coil, the curvature of the arc is large, the red dragon swings through a large angle, and is more likely to collide with the blue dragon body. On the other hand, on the outside of the coil, the curvature of the arc is small, and the angle of rotation of the dragon's body is small, so there will be no collision. Secondly, the collision cannot be an edge-to-edge collision, but a top corner-to-edge collision. Because the two handles are located on the same thread, the distance between the two threads is 55 cm, and the distance between the center axes of the board is $2 * 15 = 30$ cm, which is smaller than the distance between the threads, so it is impossible for the board to have an edge-to-edge collision. We start looking for the collision point after 300 s. The first step is to determine the location of the faucet. First, we determine the location of the faucet. According to the model established in the previous chapter, the location of the faucet corresponds to the time, so the termination moment of the disk-in can be determined by the location of the faucet collision.

Discretize the trajectory of the dragon head after 300s, and gradually increase the degree of discretization of the inner circle. The angle of the inner circle is uniformly discretized into 1000 copies, and the front handles of the faucet are placed at the discretized points in turn, so that the geometric positions of the faucet and the dragon body are determined, and then the collision detection is carried out.

Let the positions of the two handles be $(x_1, y_1), (x_2, y_2)$ respectively, and the two auxiliary direction vectors are $\vec{e}_1, \vec{e}_2$ as shown in the figure, and we utilize the property of vector addition and subtraction to solve for the position of the top angle, which is derived as follows:

The two handles form a vector $\vec{a} = (x_1 - x_2, y_1 - y_2)$ and the unit vector in that direction is given in the following equation:

$$\vec{e}_1 = \frac{(x_1 - x_2, y_1 - y_2)}{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}} \tag{14}$$

Similarly, the unit vector perpendicular to it is given by the following equation:

$$\vec{e}_2 = \frac{(y_1 - y_2, x_2 - x_1)}{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}} \tag{15}$$

The distance of the handle from the front of the dragon is 27.5 cm, and the direction is $\vec{e}_1$; half of the width of the boarder is 15 cm, and the direction is $\vec{e}_2$, so we can get the vector $\vec{b}$ of the top angle as follows:

$$\vec{b} = \vec{a} + 27.5\vec{e}_1 + 15\vec{e}_2 \tag{16}$$

After obtaining the coordinates of the top corners, it is sufficient to use the four top corners of the dragon head to detect the collision with the dragon body of the innermost and penultimate circles without using the outer dragon body for detection.

Since the innermost and penultimate circles have more loci after discretization and it takes longer computation time to detect the collision one by one, we do a round of screening first, taking the distance s_{ij} from the i-th vertex corner of the dragon's headboard to the side of the j-th boardboard as the basket selection criterion, and set a smaller threshold m, and perform the second round of collision modeling only when $s_{ij} < m$.

s_{ij} is given by the equation of the distance from a point to a straight line, which passes through the top corner $(x_1, y_1), (x_2, y_2)$ points, then the equation of the straight line can be expressed in the coordinates of the two points. Let the equation of the line for $y = kx + b$, the coordinates of the top corner of (x_0, y_0), then

$$s_{ij} = \frac{|kx_0 + b - y_0|}{\sqrt{k^2 + 1}} \tag{17}$$

In this paper, a geometric method is used to construct a collision conflict detection model. For any point in the plane of the dragon body boarding, respectively, connecting its four vertices with the dragon body boarding, constituting four triangles, if the sum of the area of these four triangles is equal to the boarding rectangle area, the point is located in the boarding rectangle, that is, the collision occurs; if the sum of the area of the triangle is greater than the boarding rectangle area, it is located outside the rectangle, that is, the collision has not occurred.

The area of a triangle is given by the coordinates of three points, which are (in any order) $(x_1, y_1), (x_2, y_2), (x_3, y_3)$:

$$S = \frac{1}{2}|(x_3 - x_1)(y_2 - y_1) + (x_2 - x_1)(y_1 - y_3)| \tag{18}$$

Using this model, this collision problem is converted into a coordinate solving problem. From the above analysis, we can see that we only need to determine the collision of the innermost circle and the penultimate circle of the boarden. The termination moment before the collision that we solved for is 416.4578 s. Table 3 gives the specific positions and velocities of the benches at this time.

Table 3. Position and velocity of the bench at the moment of termination of the collision

	horizontal coordinate x (m)	vertical coordinate y (m)	velocity (m/s)
Mixer	1.889317	-1.03866	1.000000
Section 1 of the Dragon's Body	1.420626-	1.781327	0.988705
Section 51 of the Dragon's Body	4.494389	3.263509	0.924827
Section 101 of the Dragon's Body	-5.552990	-5.02482	0.920103
Section 151 of the Dragon's Body	8.233446	0.616415	0.918579
Section 201 of the Dragon's Body	8.056790	2.695702	0.916375
Dragon's Tail	8.7005811233970	1233970	0.914978

4. Conclusions

In this paper, we propose a mathematical model based on isometric solenoids and discrete simulation by studying the position, velocity change and collision detection problems during the traveling process of the bench dragon. First, we calculate the position of each section of the bench during the traveling process of the bench dragon by iterative recursion using the equations of isometric solenoids and the robotic arm model. This method not only takes into account the physical structure of the bench dragon, but also makes full use of the mathematical properties of isometric solenoids, so that the bench dragon can maintain uniform speed and smooth dynamics during the traveling process. Then, we further optimize the position solution process of the bench dragon by combining the velocity analysis of rigid body rod. By introducing the rigid-body rod model, we are able to more accurately analyze the velocity and acceleration changes of the bench dragon during the traveling process, which provides more specific operational guidance for the dragon dancers. In addition, we also consider the collision problem that the bench dragon may encounter during the marching process, and through discrete simulation technology, we discretize the trajectory of the disk entry and perform collision detection, so as to obtain key information such as the time, position and speed of the termination moment. This not only improves the safety of the bench dragon performance, but also provides an important reference for the optimization of the performance.

In summary, the model proposed in this paper can effectively solve the optimization problem in the process of bench dragon marching and provide scientific theoretical support for the smooth progress of bench dragon performance. Through the application of mathematical modeling and simulation technology, we can not only improve the artistic effect of the bench dragon performance, but also ensure the safety and ornamental performance. In the future, we will continue to study the dynamics of the bench dragon and explore more ways to optimize the bench dragon performance, so as to make greater contributions to the inheritance and development of Chinese traditional folk culture.

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