

Study of Non-Periodical Mechanical Metamaterials: Design and Application

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Abstract: We studied a typical mechanical metamaterial with different geometry patterns to demonstrate its effect in wave transmission. An inclusion geometry described by the trigonometric function is employed to generate local resonance under wave propagation. It has been found that the inclusion geometry plays an important role in the bandgap formation and attenuation of sound wave. More importantly, for a hybrid unitcell, the existing of flat and negative-slope bands indicates the translational mode of the dense core, which is critical to understand the wave reflection through non-periodical metamaterials. Furthermore, we propose a concept of velocity tuning of its individual components, which gives rise to local high strain energy, to explain why the absorptivity of sound wave is high. With help of embedded electronic units and dielectric materials, we can realize the active control of the deformation and reconfiguration of the unitcell, thus, to alter its band structure properties. The fabrication of such metamaterials can be realized by plasma etching, laser printing and nanofabrication from centimeter scale to nanometer scale. Therefore, the applications of mechanical metamaterials can be extended from sound filtering in centimeter scale to thermal management in nanometer scale.

Keywords: Mechanical metamaterial, Wave transmission, Bandgap, Velocity tuning, Nanofabrication.

1. Introduction

Mechanical metamaterials (MMs) are artificially structured periodic composites with extraordinary dynamic property [1-3]. Due to the capability to control wave propagation like sound and heat, MMs have received great interest in many applications such as noise insulators [4], deep-subwavelength focusing[5] and energy harvesting [6]. The existence of band gap allows MMs to redirect or stop the elastic waves in very low frequency. In general, the geometry, material property and space patterns of the component are three major factors in determining the performance of MMs.

The empirical design of MMs as solid crystals of ternary or quaternary have been extensively explored [7]. The optimization of a single unit cell has been performed to maximize the wave attenuation at a desired frequency range using a level set-based topology method [7]. To maximize the width of band gaps, some studies have investigated the design for phononic materials with different material phases [8]. The classical MMs are periodic and composed of uniform unit cells. Some studies[9] show that the non-periodic MMs are restricted to the influence of non-periodic arrangement of inclusions on the transmission curve. For instance, MMs made of different types of unit cell have a wider range of frequencies gap for attenuated transmission [7]. Besides, the MMs consisted of unitcells with different sizes can generate more band gaps and even negative-slope bands [10]. However, the dispersion property for MMs with hybrid enlarged unit cell is of great interests for understanding the wave reflection and needs more study.

Another important feature of this mechanical metamaterials is its fabrication. There are many popular methods can be employed to fabricate the metamaterials, like plasma etching, laser printing and nanofabrication[11-13]. During the fabrication process, it is possible to enable active control of metamaterials by using dielectric materials and

embedding electric components. So, this paper will first present the methodology and the results of negative and flat bands and velocity tuning in section 2. Concluding remarks are given in section 3.

2. Results

2.1. Fundamental method

Here, we first consider a classical two-dimensional solid ternary unit cell to illustrate the phenomena of bandgap formation of non-periodical metamaterials and negative-slope band. An array of inclusion geometries with four-fold symmetry are considered [14]. Besides, it is necessary to ensure the structure reliability that means the minimum thicknesses of matrix and coating should be larger than zero. By modeling the MMs as a linear elastic periodic medium [15], the governing equation of a deformed continuous medium is

$$\nabla \cdot \boldsymbol{\sigma} + \rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = \mathbf{f} \quad (1)$$

where $\boldsymbol{\sigma}$ is stress tensor, \mathbf{u} is the displacement vector and ρ is the material density. For a periodical solid, in response to an incident wave, the displacement field is governed by Bloch-Floquet wave. For non-periodical solid, there is no theoretical method for boundary conditions[16]. In this work, we consider the enlarged unitcell which is composed hybrid unitcell, to investigate the non-periodical effect. The used materials include lead core, rubber coating and epoxy substrate. We calculate the band structures for non-periodical MMs using the modal analysis. The goal of this analysis is to understand how the inclusion geometry band structure changes as the materials become non-periodical.

According to the previous studies [17], the bandgap overlapping and negative-slope bands are induced by local resonance duo to the interactions between different-phases

materials. When the inclusion geometry becomes more complex, the resonant frequency of rubber coating increases faster than that of the core, resulting in the enlarged width of the bandgap.

2.2. Non-periodical effect

For periodic MMs, one feature for their band structure is the existence of whole flat band, indicating the torsional mode of the core [10]. The torsional mode is localized and has no interaction with an external wave field. Thus these flat bands cannot affect wave propagation within the MMs.

For MMs with hybrid enlarged unit cell, another type of whole flat band exists, representing the translational mode of core which can interact with wave propagation. In order to illustrate this idea, a hybrid enlarged unit cell composed of two types of unit cell as shown in the insert of Figure 1 (b) is calculated for its band structure. In this case, we find the flat bands characterizing transitional mode of the core. For the band structure calculation, since the hybrid unit cell only has two symmetry axes, we need to consider more wave vectors in the first Brillouin zone.

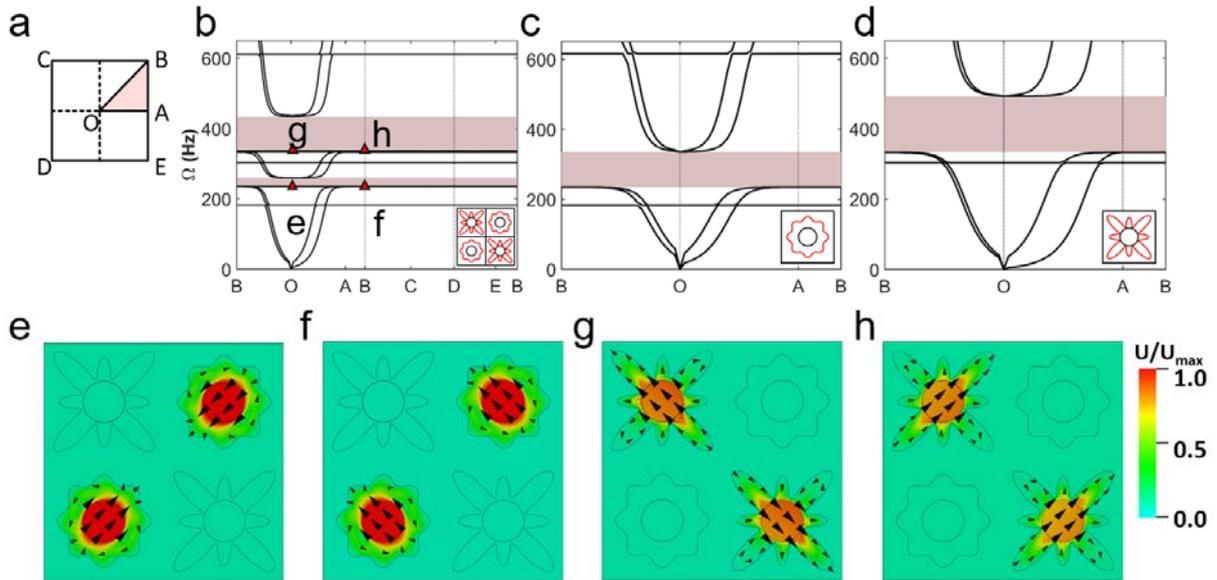


Figure 1. Enlarged hybrid enlarged unit cell, crosswise arranged. (a) The first Brillouin zone. (b) The band structure for hybrid unit cell; the insert is the sketch of the hybrid enlarged unit cell. (c, d) The band structure of the component unitcells. (e, f, g, h) Modal shapes corresponding to red triangle marks in the band structure.

According to the results given in Figure 1 (b), there are two whole flat bands that represent the translational mode of the core, labeled by red triangles. These two flat bands in Figure 1 (b) are relevant to the bands in Figure 1 (c and d). The lower flat band represent the core transitional mode of first component unit cell, which corresponds to lower bound of the first order bandgap. This flat band can interact with wave propagation. When wave propagates through the MMs, the core transitional mode direction can be adjusted to satisfy the boundary condition induced by wave vector and the mode frequency remain unchanged, e.g. shown in Figure 1 (e, f), dipole mode evolves to monopole mode when \mathbf{k} vector shifts from O to B, which is the mechanism for the existence of this kind of flat bands. Similarly, the second flat band (g and f) corresponds to the lower bound of the first order band gap in Figure 1 (d).

Because the core resonance in each component unit cells can change the motion direction to meet different boundary conditions (defined by wave vector), thus vibration frequency doesn't change for different wave vectors and the slope of the bands is zero (labeled by red triangles in Figure 1 (b)). For the same frequency band in Figure 1 (c) and (d), when wave vector \mathbf{k} shifts from A (or B) to O (points on first Brillouin zone in Figure 1 (a)), the frequency will decrease, thus the slope of the bands is positive. In addition, if the hybrid enlarged unit cell is made up of more hybrid unitcells (like 3

times 3), the dynamic property of this hybrid enlarged unit cell can reveal the mechanism of wave propagation through non-periodic MMs.

2.3. Negative-slope band

Another phenomenon is the observation of negative slope bands, under which the wave will be deflected or reflected. From the modal shapes in Figure 2, the emergence of negative slope bands can be interpreted by the modal shape analysis. Taking Figure 2 (c) for example, the model shape indicates the large matrix deformation, and the core remains stationary. This extraordinary phenomenon shows that the resonance of matrix material (instead of the inclusion) results to the emergence of negative slope bands. Furthermore, since the upper bound of the first order band gap corresponds to \mathbf{K} vector at point A rather than at point O in Figure 2 (c), the band gap widening effect comes from the coupled Bragg band gap with local resonance of the matrix material [18]. Consequently, the band structure is dramatically changed due to this phenomenon [18], which reflects wave as the negative phase velocity. The emergence of negative slope bands enlarged band gap suggests that matrix property has big effects on MMs. Thus, structured matrix with low density may be a good option for the metamaterials design.

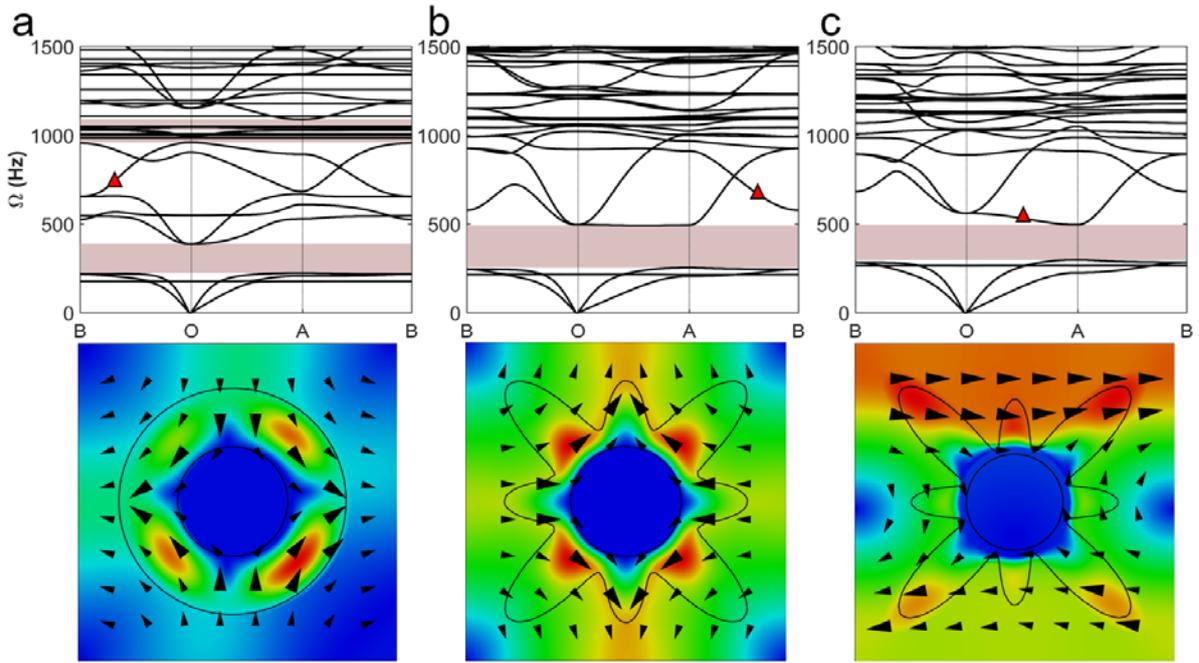


Figure 2. Band structure and mode shape for different types of unitcells. Top three charts are the band structures of the unitcells. The negative slope band is labelled by red triangle; the bottom is the corresponding vibration modes.

Since the frequencies of the incident wave propagating through the matrix is different from the inclusion resonance, the motion phase and velocity of the core and matrix are generally different. As shown in **Figure 3** When the unit cell vibrates at its equilibrium position, the shear deformation will synchronize the velocity of the core and matrix. On the other hand, when the unit cell vibrates to its valley or peak position,

the constraint for motion sync will be released. Because of this effect, S wave amplitude is largely attenuated when propagating through MMs. Clearly, more complex the inclusion geometry is, the larger effect from velocity tuning will be. Similarly, when P wave propagates through MMs, there will be also velocity tuning induce by tension-compression deformation.

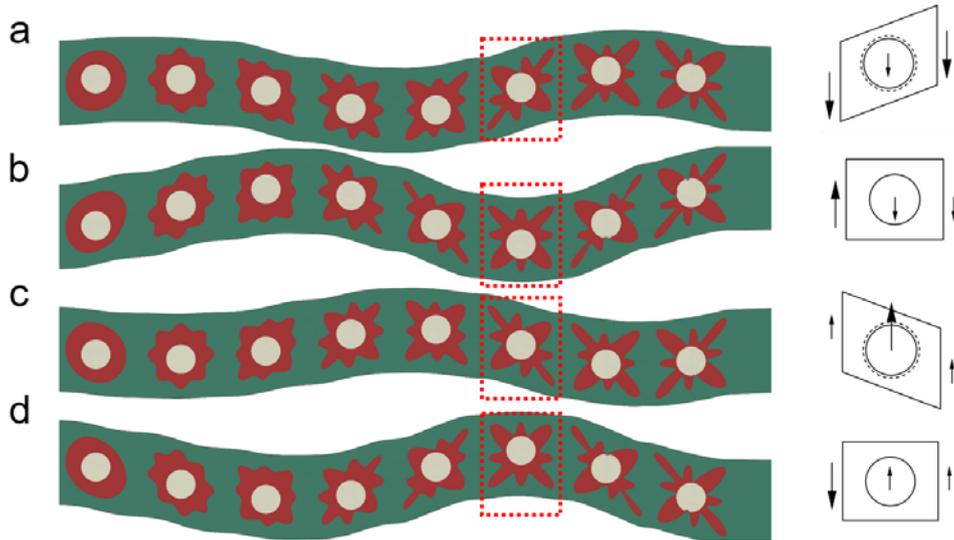


Figure 3. Illustration of velocity tuning, where the arrow indicates the velocity vector; black dotted circle represents the constraint on core motion in the matrix. (a, c) The unit cell in is the shear deformation that the core motion in the matrix is constrained. (b, d) The unit cell vibrates vertically and the constraint for the core is released.

2.4. Fabrication and Active Control

Here, we will brief introduce the current fabrication technique for the periodic metamaterials. The nano-scale fabrication enables the wide application of mechanical metamaterials with sub-wavelength structures. The flexible structure gives more freedom to reconfigure the unitcell and thus to achieve tunable features, which is closely linked to the properties of the component substrate. For example, using the

impurities, like nanoparticles, can further enhance the capability of controlling heat-carrying phonon modes[19], which can block the high-frequency thermal phonons and provide more interface to block low-frequency phonons, as shown in **Figure 4**. For the nano-scale fabrication, plasma etching, and laser printing are popular methods. Plasma etching involves the plasma-solid interactions that the ionization of gas medium will generate the high-speed particles which bombard the substrate surface to fabricate

none-scale structures. The unwanted effects are the potential drop[20-22] across plasma-solid interface the charged particles may accumulate on the surface that gives rise to the negative-resistance and causing a surface pattern effects[23-26]. Thus, it should only consider the plasma with very low discharge current during the fabrication process. Besides, the application of dielectric materials[11, 12, 27, 28] as a key factor in the active control metamaterials and can also be good for energy harvesting from mechanical vibration. For example, the dielectric elastomer has been used to make giant

deformation [29], which provides an effective way to manipulate the unitcell re-configuration. Besides, an external applied voltage can be used to control the deformation of the unitcell with prior charge injection into the substrate front surface, by ion-boosting[30], and using the conductive fabric layer on the opposite surface. These are the ongoing research work[3]. Since the local resonance vibration amplitude are relatively small, there will no materials failure and fatigue issues[31].

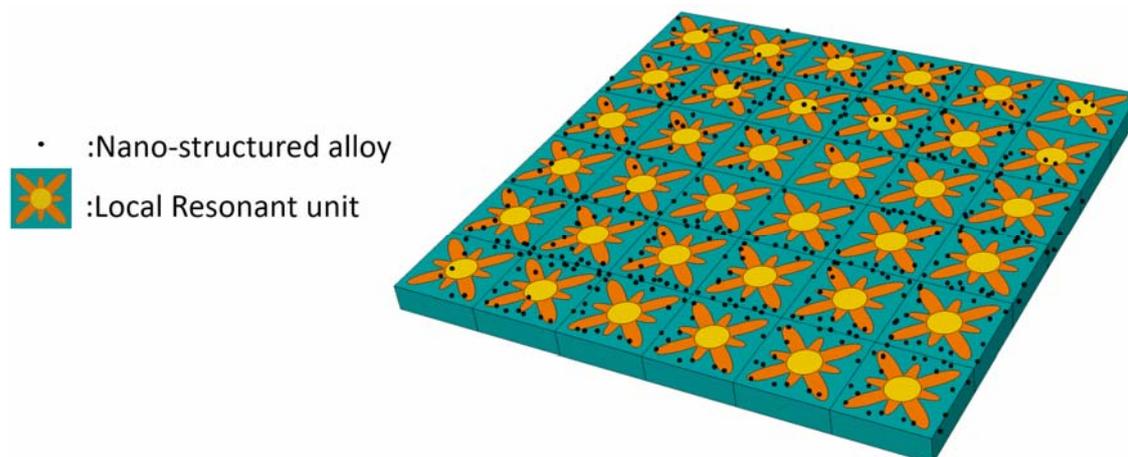


Figure 4. The illustration of metamaterials of a periodical array will three-phase materials and patterned on the nanoparticles filled substrate. The non-structured alloy is able to block high-frequency thermal phonons and unitcell are able to block local frequency phonons.

3. Conclusion

In this study, we investigated the non-periodical effect on the mechanical metamaterials and the formation of negative-slope bands. Results suggest that the inclusion geometry and unitcell pattern have a significant influence on the band structure properties, like the bandgap formation and negative slope band. Therefore, the inclusion geometry can be optimized to achieve wave attenuation at targeted frequency range. Besides, MMs with hybrid enlarged unit cell can generate whole flat bands, representing the translational mode of the core. Our study shows that the hybrid configuration, like the MMs composed of different types of unitcells as an example of non-periodic MMs, can amplify the attenuation effect. With aforementioned inclusion geometry, MMs exhibits velocity tuning of its components under wave propagation. With more complex inclusion geometry, the velocity tuning is more obvious, which verifies the proposed inclusion geometry as an effective design guidance for wave manipulation. Further investigations are needed to gain insight into the nature of wave propagation through non-periodic MMs. The fabrication of such metamaterials can be realized by plasma etching, laser printing and nanofabrication at different length scales, which are readily applied in engineering practices.

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