Friedmann Equation in Era of Multi-Messenger Astronomy: Neutrinos, Gravitational Waves and Cosmic Rays

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Abstract. The Friedman equation is the cornerstone of modern cosmology, governing the evolution of the universe and providing us with insights into its expansion, energy composition, and large-scale structure. Over the past few decades, the advancement of multi-messenger astronomy integrating data from neutrinos, gravitational waves, and cosmic rays which have greatly enriched the understanding of the universe. Neutrinos from the early universe contribute to the radiation density, and continuous observations tighten the constraints on their mass and interactions even more. Gravitational waves generated by cosmic events such as black hole mergers and cosmic inflation contribute to the energy budget of the universe, thereby refining the models of cosmic expansion and structure formation. Cosmic rays, especially the ultra-high-energy particles detected by observatories like the Pierre Auger Observatory, contribute to the energy density and affect galaxy formation and intergalactic heating.

Keywords: Friedmann equation; multi-messenger astronomy; neutrinos; gravitational waves; cosmic rays.

1. Introduction

Multi-messenger astronomy focuses on studying astronomical sources through different kinds of "messengers," including photons, neutrinos, cosmic rays, and gravitational waves [1].

The understanding of the Universe was primarily built on the detection of electromagnetic signals from various celestial objects over centuries. This has changed in 2013 with the discovery of high-energy neutrinos arriving from sources beyond the Solar System which is a new astronomical messenger. In 2015, there is another new messenger been detected - gravitational waves, this is also a great breakthrough in this region [1].

The primary goal of this paper is to explore how multi-messenger observations—neutrinos, gravitational waves, and cosmic rays—can be used to refine and constrain the Friedmann equations. By integrating these complementary data sources, the study aims to improve the accuracy of key cosmological parameters such as the Hubble constant and the density of dark energy. This approach shows the synergy between theoretical modeling and observational astronomy in multi-messenger cosmology, helping reduce the difference between different methodologies and deepen the understanding of cosmic evolution, fundamental physics, and the influence of dark matter and dark energy [2].

2. The Friedmann Equation

2.1. Derivation of the Friedmann Equation

According Joan's work, he starts from Einstein equations which are

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{1}$$

The first two terms on the left-hand side of (1) is the Einstein tensor ($G_{\mu\nu}$) in which he has plugged an extra term that includes the cosmological constant (ignore the effect of c, the speed of light). G is the universal gravitational constant and $T_{\mu\nu}$ is the energy-momentum tensor, its explicit expression is

$$T_{\mu\nu} = (\rho + p)u_{\mu}u_{\nu} - pg_{\mu\nu} \tag{2}$$

$g_{\mu\nu}$ is the metric of the manifold where the equations apply and u_{α} is the macroscopic speed of the medium.

The foundation of the derivation is the Cosmological Principle: the universe is homogeneous and isotropic on large scales. This assumption simplifies the universe's geometry a lot. The spacetime metric that satisfies this principle is the Friedmann-Lemaître-Robertson-Walker (FLRW) metric, which has the following form

$$ds^2 = dt^2 - a^2(t)\left(\frac{1}{1-\frac{r^2}{K^2}}dr2 + r^2\,d\theta^2 + r^2\sin^2\theta\,d\phi^2\right) \tag{3}$$

Firstly he to calculate the Christoffel symbols of Robertson-Walker metric.

$$\Gamma^l_{ji} = \frac{1}{2}\,g^{lm}\left(\partial_j g_{mi} + \partial_i g_{mj} - \partial_m g_{ij}\right) \tag{4}$$

Once the Christoffel symbols have been calculated, then Riemann tensor can be calculated

$$R^l_{kji} = \partial_i\Gamma^l_{kj} - \partial_j\Gamma^l_{ki} + \Gamma^m_{kj}\Gamma^l_{mi} - \Gamma^m_{ki}\Gamma^l_{mj} \tag{5}$$

The only components of the Ricci tensor that are different from 0 are R_{tt}, R_{rr} , $R_{\theta\theta}$ and $R_{\varphi\varphi,}$ and for $R_{ii} = R^m_{imi}$ and put them back into (1):

$$-\frac{3\ddot{a}}{a} + \frac{3\ddot{a}}{a} + 3\left(\frac{\dot{a}}{a}\right)^2 + 3\frac{1}{K^2a^2} - \Lambda = 8\pi G\rho(t) \tag{6}$$

Rearrange

$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi G}{3}\,\rho(t) + \frac{\Lambda}{3} - \frac{1}{K^2a^2(t)} \tag{7}$$

And for the spacial part, each spacial component would reach the same equation and removing the metric from both sides to obtain

$$\frac{\ddot{a}(t)}{a(t)} + \frac{1}{2}\left(\frac{\dot{a}(t)}{a(t)}\right)^2 = -4\pi Gp + \frac{\Lambda}{2} - \frac{1}{2}\frac{1}{K^2a^2(t)} \tag{8}$$

Then do 2 · (8) − (7) to get

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4\pi G}{3}(\rho(t) + 3p) + \frac{\Lambda}{3} \tag{9}$$

The Friedmann equation [3].

2.2. Cosmic Messengers in the Friedmann Framework

Cosmic messengers, including neutrinos, gravitational waves, and cosmic rays, play an increasingly important role in cosmology by providing direct information about the universe's energy content. They carry signals from different epochs in the universe's history and offer new ways to refine the Friedmann equation. For example, neutrinos are important for understanding the thermal history of the early universe, gravitational waves reveal extreme astrophysical events like black hole mergers, and cosmic rays trace high-energy processes from supernovae and other galactic sources. By incorporating data from these messengers into the Friedmann framework, researchers can evaluate density parameters such as Ω_{ν}, Ω_{GW}, and Ω_{CR} better, improving the understanding of how the universe's expansion and energy composition evolve over time [4].

3. Neutrinos and the Friedmann Equation

3.1. Role of Neutrinos in Cosmology

The relic cosmic neutrino background, though never directly observed, is indirectly predicted by the standard Hot Big Bang model to have a number density comparable to that of the CMB—and this prediction is now regarded as firmly established. The strongest support comes from Big Bang Nucleosynthesis (BBN), which successfully reproduces the observed light element abundances at MeV temperatures, a process that depends sensitively on the neutrino population. Yet BBN is not the only line of evidence. The dynamical impact of the neutrino background is also tightly constrained by precise measurements of CMB anisotropies and the distribution of Large-Scale Structure (LSS). Taken together, these independent datasets converge on a single conclusion: neutrinos make a real and measurable contribution to the total radiation density of the universe [5].

3.2. Observational Constraints on Neutrino Properties

The Cosmic Microwave Background (CMB) and the Cosmic Neutrino Background (CνB) are two relics of the Big Bang that still survive today. Neutrinos, in particular, are the second most abundant particles in the universe, and for that reason alone their role in cosmic evolution cannot be ignored. From the earliest times to the present, they have left clear imprints on several key processes. The universe's expansion rate, the primordial chemical and isotopic composition, the anisotropies observed in the CMB, and even the formation and growth of large-scale structures have all been, in one way or another, shaped by neutrinos [6].

4. Gravitational Waves and the Friedmann Equation

4.1. Gravitational Waves as a Cosmic Messenger

Gravitational waves are a crucial messenger in modern cosmology, showing a unique way to study the most energetic events in universe. These waves are generated by events such as mergers of compact objects like black hole, inflationary processes, and phase transitions in the early universe. The energy density of gravitational waves, denoted Ω_GW, contributes to the Friedmann equation, which governs the expansion of the universe. By detecting gravitational waves, it can learn more about the energy budget of the universe and refine cosmological models. These observations data would help in understanding both the large-scale structure and the detailed evolution of cosmic parameters [4].

4.2. Primordial Gravitational Waves

Primordial gravitational waves are expected to leave indirect signatures on the CMB, most notably through polarization anisotropies. While temperature anisotropies have already been observed, current experiments can only set upper limits on polarization anisotropies caused by a gravitational wave background. Inflationary models strongly suggest that such a background exists, as observations of the universe's total energy density are consistent with these predictions. A primary goal is the detection of B-mode polarization, whose spectrum is well predicted theoretically. Polarization occurs only during specific epochs when electrons scatter photons without being too tightly coupled: during recombination ($t \approx 380{,}000$ years) and during reionization at much later times. These epochs leave signals at characteristic angular scales of roughly 2° and 50°. Gravitational lensing of E-modes also generates B-modes on smaller scales, but this effect limits the ability to detect higher B-mode harmonics [7].

4.3. Astrophysical Gravitational Waves

Using data from the third and fourth science runs of the LIGO and GEO600 detectors, researchers placed upper limits on gravitational wave emissions from 78 radio pulsars. The most stringent strain limit, 2.6×10^{-25}, was obtained for PSR J1603–7202. For PSR J2124–3358, the equatorial ellipticity limit reached as low as 10^{-6}, marking one of the tightest constraints to date. The Crab pulsar remains a special case: its strain upper limit is still about 2.2 times the fiducial spin-down limit, a reminder that even well-studied sources can resist tighter bounds. Beyond individual pulsars, a two-year LIGO dataset has also constrained the energy density of the stochastic gravitational-wave background, normalized to the universe's critical density, to 6.9×10^{-6} around 100 Hz at 95% confidence. This represents a meaningful step forward, improving upon the indirect limits previously inferred from Big Bang nucleosynthesis at the same frequency [8].

5. Cosmic Rays and the Friedmann Equation

5.1. Cosmic Rays in the Context of Cosmology

According to the work of A. W. Strong et al., the Galaxy can be treated as an approximate cosmic-ray electron calorimeter, at least when only γ-ray processes are considered. Roughly one-third of the total electron energy loss arises from synchrotron radiation, and about 10–20% of this synchrotron emission originates from secondary cosmic-ray electrons and positrons. The model also indicates that the observed link between far-infrared and radio luminosity is consistent with what is seen in other galaxies. Taken together, these findings sharpen the understanding of the connection between diffuse emissions from radio waves to γ-rays in "normal" galaxies—those not dominated by active galactic nuclei—and they also help refine estimates of the extragalactic diffuse background [9].

5.2. Observational Insights

The energy spectrum of ultra-high-energy cosmic rays above 2.5×10^{18} eV has been measured with the surface detector array of the Pierre Auger Observatory [1]. These measurements revealed a clear suppression of the flux above 4×10^{19} eV, a result later updated and confirmed by A. Aab et al. To extend the analysis to lower energies, the same team examined air showers detected simultaneously by the fluorescence detector and at least one surface detector station. Although the fluorescence detector operates with an effective duty cycle of only about 13%, the advantage lies in its lower energy threshold and high resolution. These hybrid events make it possible to measure the cosmic-ray flux in the ankle region with much greater precision [10].

5.3. Relevance to the Friedmann Equation

Cosmic rays' contribution on the radiation or matter density in the universe has implications for the Friedmann equation, which describes cosmic expansion. The energy density of cosmic rays, though relatively small, must be accounted for in precise models of the universe's evolution. Their contribution to the radiation density Ω_{CR} affects the early universe's expansion rate and could influence late-time cosmic evolution. As cosmic rays interact with the intergalactic medium, they have impact on both the thermal and ionization states of the universe, thereby affecting structure formation and the evolution of large-scale cosmic phenomena. Quantifying their contribution provides important constraints on the Friedmann equation and enhances the accuracy of cosmological models describing the universe's energy budget [11].

6. Synergies Between Messengers

6.1. Combined Constraints on Cosmological Parameters

Multi-messenger astronomy, which integrates observations from gravitational waves, neutrinos, and cosmic rays, has become a powerful approach for probing cosmological parameters. By cross-validating independent observational channels, it enables a more robust and comprehensive understanding of the universe. For example, the detection of gravitational waves from binary neutron star mergers has facilitated direct measurements of parameters such as the Hubble constant, though some discrepancies remain when compared with traditional methods. The joint analysis of multi-messenger datasets also refines models of cosmic expansion and constrains the equation of state of the universe. These combined constraints are essential for testing theories of dark energy and for narrowing the range of viable cosmological models, particularly those related to the expansion history of the universe [12].

6.2. Probing Dark Energy and Modified Gravity

Multi-messenger observations offer significant potential for advancing the understanding of dark energy and theories of modified gravity, including alternatives to the standard ΛCDM framework. The combined use of gravitational wave and electromagnetic signals enables tests of possible deviations from the standard Friedmann equations, thereby probing whether extensions to general relativity, such as f(R) gravity, could account for observed cosmological phenomena. Furthermore, analyses of large-scale structure formation and cosmological perturbations, combined with multi-messenger data, provide stringent constraints on dynamical dark energy models such as quintessence and related candidates. As a result, multi-messenger cosmology serves as a powerful framework to test theories that challenge the cosmological constant as the sole explanation for cosmic acceleration [13].

6.3. Challenges in Multi-Messenger Cosmology

Combining datasets from different cosmic messengers is challenging. A central case is the “Hubble tension,” where estimates of the Hubble constant (H_0) from early-universe probes such as the Cosmic Microwave Background differ from late-universe measurements from the local distance ladder. This shows the necessity of combine multi-messenger data. Moreover, researchers face statistical and modeling problem especially with noisy or incomplete data. Addressing all these problems requires improved analytical tools and model-fitting methods to keep cosmological results consistent [14].

7. Future Directions and Observational Prospects

7.1.Next-Generation Observatories

The future of cosmology is bright as the next-generation observatories shows up. They will improve the precision of measurements which help researcher understand the universe better. New neutrino detectors such as IceCube-Gen2 and PINGU will give better data for cosmological models to make it more accurate. Also, for upcoming gravitational wave facilities, including LISA and Cosmic Explorer, will produce precise data for the universe’s most energetic events like black holes and neutron stars merger. In addition, advanced cosmic-ray observatories like the Cherenkov Telescope Array (CTA) and the Probe of Extreme Multi-Messenger Astrophysics (POEMMA) will broaden knowledge of ultra-high-energy cosmic rays, creating new ways to test models of cosmic structure and evolution [15].

7.2.Refining the Friedmann Equation

One of the main objects of future cosmology will be to refine the Friedmann equation, which describes how the universe expands. Getting more accurate values for parameters such as Ω_ν, Ω_{GW}, and Ω_{CR}. This will greatly improve current understanding of the universe’s energy content and its history. By combining multi-messenger observations with better theoretical models and observational methods, the author can test the Friedmann equation with higher precision. This may provide new insights into both the early and late universe and help resolve key problems like the Hubble constant tension and the role of dark energy in cosmic acceleration [11].

7.3.Unifying Multi-Messenger Cosmology

Future research will focus on using multi-messenger datasets to reduce the difference between theoretical models and observational data. By combining information from neutrinos, gravitational waves, and cosmic rays, scientists can come up with a better theory of the universe's history and evolution. They can use all these observational results to find new paths to test and refine cosmological theories, especially by studying how cosmic messengers interact with other physical components. By understanding the role of each messenger in the energy budget and cosmic dynamics, a deeper insight about dark matter, dark energy, and other key elements that shape the universe can be provided [2].

8. Conclusion

The future of cosmology will rely more on multi-messenger observations to improve fundamental models such as the Friedmann equation. By combining data from neutrinos, gravitational waves, and cosmic rays, scientist can build a better picture of the universe's energy content and its evolution. The results from these messengers will be the key to addressing problems about dark energy, the expansion of the universe, and the basic forces. By the evolution of observational technology, measurements which is the foundation of physics will be more precise. This will open a new way for the progress in both theoretical cosmology and fundamental physics.

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